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# Quasi-Dirac points in one-dimensional graphene superlattices

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ABSTRACT

Quasi-Dirac points (QDPs) with energy different from the traditional Dirac points (TDPs) have been found for the first time in one-dimensional graphene superlattices. The angular-averaged conductance reaches a minimum value at the QDPs, at which the Fano factor approaches 1/3. Surprisingly, the minimum conductance at these QDPs may be lower than that at the TDPs under certain conditions. This is remarkable as the minimum conductance attainable in graphene superlattices was believed to appear at TDPs.



## 1. Introduction

Graphene has attracted great interest in the scientific community since its realization in the laboratory [1,2], including its possible applications in spintronics [3]. Study has shown that the electrical transport in graphene is ballistic up to lengths ∼0.5 to 1 μm, which is consistent with results from quantum Hall measurements [4]. As the Dirac point is a very special point in graphene, the properties of this point, such as transport and the Fano factor (the ratio of the noise power to the mean current), have been studied extensively in both monolayer and bilayer graphene [5–10]. On the other hand, superlattices have been used in fields such as electronics [11,12], spintronics [13–15], and photonics [16–19]. Since the realization of graphene superlattices [20,21], the transport properties at the Dirac point in graphene superlattices have also been studied [22–27]. Some studies [28–30] have shown that 1D superlattice systems may be achieved experimentally by using electrodes that modulate the Dirac cone shift into a superlattice potential.

The Fano factor of graphene has been the focus of theoretical and experimental studies of graphene, since it is strongly related to a unique spectrum of excitations in the material, namely the two-dimensional relativistic spectrum of the Dirac point. One of the remarkable features of graphene is its finite minimum conductivity. This is not only attractive conceptually, but is also important for possible applications, such as ballistic field-effect transistors [4]. It has been found that the minimum conductivity of order $e^2/h$ at the Dirac point is associated with a maximum of the Fano factor [9]. Moreover, the Fano factor at the Dirac point is $F = 1/3$ for short and wide graphene strips. This is the same value as for a disordered metal. This result is remarkable because the classical dynamics of the Dirac fermions are ballistic. Interestingly, the Fano factor remains $F = 1/3$ at the Dirac point in bilayer graphene [10]. At the Dirac point of charge neutrality, the bilayer transmits as two independent monolayers in parallel. Both the current and noise are resonant at twice the monolayer value, so that their ratio, the Fano factor, has the same value as in monolayer graphene. For graphene superlattices, the Dirac point appears, at which the conductance is a minimum and the Fano factor reaches its maximum value of 1/3. These conclusions are the same as those for monolayer and bilayer graphene [9,10]. In other words, the conductance minimum with a Fano factor equal to 1/3 at the Dirac point is observed in monolayer, bilayer, and superlattice graphene. In addition, emergence of extra Dirac points in the 1D graphene superlattices has been proposed [31–33]. The energy of the extra Dirac point is the same with that of the original Dirac point. Since these Dirac points have been proposed, they are called TDPs in this study. For 2D graphene superlattices with triangular or hexagonal potential [22, 34], new Dirac point with energy different from the TDP has been proposed. One may expected this result since the geometry is similar to graphene.

Since the TDPs are very special points for graphene, it is interesting to search for other points with similar transport properties. Is it possible to find that the conductance minimum also occurs at a point other than the TDP? If the answer is yes, the concept of a minimum conductance at the TDP needs to be modified. What are the conditions required for this intriguing phenomenon? Although the conductance and Fano factor of graphene superlattices

