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Complexity multiscale asynchrony measure and behavior for interacting financial dynamics

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ABSTRACT

A stochastic financial price process is proposed and investigated by the finite-range multitype contact dynamical system, in an attempt to study the nonlinear behaviors of real asset markets. The viruses spreading process in a finite-range multitype system is used to imitate the interacting behaviors of diverse investment attitudes in a financial market, and the empirical research on descriptive statistics and autocorrelation behaviors of return time series is performed for different values of propagation rates. Then the multiscale entropy analysis is adopted to study several different shuffled return series, including the original return series, the corresponding reversal series, the random shuffled series, the volatility shuffled series and the Zipf-type shuffled series. Furthermore, we propose and compare the multiscale cross-sample entropy and its modification algorithm called composite multiscale cross-sample entropy. We apply them to study the asynchrony of pairs of time series under different time scales.

1. Introduction

Recently, statistical behaviors and fractal behaviors of market fluctuations have been a focus of economic research for a more clear understanding of financial market dynamics. Some fluctuation statistical properties, such as the fat-tail distribution of price changes, the power law of logarithmic return and volume, the volatility clustering which is described as on-off intermittency in literature of nonlinear dynamics, and the multifractality of volatility, are uncovered from empirical or theoretical research. Capturing these features of financial markets is becoming a key issue to model the dynamics of the forwards prices in the risk management, physical assets valuation, and derivatives pricing [1–8]. Due to the fact that a financial market can be considered like a strongly fluctuating complex system with a large number of interacting elements, many methods developed in statistical physics can be applied to characterize the time evolution of stock prices. Some research work has applied statistical physics (or interacting particle systems) to study statistical behaviors of fluctuations of financial markets [9–14]. Most of the models are based on the perspective that the price movements are caused primarily by the arrival of new information, or the idea that the price fluctuations are due to the interaction among the market investors, see [15–20]. For instance, the agent based financial models have been developed by Ising dynamic systems [21–23] to investigate the fluctuation behaviors of returns. These financial models describe the interaction strength among the agents, where the Ising system is supposed as a model of imitative behavior, in which individuals modify their behavior so as to conform to the behavior of other individuals in their vicinity. Contact dynamic systems are introduced to the financial time series models [24–26], to investigate the long-term memory, the nonlinear correlations, the Zipf distribution and the multifractal phenomenon of returns, where the contact system is a model for epidemic spreading that mimics the interplay of local infections and recovery of individuals. In the present paper, the finite-range multitype contact (FRMC) process is applied to develop a random agent-based financial time series model [9–11,27]. Compared with the contact system, the FRMC process has three types of particles, which can also be explained as a model for the spreading that mimics the interplay of heavy infection, slight infection and recovery of individuals. Heavy infection and slight infection spread at the rates proportional to the number of heavily and slightly infected neighbors respectively, and both heavily and slightly infected individuals recover at a constant rate to become healthy individuals. What is interesting in this process is that a heavily infected individual can affect and change its slightly infected neighbors into heavily infected ones, while the converse process is not possible.

In the statistical exploration of financial properties, the proposed random agent-based financial model is utilized to study the relationship between the relative complexity and the order of re-

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turn series. The multiscale entropy (MSE) analysis is employed to investigate the complexity behaviors for different shuffled return series, including the original return series, the corresponding reversal series, the random shuffled series, the volatility shuffled series and the Zipf-type shuffled series. Furthermore, according to the composite multiscale entropy (CMSE) method as the modification of MSE approach by Wu et al. [28,29], we can also propose the composite multiscale cross-sample entropy (CMSCE) as a modification of multiscale cross-sample entropy (MSCE) approach, to analyze the asynchrony of pairs of series under different time scales in Section 4. The comparison analysis between these two methods shows that the CMSCE method can be more advantageous in decreasing the variance of cross-sample entropy estimation for short-term time series than the MSCE method. Besides, in the empirical research, we quantify the asynchrony of pairs of return series and various volatility series for Shanghai Stock Exchange Composite Index (SSE), Shenzhen Stock Exchange Component (SZSE) and the simulation data derived from the FRMC system with different parameters by utilizing the CMSCE method.

