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Phase transition in kinetic exchange opinion models with independence

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ABSTRACT

In this work we study the critical behavior of a three-state (+1, −1, 0) opinion model with independence. Each agent has a probability q to act as independent, i.e., he/she can choose his/her opinion independently of the opinions of the other agents. On the other hand, with the complementary probability $1-q$ the agent interacts with a randomly chosen individual through a kinetic exchange. Our analytical and numerical results show that the independence mechanism acts as a noise that induces an order–disorder transition at critical points q_c that depend on the individuals' flexibility. For a special value of this flexibility the system undergoes a transition to an absorbing state with all opinions 0.

1. Introduction

In the recent years, the statistical physics techniques have been successfully applied in the description of socioeconomic phenomena. Among the studied problems we can cite opinion dynamics, language evolution, biological aging, dynamics of stock markets, earthquakes and many others [1–3]. These interdisciplinary topics are usually treated by means of computer simulations of agent-based models, which allow us to understand the emergence of collective phenomena in those systems.

Recently, the impact of nonconformity in opinion dynamics has attracted attention of physicists [4–8]. Anticonformists are similar to conformists, since both take cognizance of the group norm. Thus, conformists agree with the norm, anticonformers disagree. On the other hand, we have the independent behavior, where the individual tends to resist the groups' influence. As discussed in [7,8], independence is a kind of nonconformity, and it acts on an opinion model as a kind of stochastic driving that can lead the model to undergo a phase transition. In fact, independence plays the role of a random noise similar to social temperature [5,7,8].

In this work we study the impact of independence on agents' behavior in a kinetic exchange opinion model. For this purpose, we introduce a probability q of agents to make independent decisions. Our analytical results and numerical simulations show that the model undergoes a phase transition at critical points q_c that depend on another model parameter, related to the agents' flexibility.

This work is organized as follows. In Section 2 we present the microscopic rules that define the model and in Section 3 the numerical and analytical results are discussed. Finally, our conclusions are presented in Section 4.

2. Model

Our model is based on kinetic exchange opinion models (KEOM) [9–12]. A population of N agents is defined on a fully-connected graph, i.e., each agent can interact with all others, which characterizes a mean-field-like scheme. In addition, each agent i carries one of three possible opinions (or states), namely $o_i = +1$, -1 or 0. The following microscopic rules govern the dynamics:

(1) An agent i is randomly chosen;
(2) With probability q, this agent will act independently. In this case, with probability g he/she chooses the opinion $o_i = 0$, with probability $(1-g)/2$ he/she adopts the opinion $o_i = +1$ and with probability $(1-g)/2$ he/she chooses the opinion $o_i = -1$;
(3) On the other hand, with probability $1-q$ we choose another agent, say j, at random, in a way that j will influence i. Thus, the opinion of the agent i in the next time step $t+1$ will be updated according to

$$o_i(t+1) = \mathrm{sgn}\big[o_i(t) + o_j(t)\big], \quad (1)$$

where the sign function is defined such that $\mathrm{sgn}(0) = 0$.

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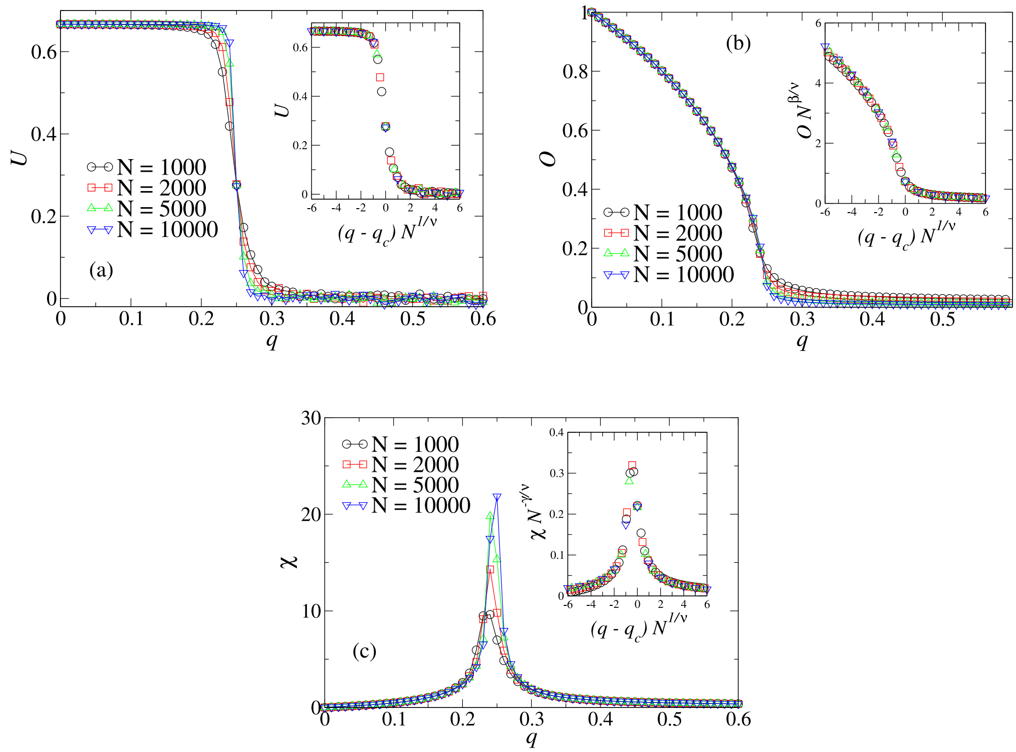