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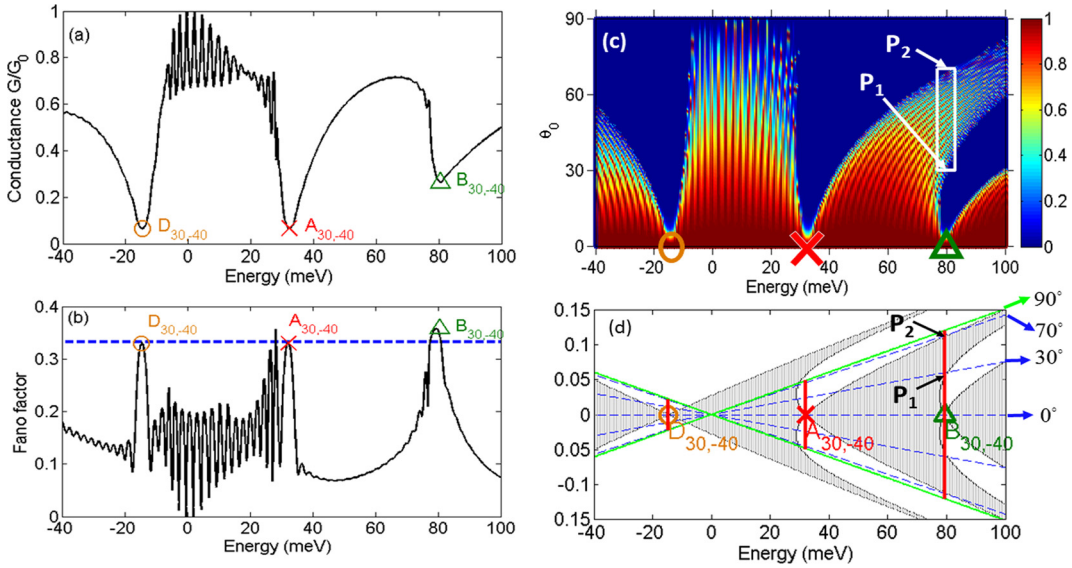


**Fig. 1.** The conductance and (b) Fano factor as a function of Fermi energy for the graphene superlattice $(AB)^{20}$. $D_{d_B,U_B}$, $A_{d_B,U_B}$, and $B_{d_B,U_B}$ denote the locations of the first, second, and third local conductance minima for specific $d_B$ (nm) and $U_B$ (meV). The other parameters are $d_A = 14$ nm, $U_A = 40$ meV. The location of the first local conductance minimum corresponds to the TDP. (c) The transmission spectrum and (d) the corresponding band structure for the graphene superlattice. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

have been studied, these questions have not yet been exploited. In this study, these questions will be investigated.

## 2. Model and formulation

Consider a periodic monolayer graphene superlattice structure $(AB)^n$, where $n$ is the number of cells, A and B are different barriers with barrier widths $d_A$ and $d_B$ and electrostatic potentials $U_A$ and $U_B$, respectively. It is assumed that there is no impurities or defect in graphene. Study has shown that the theory under the assumption of impurity-free transport still gives a qualitative agreement with experiment [35]. Therefore, the effect of this interaction on the unique transport property of QDP can be limited, even if the electrons near QDPs can interact with electrons in the bulk states via unavoidable impurity scattering. The graphene superlattice is placed in the $x$–$y$ plane with the growth direction along the $x$-axis. In the presence of a potential $U(x)$, the Hamiltonian [8,36] for carriers near the K point can be written as