2. Financial price model from FRMC process

The FRMC process, as its name implied, is different from ordinary contact process for it has more than one type of particles in the process. Generally, the "tree, bush and grass" can help us have a better understanding on the FRMC process [30–32]. In the description, there are three plants which are tree, bush and grass, and at each site only one plant can grow up, which means if a new plant grows up at a site, the previous plant at this site would die or can be comprehend as the previous plant becomes the new plant. A grass can grow up to a bush (or tree), and a bush can grow up to a tree or become a grass, while a tree can not become a bush but only a grass. The death rate of bush becoming grass is as the same as the death rate of tree becoming grass, but the birth rate of a new bush is different from that of a new tree. In order to make it easy to understand, let $2 = \text{tree}$, $1 = \text{bush}$ and $0 = \text{grass}$, a brief mathematical description of FRMC process is introduced as follows, for details see Refs. [11,31,33]. The FRMC process on $\mathbb{Z}^d$ is a continuous time Markov process $\{\eta_s, s > 0\}$ with infection parameters λ_1 and λ_2 (the λ_1 and λ_2 are the rates of Poisson processes) on the configuration space $\{0,1,2\}^{\mathbb{Z}^d}$, i.e., $\eta_s(x) \in \{0,1,2\}$ for $x \in \mathbb{Z}^d$, $\eta_s(x) = 0$ is explained as vacant at site x, and $\eta_s(x) = i$ $(i = 1,2)$ is explained as that there is an "i" type of particles at site x. In this model, the process evolves according to the following rules: (i) particles die at rate 1; (ii) particles of type i give birth at rate λ_i $(i = 1,2)$; (iii) a new particle from x is sent to y chosen at random from $\{y : 0 < |y - x| \le L\}$ (where the finite-range L is a positive integer); (iv) when $\eta_s(y) \ge \eta_s(x)$, then the birth is prohibited.

From the above description of FRMC process, the process has something in common with the stock market, where the viruses spreading in a finite-range multitype system is similar to the interaction and dispersal of different types of investment attitudes in the stock market. Therefore, as a common sense, the traders' investment decisions towards the stock market lead to the fluctuations of stock price, causing the stock price up or down. Suppose that investment attitudes are displayed by the particles of the FRMC process, which are classified into three categories based on the standards of strength: strong attitudes, weak attitudes and neutral attitudes, corresponding to "2" type particles ("tree"), "1" type particles ("bush") and "0" type particles ("grass") respectively. In the real stock market, the above assumption is also existent and reasonable, since the investment attitudes of different traders in a real market are not always in the same strength, which means some investors (usually regarded as institutional investors) are more certain in their decisions while others (regarded as individual investors) are not so sure and relatively easy to change their investment decisions. Then, according to the mathematical description of FRMC process in previous paragraph, let $\eta_s^{A_1,A_2}$ denote the state at time s with initial state $A_1 = \{x \in \mathbb{Z}^d : \eta_0(x) = 1\}$ and $A_2 = \{x \in \mathbb{Z}^d : \eta_0(x) = 2\}$. More generally, we consider the initial distribution as ν_{θ_1} and ν_{θ_2}, that is, each site is independently occupied by type 1 particles with probability θ_1 and by type 2 particles with probability θ_2 $(\theta_1 + \theta_2 \le 1)$, and let $\eta_s^{\theta_1,\theta_2}$ denote the corresponding FRMC model with initial distribution ν_{θ_1} and ν_{θ_2}. In this paper, we treat the initial distribution ν_{θ_1} and ν_{θ_2} as two constants labeled as the initial density θ_1 and θ_2 for short, where they represent the probability of each type particles respectively at time 0. The most significant feature of the FRMC process is that survival and extinction can both occur, where one occurrence depends on the values of the rates λ_1 and λ_2, those represent the propagation rates of strong attitudes and weak attitudes in this agent-based financial price model.

