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Wave diffraction by a cosmic string

Isabel Fernández-Núñez [a,b], Oleg Bulashenko [a,*]

[a] Departament de Física Quàntica i Astrofísica, Facultat de Física, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain
[b] Institut de Ciències del Cosmos (ICCUB), Facultat de Física, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Spain

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ABSTRACT

We show that if a cosmic string exists, it may be identified through characteristic diffraction pattern in the energy spectrum of the observed signal. In particular, if the string is on the line of sight, the wave field is shown to fit the Cornu spiral. We suggest a simple procedure, based on Keller's geometrical theory of diffraction, which allows to explain wave effects in conical spacetime of a cosmic string in terms of interference of four characteristic rays. Our results are supposed to be valid for scalar massless waves, including gravitational waves, electromagnetic waves, or even sound in case of condensed matter systems with analogous topological defects.

1. Introduction

Topological defects may appear naturally during a symmetry-breaking phase transition in various physical systems. One of the examples is a cosmic string – a long-lived topologically stable structure that may have been formed at phase transitions in the early Universe [1–3]. Cosmic strings are analogous to other linear defects found in condensed matter systems: vortex lines in liquid helium [4], flux tubes in type-II superconductors [5], disclinations in liquid crystals [6], in graphene [7], or in metamaterials [8–10].

The spacetime around a straight cosmic string is locally flat, but it globally has a conical topology that can give rise to a variety of observable phenomena [2,3]. The most evident way to detect cosmic strings is by means of gravitational lensing. The conical topology should produce double images of a distant source situated behind the string [11]. The images should be undistorted but they may overlap if the split angle, which is proportional to the string tension, is small. In such a case, the wave effects are extremely important as a probe in gravitational lensing [12], that was extensively studied for compact or point-like objects [13], but only a few studies are known for the strings [14–18].

In this Letter we show that the wave propagation in conical spacetime, caused by a cosmic string or similar topological defects, can be effectively treated in the framework of the celebrated Arnold Sommerfeld's half-plane diffraction problem [19–21]. In this way, we find analytical solutions in terms of Fresnel integrals, that let us conclude that the wave effects in conical space are determined by a unique parameter, the Fresnel zone number. For the wave effects to be detectable in a compact-mass gravitational lens, the wavelength λ should be comparable with the Schwarzchild radius R_s of the lens [12]. This condition cannot be applied to a string, a non-compact object with conical topology. Instead, we show that the diffraction effects caused by a string are of the leading order with respect to geometrical optics whenever the observation point (either in space or in frequency spectrum) belongs to the low-number Fresnel zone. This is in contrast to the case of a compact-mass lens, for which diffraction scales like $O(\lambda/R_s)$. Basing on Keller's geometrical theory of diffraction [22], we suggest a simple procedure how the geometrical-optics approximation can be "improved" by adding just two additional paths corresponding to diffraction. These are waves coming from the source to the observer but hitting the string following the shortest path. In this way, the interference effects will be taken into account to the leading order. Our results imply that if a cosmic string exists, it may be identified through a characteristic diffraction pattern in the energy spectrum of the observed signal. Finally, we show that if the string is on the line of sight, the wave amplitude fits the Cornu spiral – the prominent result for the Fresnel diffraction by a straight edge or a slit.

2. Spacetime of a cosmic string

We start with a spacetime metric for a static cylindrically symmetric cosmic string [11,23]

$$ds^2 = -dt^2 + dr^2 + (1 - 4G\mu)^2 r^2 d\varphi^2 + dz^2, \qquad (1)$$

* Corresponding author.
E-mail address: oleg.bulashenko@ub.edu (O. Bulashenko).

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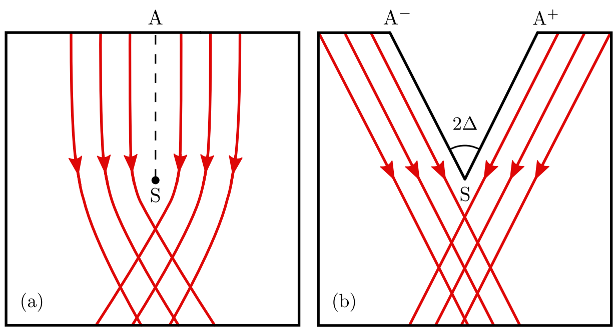

Fig. 1. Geodesics in the conical space on $z = 0$ plane: (a) curved spacetime; (b) flat spacetime with a deficit angle 2Δ. The cut half-plane SA is perpendicular to the plane of the figure with the edge S coinciding with the string. After the angular transformation the half-plane SA is converted to a wedge of two half-planes SA^- and SA^+, which should be identified.

where G is the gravitational constant, μ is the linear mass density of the string lying along the z-axis, (t, r, φ, z) are cylindrical coordinates, and the system of units in which the speed of light $c = 1$ is assumed. With a new angular coordinate $\theta = (1 - 4G\mu)\varphi$, the metric (1) takes a Minkowskian form

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + dz^2, \tag{2}$$

which is locally flat, but it globally has a conical topology, since a wedge of angular size $8\pi G\mu$ is taken out from flat space and the two faces of the wedge are identified [2,11]. By introducing the deficit angle 2Δ with

$$\Delta = 4\pi G\mu, \tag{3}$$

the angular coordinate θ spans the range $2\pi - 2\Delta$. Solutions of Hamilton's equations [24] for both geometries are depicted in Fig. 1. One can see that geodesics for the metric (1) are curved and deflected an angle Δ [11,25]. However, in coordinates (2) they are just straight lines. Since geodesics passing on opposite sides of the string eventually cross, one should expect interference or diffraction effects.

