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Environment induced entanglement in many-body mesoscopic systems

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ABSTRACT

We show that two, non-interacting, infinitely long spin chains can become globally entangled at the mesoscopic level of their fluctuation operators through a purely noisy microscopic mechanism induced by the presence of a common heat bath. By focusing on a suitable class of mesoscopic observables, the behaviour of the dissipatively generated quantum correlations between the two chains is studied as a function of the dissipation strength and bath temperature.

The presence of an external environment, typically a heat bath, modifies the dynamics of quantum systems in interaction with it, leading in general to loss of quantum correlations due to decohering and mixing-enhancing effects [1]. Nevertheless, it has also been established that suitable environments can enhance quantum entanglement instead of destroying it [2].

This mechanism of environment induced entanglement generation has been extensively studied for systems made of few qubits or oscillator modes [3], and specific protocols have been proposed to prepare predefined entangled states via the action of suitably engineered environments [4].

Instead, in this paper, we study the possibility that entanglement be created through a purely noisy mechanism in many-body systems. In a quantum system made of a large number N of constituents, accessible observables are collective ones, *i.e.* those involving the degrees of freedom of all its elementary parts. For these "macroscopic" observables, one usually expects that quantum effects fade away as N becomes large, even more so when the many-body system is in contact with an external environment. This is surely the case for the so-called "mean field" observables, *i.e.* averages over the whole system of microscopic operators; these quantities scale as $1/N$ and as such behave as classical observables when the number of system constituents becomes large.

Nevertheless, other collective observables exist that scale as $1/\sqrt{N}$ and that might retain some quantum properties as N increases [5–7]. These observables have been called "fluctuation operators", since, as we shall see, they physically represent some sort of deviations from mean values. The set of all these fluctuation operators form an algebra that, irrespective of the nature of the microscopic many-body system, turns out to be always non-commutative and of bosonic type, thus showing a quantum behaviour. Being half-way between microscopic observables (as for instance the individual spin operators in a generic spin systems) and truly macroscopic ones (*e.g.* the corresponding mean magnetization), the fluctuation operators have been named "mesoscopic": they are the place where to look for truly quantum signals in the dynamics of "large" systems, *i.e.* in systems in which the number of microscopic constituents is let to arbitrarily grow at fixed density (thermodynamical limit).

Although the characteristics and time evolution of the fluctuation algebra have been extensively studied in many systems [6], very little is known of its behaviour in open many-body systems, *i.e.* in systems immersed in an external bath. This is the most common situation encountered in actual experiments, typically involving cold atoms, optomechanical or spin-like systems [8,9], that can never be thought of as completely isolated from their thermal surroundings. Actually, the repeated claim of having detected "macroscopic" entanglement in those experiments [10,11] poses a serious challenge in trying to interpret theoretically those results [12,13].

Motivated by these experimental findings, in the following we shall show that quantum behaviour can indeed be present at the mesoscopic level in open many-body systems provided suitable fluctuation operators are considered and, even more strikingly, that entanglement can be induced in mesoscopic observables by the presence of the external bath.

We shall consider a many-body system composed of two spin-1/2 chains, one next to the other, immersed in a heat bath at a given inverse temperature $\beta = 1/T$. Each site of this double chain, actually composed of the corresponding couple of sites in the two chains, will be labelled by an integer $k = 1, 2, \ldots, N$. In this

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situation, the thermodynamical limit corresponds to letting the total number of sites N going to infinity.

A spin algebra $\mathcal{M}$, corresponding to the tensor product of two spin-1/2 algebras, is attached to each site; its elements at site k, $x^{(k)} \in \mathcal{M}^{(k)}$, are then of the form $x^{(k)} = x_1^{(k)} \otimes x_2^{(k)}$, where $x_1^{(k)}$, $x_2^{(k)}$ are spin algebra elements pertaining to the first, second chain, respectively. The algebra $\mathcal{M}$ clearly coincides with the algebra of 4×4 complex matrices and a convenient basis in it is given by $\sigma_\mu \otimes \sigma_\nu$, $\mu, \nu = 0, 1, 2, 3$, where σ_i, $i = 1, 2, 3$ are the usual Pauli matrices, while σ_0 is the unit matrix. For any finite set I of contiguous sites, one defines the finite-size tensor algebra $\mathcal{A}_I = \bigotimes_{k\in I} \mathcal{M}^{(k)}$; the union $\mathcal{A}$ of all these algebras, $\mathcal{A} = \bigcup_I \mathcal{A}_I$, is called the *quasi-local algebra* and the observables of the system clearly belong to it.