Fig. 1. (Color online.) Binder cumulant U (a), order parameter O (b) and susceptibility χ (c) as functions of the independence probability q for the homogeneous case ($g = 1/3$) and different population sizes N. In the inset we exhibit the corresponding scaling plots. The estimated critical quantities are $q_c \approx 0.25$, $\beta \approx 0.5$, $\gamma \approx 1.0$ and $1/\nu \approx 0.5$. Results are averaged over 300, 250, 200 and 150 samples for $N = 1000, 2000, 5000$ and $10\,000$, respectively.

In the case where the agent i does not act independently, the change of his/her state occur according to a rule similar to the one proposed recently in a KEOM [12]. Notice, however, that in Ref. [12] two randomly chosen agents i and j interact with competitive couplings, i.e., the kinetic equation of interaction is $o_i(t+1) = \text{sgn}[o_i(t) + \mu_{ij}\, o_j(t)]$. In this case, the couplings μ_{ij} are random variables presenting the value -1 ($+1$) with probability p ($1 - p$). In other words, the parameter p denotes the fraction of negative interactions. In this case, the model of Ref. [12] undergoes a nonequilibrium phase transition at $p_c = 1/4$. In the absence of negative interactions ($p = 0$), the population reaches consensus states with all opinions $+1$ or -1.

Thus, our Eq. (1) represents the KEOM of Ref. [12] with no negative interactions, and the above parameter g can be related to the agents' flexibility [6]. In this case, for $q = 0$ (no independence) all stationary states will give us $O = 1$, where O is the order parameter of the system,

$$O = \left\langle \frac{1}{N} \left| \sum_{i=1}^{N} o_i \right| \right\rangle, \tag{2}$$

and $\langle \dots \rangle$ denotes a disorder or configurational average taken at steady states. Eq. (2) defines the "magnetization per spin" of the system. We will show by means of analytical and numerical results that the independent behavior works as a noise that induces a phase transition in the KEOM in the absence of negative interactions.

The three states considered in the model can be interpreted as follows [13–15]. We have a population of voters that can choose among two candidates A and B. Thus, the opinions represent the intention of an agent to vote for the candidate A (opinion $+1$), for the candidate B (opinion -1), or the agent may be undecided (opinion 0). In this case, notice that there is a difference among the undecided and independent agents. An agent i that decide to behave independently (with probability q) can make a decision to change or not his/her opinion based on his/her own conviction, whatever is the his/her current state o_i (decided or undecided). In other words, an interaction with an agent j is not required. On the other hand, an undecided agent i can change his/her opinion o_i in two ways: due to an interaction with a decided agent j (following the rule given by Eq. (1), with probability $1 - q$) or due to his/her own decision to do that (independently, with probability q).

Regarding the independent behavior, one can consider the homogeneous case ($g = 1/3$) and the heterogeneous one ($g \neq 1/3$). These cases will be considered separately in the next section.

3. Results

3.1. Homogeneous case: $g = 1/3$

One can start studying the homogeneous case $g = 1/3$. In this case, we have that all probabilities related to the independent behavior, namely g and $(1 - g)/2$, are equal to 1/3. Thus, the probability that an agent i chooses a given opinion $+1$, -1 or 0 independently of the opinions of the other agents is $q/3$. For the analysis of the model, we have considered the order parameter O defined by Eq. (2), as well as the susceptibility χ and the Binder cumulant U [16,17], defined as

$$\chi = N\left(\langle O^2\rangle - \langle O\rangle^2\right) \tag{3}$$

$$U = 1 - \frac{\langle O^4\rangle}{3\langle O^2\rangle^2}. \tag{4}$$

Notice that the Binder cumulant defined by Eq. (4) is directly related to the order's parameter *kurtosis* k, that can be defined as $k = \langle O^4\rangle/3\langle O^2\rangle^2$. The initial configuration of the population is fully disordered, i.e., we started all simulations with an equal fraction of each opinion (1/3 for each one). In addition, one time step in the simulations is defined by the application of the rules defined in the previous section N times. In Fig. 1 we exhibit the quantities of interest as functions of q for different population sizes N. All results