$$\hat{H} = v_F \sigma \cdot \hat{p} + U(x)\hat{I}, \tag{1}$$

where $v_F \approx 10^6$ m/s is the Fermi velocity, $\sigma = (\sigma_x, \sigma_y)$ are the Pauli matrices, $\hat{p} = (p_x, p_y)$ are the momentum operators, and $\hat{I}$ is a $2 \times 2$ matrix. The system studied in this paper is homogeneous in the $y$ direction. The wave function inside the $j$th potential can be expressed as $\Psi_j = \left\{ a_j \begin{pmatrix} 1 \\ e^{i\theta_j} \end{pmatrix} e^{iq_{x,j}\cdot x} + b_j \begin{pmatrix} 1 \\ -e^{-i\theta_j} \end{pmatrix} e^{-iq_{x,j}\cdot x} \right\} e^{ik_y \cdot y}$, where $a_j$ and $b_j$ are the coefficients of incident and reflected waves, $\cos\theta_j = q_{x,j}/k_j$, $k_j = (E - U_j)/\hbar v_F$, $k_y$ is the $y$ component of the wave vector. $q_{x,j}$ is the $x$ component of the wave vector, which is $q_{x,j} = sign(k_j)\sqrt{k_j^2 - k_y^2}$ for $k_j^2 > k_y^2$ and $q_{x,j} = i\sqrt{k_y^2 - k_j^2}$ for $k_j^2 < k_y^2$. The transmission probability of the system is given by $T_m = 1 - |t_{21}/t_{22}|^2$, where $t_{ij}$ is an element of the total transfer matrix $T$. The total transfer matrix is expressed as $T = N_S^{-1}(\sum_j M_j N_j^{-1})N_C$, where the subscripts $S$ and $C$ denote the incident and exit region, $M_j = \begin{pmatrix} e^{iq_{x,j}\cdot d_j} & e^{-iq_{x,j}\cdot d_j} \\ e^{i(q_{x,j}\cdot d_j + \theta_j)} & -e^{-i(\theta_j + q_{x,j}\cdot d_j)} \end{pmatrix}$ and $N_j = \begin{pmatrix} 1 & 1 \\ e^{i\theta_j} & -e^{-i\theta_j} \end{pmatrix}$. The normalized conductance of the system at zero temperature can be written as $G/G_0 = \int_0^{\frac{\pi}{2}} T_m(E, k_y)\cos\theta_0 d\theta_0$. The Fano factor is given by $F = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} T_m(E, k_y)[1 - T_m(E, k_y)]\cos\theta_0 d\theta_0 / \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} T_m(E, k_y)\cos\theta_0 d\theta_0$. Based on the Bloch theorem, the band structure for a graphene superlattice with $n = \infty$ can be obtained as $\cos(KD) = (p_{11} + p_{22})/2$, where $p_{ij}$ is the element of the matrix $P = N_A^{-1}(M_A N_A^{-1} M_B N_B^{-1})N_A$, $D = d_A + d_B$ is the width of a unit cell and $K$ is the Bloch wave number.

## 3. Results and discussion

Fig. 1 shows the conductance, Fano factor, and transmission spectra for the graphene superlattice. The location of the TDP can be calculated as $E = (U_A + U_B)/2 + [d_A/(d_A + d_B) - 1/2](U_A - U_B)$, which is $E = -14.5$ meV in the present case. In Fig. 1(a), there are three local conductance minima denoted as $D_{d_B,U_B}$, $A_{d_B,U_B}$, and $B_{d_B,U_B}$. The conductance minimum at $D_{d_B,U_B}$ is caused by the TDP. The Fano factor at $D_{d_B,U_B}$ is approximately 1/3, as shown in Fig. 1(b). This is not a new discovery for this paper, since many papers have shown that the conductance minimum typically occurs at the TDP [9,10,22–26]. However, it is interesting to find that there is another conductance minimum denoted as $A_{30,-40}$, at which the conductance is approximately equal to the one at $D_{30,-40}$. Moreover, the Fano factor at $A_{30,-40}$ is the same as the one at $D_{30,-40}$. Since the latter is known to be the TDP, the former is thus named QDP due to its similarity to the TDP. The energy of the QDP is different from that of the TDP. In Fig. 5(a) of Ref. [24], although there is a conductance minimum at $E = 92$ meV, the Fano factor at this point is not 1/3. Therefore, this point is not a QDP. The QDP in Fig. 1 is achieved through a proper design of the parameters of graphene superlattices, such as the barrier width and barrier height. In addition, there is a special point denoted by $B_{30,-40}$.

However, the Fano factor at $B_{30,-40}$ is not 1/3. Therefore, this point is not a QDP. Fig. 1(c) shows the transmission spectrum of the graphene superlattice. At the energies corresponded to $D_{30,-40}$ and $A_{30,-40}$ in Fig. 1(a), a high transmission probability only occurs at small angles, leading to a conductance minimum. Although the transmission probability decreases to zero quickly as the angle increases from zero at the energy corresponding to $B_{30,-40}$, the transmission probability becomes non-zero again (denoted by the white rectangle). Therefore, the conductance at $B_{30,-40}$ is larger than those at $D_{30,-40}$ and $A_{30,-40}$. The interesting angular dependence of the transmission spectrum at $B_{30,-40}$ can be explained by