A state ω for the system is a linear, positive, normalized functional on the algebra $\mathcal{A}$, $\omega : \mathcal{A} \to \mathbb{C}$, assigning the expectation value $\omega(X)$ to each elements X of $\mathcal{A}$. For finite N, it can be represented by a density matrix ρ through the identification $\omega(X) = \mathrm{Tr}[\rho X]$; however, since we are interested in the thermodynamical limit, it is more convenient to work in the abstract algebraic formulation [14].

Since the two chains can be thought to be initially at equilibrium with the bath, as reference state for our system we take a product state

$$\omega = \omega^{(1)} \otimes \omega^{(2)} \otimes \omega^{(3)} \otimes \ldots, \tag{1}$$

where $\omega^{(k)}$, $k = 1, 2, 3, \ldots$, are single site states, that for simplicity can be assumed to be all equal to a reference thermal state, at the bath temperature. As a consequence, ω has the property that given two observables $x^{(k)}$, $y^{(l)}$ at different sites, $k \neq l$, then: $\omega(x^{(k)} y^{(l)}) = \omega(x^{(k)})\,\omega(y^{(l)})$; this means that in practice ω is uniquely defined by the expectation values on all observables $x \in \mathcal{M}$ at one site of the double chain, that in the following will be simply called $\omega(x)$.

Most of the physical properties of many-body systems can be obtained by focusing on collective observables, *i.e.* on operators involving all system degrees of freedom, which, in the present situation, means combinations of spin variables at all N sites. In the thermodynamical limit, *i.e.* when N becomes infinitely large, a suitable scaling with N needs to be included in the definition of these observables in order to obtain meaningful limiting operators.

A well-known example of such observables is given by the averages over all sites of a given spin operator $x \in \mathcal{M}$:

$$\overline{X}_N = \frac{1}{N} \sum_{k=1}^{N} x^{(k)}. \tag{2}$$

As N grows, the sequence of operators $\{\overline{X}_N\}$ converges to the "macroscopic" observable $\overline{X} = \lim_{N\to\infty} \overline{X}_N$. This convergence should be intended in the weak sense, *i.e.* under state average.[1] In the case of the product state (1), this limit is easily computed:

$$\lim_{N\to\infty} \omega(\overline{X}_N) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \omega\big(x^{(k)}\big) = \omega(x),$$

since the expectations $\omega(x^{(k)})$ are all equal and independent of k. In practice, one obtains [5]

$$\overline{X} = \lim_{N\to\infty} \overline{X}_N = \omega(x)\,\mathbf{1}, \tag{3}$$

with $\mathbf{1}$ the identity operator. As a result, the set of all these limiting operators form an abelian algebra, since all operators commute among themselves; it is called the *mean field* algebra and it is known to represent the classical behaviour of the system.

Nevertheless, some of the system quantum properties can survive even in the large N limit: they are encoded in the so-called *fluctuation operators*. These are collective observables that scale as the square root of N,

$$\widetilde{X} = \lim_{N\to\infty} \widetilde{X}_N \equiv \lim_{N\to\infty} \frac{1}{\sqrt{N}} \sum_{k=1}^{N} \big[x^{(k)} - \omega\big(x^{(k)}\big)\big], \tag{4}$$

and represent a sort of deviation from (or fluctuation about) the average. One easily sees that the commutator of two such fluctuation observables is in general nonvanishing, since it is equal to a mean field operator

$$\big[\widetilde{X}, \widetilde{Y}\big] = \lim_{N\to\infty} \big[\widetilde{X}_N, \widetilde{Y}_N\big] = \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \big[x^{(k)}, y^{(k)}\big], \tag{5}$$

being $[x^{(k)}, y^{(l)}] = 0$ for $k \neq l$. Recalling (3), this implies that $[\widetilde{X}, \widetilde{Y}]$ is proportional to the identity operator, and therefore that the algebra formed by all fluctuation operators possesses a quantum character, being non-abelian.