Now, the graphical representation of one-dimensional multitype contact model with configuration space $\{0,1,2\}^{\mathbb{Z}}$ and two ranges $L = 2$ and $L = 3$ respectively is exhibited in Fig. 1, which is useful to clarify and imitate the model and it evolves according to the following steps. Firstly, confirm the analyzed range. Consider a financial model of auctions for a stock index in a market, and assume that the stock market is comprised of $2\mathcal{N}$ ($\mathcal{N}$ is large enough) traders, who are located in a line $\{-\mathcal{N}, \cdots, -1, 0, 1, \cdots, \mathcal{N}\}$. Secondly, initialize distribution. At the beginning of each trading day $(t = 1, 2, \cdots, T)$, we select two fractions of traders (with initial density θ_1 and θ_2) randomly in the system who hold weak attitudes and strong attitudes respectively. Thirdly, spread the information. Assume that at each day one trader can spread his/her attitude several times, and let l be the trading time length in each trading day, and $\mathscr{P}_t(s)$ be the stock price at time s in the t-th day, where $s \in [0, l]$. Consider $\mathbb{R}_+ \times \mathbb{Z}$ as the space-time, for each pair $x, y \in \{-\mathcal{N}, \cdots, -1, 0, 1, \cdots, \mathcal{N}\}$ with $0 < |x - y| \le L$, let $\{T_{1,n}^{(x,y)}, n \ge 1\}$, $\{T_{2,n}^{(x,y)}, n \ge 1\}$ and $\{U_n^x, n \ge 1\}$ be jump points of Poisson processes, with rates λ_1, λ_2 and 1 respectively. The x and y stand for different traders, while the number of neighbors of each trader is $2L$. For each ordered pair of distinct time lines from $\mathbb{R}_+ \times \{x\}$ to $\mathbb{R}_+ \times \{y\}$ with $0 < |x - y| \le L$, we place pink directed arrows at time points $\{T_{1,n}^{(x,y)}, n \ge 1\}$ from x to y in Fig. 1, according to a Poisson process with rate λ_1, independently of other Poisson processes. This indicates that if the trader x takes weak attitude then the trader y will take weak attitude if he takes neutral attitude. Similarly, we place red directed arrows at time points $\{T_{2,n}^{(x,y)}, n \ge 1\}$ from $\mathbb{R}_+ \times \{x\}$ to $\mathbb{R}_+ \times \{y\}$ in Fig. 1, according to a Poisson process with rate λ_2, independently of other Poisson processes, which indicates that if the trader x takes strong attitude then the trader y will take strong attitude whatever he takes. Along each line $\mathbb{R}_+ \times \{x\}$, we place a "$\delta$" at time points $\{U_n^x, n \ge 1\}$ according to a Poisson process with intensity 1 and independently of other Poisson processes. The effect of a δ is to recover the attitudes of the trader x to the neutral attitude. Finally, perform the statistics. In this dynamics, weak attitude investors acquire some information, which is represented by a random variable ξ_t of standard uniform distribution on $(-1, 1)$, similarly ζ_t (independent of ξ_t) is defined for strong attitude investors. The random variables ζ_t and ξ_t both have only one value at trading day t, which means all weak attitude investors acquire one type of information as well as all strong attitude investors. These investors send bullish, bearish or neutral signal to their finite-range neighbors depending on the value of their corresponding random variable. If the value of random variable is greater than 0, the signal is bullish, if it is equal to 0, the signal is neutral, if it is smaller than 0, the signal is bearish. Since strong attitude investors are more certain in their investment information, as it comes from the latest technique or sources, their decisions on trading positions are more aggressive.