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Extended thermodynamics of real gases with dynamic pressure: An extension of Meixner's theory

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ABSTRACT

Basing on the recent theory of extended thermodynamics of dense gases, we study a thermodynamic theory of gases with the energy transfer from molecular translational mode to internal modes as an extension of Meixner's theory. We focus our attention on the simplest case with only one dissipative process due to the dynamic pressure. The dispersion relation for sound derived from the present theory is compared with that from Meixner's theory. Kinetic theoretical basis of the present approach is also discussed.

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1. Introduction

Energy transfer from molecular translational mode to internal modes, such as rotational and vibrational modes, affects the propagation speed and attenuation of a sound in a gas composed of polyatomic molecules. Especially when the frequency ω of the sound is in the same order of magnitude as the inverse of the relaxation time of the energy transfer, $1/\tau$, the effect on the sound is prominent. Such nonequilibrium phenomena are usually observed in the ultrasonic frequency range.

The thermodynamic theory with nonequilibrium parameters governed by the relaxation equations [1–3] has been utilized to describe the phenomena for many years. In order to grasp the essence of the theory, let us consider the simplest case where only one relaxation equation for a nonequilibrium parameter ξ is present in addition to the system of Euler equations for a gas that expresses the mass, momentum and energy conservation laws. That is, we neglect all dissipative processes but we take into account the relaxation process. The relaxation equation is introduced in such a way that

$$\dot{\xi} = -\beta A, \tag{1}$$

where a dot on ξ represents the material time derivative, β is a positive coefficient, and A is the affinity of the relaxation process of the energy transfer that depends not only on ξ but also on other thermodynamic quantities, say, the mass density and the entropy density. When $\omega\tau \ll 1$, it was proved that the relaxation process may be interpreted in terms of the dynamic pressure Π, which is related to the gas velocity $\boldsymbol{v}$ as

$$\Pi = -\nu^{\text{eff}} \operatorname{div} \boldsymbol{v}$$

with ν^{eff} being the effective bulk viscosity.

Although Meixner's theory mentioned above seems to be natural, there remain some problems that should be overcome: (i) In Meixner's theory, the relaxation equation (1) is not fully congruous with the Euler equations. It has not been introduced on the same ground of the Euler equations as one of the general thermodynamic basic field equations. In fact, in a rarefied gas limit, the relaxation equation is not consistent with its counterpart of the moment equations derived from the kinetic theory of gases [4]. See also Section 4.2 below. (ii) Meixner's theory is formulated within the framework of thermodynamics of irreversible processes [3]. The local equilibrium assumption is premised from the beginning. However, in such phenomena as ultrasonic wave propagation where temporal and spatial changes are rapid and steep, this assumption is not well-satisfied [5].

In this Letter, we propose a fully-consistent thermodynamic theory of the sound propagation in a gas with the energy transfer where the local equilibrium assumption is not necessarily valid, and thereby try to extend Meixner's theory. We adopt the theory

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of extended thermodynamics (ET) [6] of dense gases [7]. As before, the essence of our theory can be most clearly shown by studying the simplest case where only one dissipative process due to the dynamic pressure exists. In Section 2, we derive the closed system of field equations for gases. In Section 3, we study the dispersion relation for sound and compare it with that derived from Meixner's theory. The last section is devoted to concluding remarks with the discussions of subsystems and the kinetic theoretical basis of the present theory.

2. Extended thermodynamics of real gases with 6 fields

First of all, let us recall that, for a rarefied monatomic gas, it is simple to construct a rational theory of extended thermodynamics as a hyperbolic counterpart of the Navier–Stokes–Fourier theory because the hierarchy of the basic system of differential equations is properly dictated by the kinetic theory [6]. While, for rarefied polyatomic gases and for dense gases, a satisfactory theory was not established [8–13] until the appropriate binary hierarchy of the differential equations for dense gases with 14 fields has been proposed by the present authors [7]. In fact, using only general principles such as the Galilean invariance and the entropy principle, we proved that the system of field equations can be closed with respect to the independent field variables and the constitutive functions are determined explicitly by the equilibrium thermal and caloric equations of state.

2.1. Binary hierarchy of the differential equations

As mentioned above, we restrict our study within the simplest case of 6 independent field variables, that is,

mass density:	$F\ (=\rho)$,
momentum density:	$F_i\ (=\rho v_i)$,
energy density:	G_{ii},
trace part of momentum flux:	F_{ii}.

We adopt the following binary hierarchy (F-series and G-series, see also Section 4.2) of the balance equations [7]:

$$\begin{aligned}
&\frac{\partial F}{\partial t}+\frac{\partial F_k}{\partial x_k}=0,\\
&\frac{\partial F_i}{\partial t}+\frac{\partial F_{ik}}{\partial x_k}=0,\\
&\frac{\partial F_{ii}}{\partial t}+\frac{\partial F_{iik}}{\partial x_k}=P_{ii},\qquad \frac{\partial G_{ii}}{\partial t}+\frac{\partial G_{iik}}{\partial x_k}=0,
\end{aligned}\tag{2}$$

where F_{ik} is the momentum flux, F_{iik} is the flux of F_{ii}, G_{iik} is the energy flux, and P_{ii} is the production with respect to F_{ii}. The equations with no production term represent the mass, momentum and energy conservation laws.