The fluctuation algebra is clearly bosonic and look very similar to the Heisenberg algebra of position and momentum operators; as in that case, the algebra elements $\widetilde{X}$ in (4) turn out to be unbounded operators, their norm diverging as $\sqrt{N}$ in the thermodynamical limit. To avoid convergence problems, it is then convenient to work with the corresponding Weyl operators, $\lim_{N\to\infty} e^{i\widetilde{X}_N}$, whose existence in the weak sense is guaranteed by the so-called *quantum central limit* [5–7]. Indeed, defining the following sesquilinear form on the algebra of fluctuations:

$$\big\langle \widetilde{X}, \widetilde{Y}\big\rangle_\omega = \lim_{N\to\infty} \omega\big(\widetilde{X}_N^\dagger \widetilde{Y}_N\big), \tag{6}$$

one shows that, for any hermitian spin operator x, the following result holds:

$$\lim_{N\to\infty} \omega\big(e^{i\widetilde{X}_N}\big) = e^{-\frac{1}{2}\langle \widetilde{X}, \widetilde{X}\rangle_\omega}. \tag{7}$$

Similarly, products of any number of Weyl operators can be analogously computed; in particular, one has

$$\lim_{N\to\infty} \omega\big(e^{i\widetilde{X}_N}\, e^{i\widetilde{Y}_N}\big) = e^{-\frac{1}{2}(\langle \widetilde{X}+\widetilde{Y}, \widetilde{X}+\widetilde{Y}\rangle_\omega + [\widetilde{X}, \widetilde{Y}])}, \tag{8}$$

with

$$\big[\widetilde{X}, \widetilde{Y}\big] = 2i\,\mathcal{I}m\big(\big\langle \widetilde{X}, \widetilde{Y}\big\rangle_\omega\big)\mathbf{1}. \tag{9}$$

In other terms, in the large N limit, the set of hermitian fluctuation operators $\{\widetilde{X}\}$ form a well defined bosonic algebra, characterized by the commutation relations (9). This algebra can be appropriately described in terms of the Weyl operators $W(x) = e^{i\widetilde{X}}$ and a suitable Gaussian state $\widetilde{\omega}$ reproducing all higher order correlations[2]:

$$\widetilde{\omega}\big(W(x)W(y)\ldots\big) = \lim_{N\to\infty} \omega\big(e^{i\widetilde{X}_N}\, e^{i\widetilde{Y}_N} \ldots\big). \tag{10}$$

[1] More precisely, weak convergence means that, given any couple of local operators Y and Z having support only on a finite number of sites, the sequence $\omega(Y\,\overline{X}_N\,Z)$ converges in the limit of large N; because of the assumed form of the state ω, one further has: $\lim_{N\to\infty} \omega(Y\,\overline{X}_N\,Z) = \omega(YZ)\,\omega(x)$, and thus $\lim_{N\to\infty} \overline{X}_N = \omega(x)\,\mathbf{1}$.

[2] Given a state ω over an algebra of operators $\mathcal{A}$, a standard procedure, the so-called *GNS construction* [14], allows to build a Hilbert space $\mathcal{H}_\omega$, generated by a cyclic "vacuum" vector $|\Omega_\omega\rangle$, and a representation π_ω of $\mathcal{A}$ into the bounded operators on $\mathcal{H}_\omega$; further, the expectation of any element $X \in \mathcal{A}$ is given by the corresponding vacuum mean value, *i.e.* $\omega(X) = \langle\Omega_\omega|\pi_\omega(X)|\Omega_\omega\rangle$. Therefore, the Weyl correlations in (10) reproduce the mean value of any fluctuation observable in any state of the corresponding Hilbert space.