3. Wave equation in conical space

We consider the question of finding a solution of the wave equation in background (1) corresponding to a time harmonic distant source, so that the incident waves are plane waves. In order to reduce the problem to two dimensions, the waves are assumed to be emitted in the direction orthogonal to the string. Similarly to Ref. [17], we write the wave equation for a scalar field $U(r, \varphi)$ as

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{\beta^2 r^2}\frac{\partial^2}{\partial \varphi^2} + \omega^2\right) U = 0, \tag{4}$$

where we denoted $\beta = 1 - \Delta/\pi$. We assume that Eq. (4) is valid for electromagnetic waves, as well as for gravitational waves when the effect of gravitational lensing on polarization is negligible and both types of waves can be described by a scalar field [26]. A plane wave of unit amplitude incident from the direction φ_0 is described by

$$U = e^{ikr\cos\{\beta(\varphi - \varphi_0)\}}. \tag{5}$$

Next, unlike Ref. [17], we perform the coordinate transformation taking advantage of the flat background (2). To do that, we place the cut line SA strictly perpendicular to the wavefront of the incident wave, as shown in Fig. 1(a), so we get $\partial_\varphi U = 0$ at the cut. Then, we assign the values $\varphi_0^- = -\pi$ to the left and $\varphi_0^+ = \pi$ to the right of the line SA. After angular transformation $\theta = \beta\varphi$, the line SA converts to the wedge SA^-, SA^+ given by the angles $\pm(\pi - \Delta)$. The incident field (5) will now be represented by two plane waves

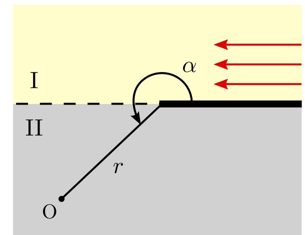

Fig. 2. Plane wave grazing a half-plane screen (thick line). The entire space is split into two regions: illuminated (I), shadow (II).

$$U = e^{ikr\cos(\theta \pm \Delta)} \tag{6}$$

incoming from the directions $\pm(\pi - \Delta)$ with wavefronts perpendicular to the faces of the wedge and propagating in a flat background [see Fig. 1(b)]. As we are going to show now, this problem can be reduced to the canonical problem of diffraction on a perfectly conducting half-plane screen solved exactly by Sommerfeld [19–21].

Let us consider a single plane wave grazing an infinite half-plane screen, as shown in Fig. 2. Following Sommerfeld [20], the exact solution for the field at any point $O(r, \alpha)$ can be written in the compact form

$$U = e^{-ikr\cos\alpha}\,\mathcal{F}\left(\sqrt{2kr}\cos\frac{\alpha}{2}\right), \tag{7}$$

where r is the distance from the screen edge, α is the angle measured from the surface of the screen facing the source, and $\mathcal{F}(u) = e^{-i\pi/4}\pi^{-1/2}\int_{-\infty}^{u} e^{is^2} ds$ is the Fresnel integral [27]. In Eq. (7) we have taken into account the zero angle of incidence and the Neumann boundary condition on the screen, $\partial_\alpha U(r, 0) = 0$. It can be verified that solution (7) contains both the geometrical-optics (GO) and the diffracted (D) fields. Indeed, for the angles $0 < \alpha < \pi$, in the limit $kr \to \infty$ far away from the edge, one gets $\mathcal{F} \to 1$ and $U = e^{-ikr\cos\alpha}$, which is the GO incident field. Whereas, for the angles $\pi < \alpha < 2\pi$, one obtains $\mathcal{F} \to 0$ giving $U = 0$ at infinity. The Fresnel function $\mathcal{F}$ smooths the discontinuity of the GO solution across the shadow boundary $\alpha = \pi$ making the total field continuous everywhere. This smooth transition constitutes the diffraction phenomenon [20]. It should also be noted that the original Sommerfeld problem treats two possible boundary conditions on the screen (Dirichlet or Neumann) depending on the polarization of the incident field. However, for grazing incidence, only one polarization can propagate which corresponds to the Neumann condition. On the other hand, the zero field condition is unphysical for the conical space we consider.

Having defined the solution for a single half-plane, we now construct the wave field corresponding to the geometry of Fig. 1(b), in which we have two plane waves (6) grazing the faces of the wedge. Substituting grazing angles: $\alpha = \pi - \Delta \mp \theta$ into Eq. (7), we obtain the total field $U(r, \theta)$ at the observation point O

$$U = e^{ikr\cos(\Delta + \theta)}\mathcal{F}(w^+) + e^{ikr\cos(\Delta - \theta)}\mathcal{F}(w^-) \tag{8}$$

with $w^\pm = \sqrt{2kr}\sin[(\Delta \pm \theta)/2]$. It describes the wave effects in the gravitational lensing by a cosmic string. It is easy to verify that for $\Delta = 0$, it reduces to the unlensed field $U_0 = e^{ikr\cos\theta}$, which is a usual plane wave in Minkowskian space.

For further analysis Eq. (8) can be rewritten in a more convenient form in terms of the eikonals $s^\pm = r\cos(\Delta \pm \theta)$ of the GO waves. The arguments of the Fresnel function become $w^\pm = \sigma^\pm\sqrt{k(r - s^\pm)}$ where $\sigma^\pm \equiv \mathrm{sgn}(\Delta \pm \theta)$ are sign functions giving