As the balance equations (2) should be invariant under the *Galilean transformation*, the dependence of the quantities on the velocity can be expressed as follows [14]:

$$\begin{aligned}
&F_{ij}=\rho v_i v_j+M_{ij},\\
&G_{ii}=\rho v_i v_i+m_{ii},\\
&F_{iik}=\rho v_i v_i v_k+3M_{(ik}v_{i)}+M_{iik},\\
&G_{iik}=\rho v_i v_i v_k+m_{ii}v_k+2M_{ik}v_i+m_{iik},
\end{aligned}\tag{3}$$

where M_{ij}, m_{ii}, M_{iik} and m_{iik} do not depend on the velocity. Parentheses around a set of indices represent the symmetrization with respect to the indices. The production P_{ii} is also independent of the velocity.

With Eq. (3), the balance equations (2) can be rewritten as

$$\begin{aligned}
&\dot\rho+\rho\frac{\partial v_k}{\partial x_k}=0,\\
&\rho\dot v_i+\frac{\partial M_{ij}}{\partial x_j}=0,\\
&\dot m_{ii}+m_{ii}\frac{\partial v_k}{\partial x_k}+\frac{\partial m_{iik}}{\partial x_k}+2\frac{\partial v_i}{\partial x_k}M_{ik}=0,\\
&\dot M_{ii}+M_{ii}\frac{\partial v_k}{\partial x_k}+\frac{\partial M_{iik}}{\partial x_k}+2\frac{\partial v_i}{\partial x_k}M_{ik}=P_{ii}.
\end{aligned}\tag{4}$$

We notice that the quantities M_{ij}, m_{ii} and m_{ppi} have the following conventional meanings:

stress:	$t_{ij}=-M_{ij}\ (=-(p+\Pi)\delta_{ij}-M_{\langle ij\rangle})$,
specific internal energy:	$\varepsilon=\frac{1}{2\rho}m_{ii}$,
heat flux:	$q_i=\frac{1}{2}m_{ppi}$,

where the pressure p depends only on ρ and m_{ii}, Π is the dynamic pressure, and angular brackets denote the symmetric traceless part.

We may now adopt $\{\rho, v_i, m_{ii}, \Pi\}$ as a set of independent variables instead of $\{F, F_i, G_{ii}, F_{ii}\}$. The balance equation of M_{ii} (Eq. $(4)_4$) is then rewritten as

$$\begin{aligned}
\dot\Pi&+\left(\frac{5}{3}p-\rho\left(\frac{\partial p}{\partial\rho}\right)_{m_{ii}}-(m_{rr}+2p)\left(\frac{\partial p}{\partial m_{qq}}\right)_\rho\right)\frac{\partial v_k}{\partial x_k}\\
&+\left(\frac{5}{3}-2\left(\frac{\partial p}{\partial m_{qq}}\right)_\rho\right)\Pi\frac{\partial v_k}{\partial x_k}+2\left(\frac{1}{3}-\left(\frac{\partial p}{\partial m_{qq}}\right)_\rho\right)\frac{\partial v_r}{\partial x_k}M_{\langle rk\rangle}\\
&+\frac{1}{3}\frac{\partial M_{rrk}}{\partial x_k}-\left(\frac{\partial p}{\partial m_{qq}}\right)_\rho\frac{\partial m_{rrk}}{\partial x_k}=\frac{P_{rr}}{3}.
\end{aligned}$$

2.2. Constitutive equations

We need the constitutive equations in order to set up the closed system of field equations. We assume that the constitutive equations at one point and time depend on the independent fields at that point and time. Therefore the constitutive quantities $\{M_{\langle ij\rangle}, M_{iik}, m_{iik}, P_{ii}\}$ are expressed as functions of

$$(\rho, m_{ii}, \Pi).$$

We apply the constitutive theory of ET [6] where the following universal physical principles (A)–(C) are imposed on the constitutive equations:

(A) *Material frame indifference principle*: This requires that constitutive equations are independent of an observer. This principle and the Galilean invariance for the balance laws constitute the objectivity principle (the principle of relativity).

(B) *Entropy principle*: All solutions of the system of field equations must satisfy the entropy balance law:

$$\frac{\partial h}{\partial t}+\frac{\partial(hv_k+\varphi_k)}{\partial x_k}=\Sigma\geqq 0\Leftrightarrow \dot h+h\frac{\partial v_k}{\partial x_k}+\frac{\partial\varphi_k}{\partial x_k}=\Sigma\geqq 0,$$

where h is the entropy density, h_k is the entropy flux ($h_k = hv_k+\varphi_k$: φ_k is the nonconvective entropy flux), and Σ is the entropy production. Here h and φ_k are constitutive quantities:

$$h\equiv\hat h(\rho, m_{ii}, \Pi),\qquad \varphi_k\equiv\hat\varphi_k(\rho, m_{ii}, \Pi).$$

(C) *Causality*: This requires the concavity of the entropy density and guarantees the hyperbolicity of the system of field equations. This also ensures the well-posedness (local in time) of a Cauchy problem and the finiteness of the propagation speeds of disturbances.

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