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Quantum critical points in tunneling junction of topological superconductor and topological insulator

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ABSTRACT

The tunneling junction between one-dimensional topological superconductor and integer (fractional) topological insulator (TI), realized via point contact, is investigated theoretically with bosonization technology and renormalization group methods. For the integer TI case, in a finite range of edge interaction parameter, there is a non-trivial stable fixed point which corresponds to the physical picture that the edge of TI breaks up into two sections at the junction, with one side coupling strongly to the Majorana fermion and exhibiting perfect Andreev reflection, while the other side decouples, exhibiting perfect normal reflection at low energies. This fixed point can be used as a signature of the Majorana fermion and tested by nowadays experiment techniques. For the fractional TI case, the universal low-energy transport properties are described by perfect normal reflection, perfect Andreev reflection, or perfect insulating fixed points dependent on the filling fraction and edge interaction parameter of fractional TI.

1. introduction

Recently, the study of topological superconductors which support Majorana fermion excitations has been a focus of theoretical and experimental studies in condensed matter physics [1–3]. Majorana fermions being their own anti-particles have exotic non-Abelian braiding statistics and great potential in the applications of fault-tolerant topological quantum computation [4]. There are many proposals which allow us to engineer topological superconductor (TSC), based on proximity coupling to s-wave superconductors. These include topological insulators [5,6], semiconductor quantum wires [7,8], and chains of magnetic adatoms [9–13]. Among these proposals, the most promising candidate for the experimental realization is the semiconductor quantum wires proposal [1,14]. The experimental evidences of Majorana fermions have been shown in spin–orbit coupled quantum wire model [15–17]. All other proposals are being actively pursued [18,19].

Because of these intrinsically fascinating of Majorana fermions, there are many interesting transport properties and critical points when TSC couples to other materials [20–33]. A junction between a TSC and a Fermi lead (or interacting lead) is predicted to exhibit perfect Andreev reflection at low energies [20,23]. Further, a novel type of quantum frustration and quantum critical points appear at low energies when one-dimensional (1D) TSC couples to two interacting leads or an interacting lead with two channels [23–25]. At this critical point, the perfect Andreev reflection occurs in one interacting lead (one channel) and perfect normal reflection in the other. The tunneling junction between a TSC with chiral Majorana liquid at the edge and a helical Luttinger liquid is studied [28], the main conclusion of which is that at low energies, the helical Luttinger liquids is cut into two separated half wires by backscattering potential and the tunneling between the Majorana liquid and the helical Luttinger liquid is forbidden. The perfect Andreev transmission (the reflected hole goes into a different lead from where the electron arrived) can occur when the edge of topological insulator (TI) contacts with a Kramers pair of Majorana fermions in TSC [33].

Usually, the quantum wires with electron–electron interaction are described by Luttinger liquids theory [34,35] and the low-energy physics of the tunneling junctions between TSC and interacting quantum wires are analyzed by renormalization group method [23–31]. These interacting quantum wires are topological trivial systems. In contrast, the interplay of the TSC and other topological matters may result in novel and interesting transport properties. Recently, we have studied the point contact tunneling junction between 1D TSC and single-channel quantum Hall (QH) liquids [36]. For the $\nu = 1$ integer QH liquid, the perfect Andreev reflection with quantized zero-bias tunneling conductance $2e^2/h$ is predicted to occur at zero temperature and voltage, which is

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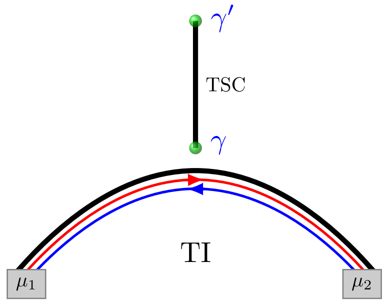

Fig. 1. Schematic illustration of the tunneling junction between TSC and TI. The edge of TI can be described in terms of two bosonic fields ϕ_α. The 1D TSC is characterized by the Majorana fermions γ and γ'.

caused by Majorana fermion tunneling not by the Cooper-pair tunneling. The quantized conductance can serve as a definitive fingerprint of a Majorana fermion. However, for the Laughlin fractional QH liquid cases, the universal low-energy transport is governed by the perfect normal reflection fixed point with vanishing zero-bias tunneling conductance.

The edges states of two-dimensional (2D) integer TI, known as helical liquid, are topologically protected by time-reversal symmetry. The localized Majorana modes emerge at interface of superconductor-ferromagnet junction on the edge of 2D TI [6,37]. The different geometries of TSC coupling with the edge of 2D TI have been investigated [28,32,33]. The fractional TI [38–43], which is the strongly interacting version of 2D TI, can be regarded as the generalization of the fractional QH liquids to time-reversal-invariant systems. The simplest case of a fractional TI consists of two decoupled copies of a Laughlin fractional QH states with opposite spin polarizations. The parafermions (fractionalizing Majorana fermions) can be obtained at the interface between a SC and a ferromagnet along the edge of fractional TI [44–47]. Due to these intriguing and exotic properties, it is of both theoretical and practical interest to investigate the transport properties of junction between the TSC and integer (fractional) TI.

The content of the paper is organized as follows. In Sec. 2, using bosonization technology and renormalization group methods, we firstly research the tunneling transport signatures of 1D TSC and integer TI. In a finite range of edge interaction parameter, the edge of TI breaks up into two sections at the junction, with one side having perfect Andreev reflection due to Majorana fermion tunneling, while the other side decouples, having perfect normal reflection. This physical picture of our setup can be tested by present experimental techniques. Next, we calculate the phase diagram of the fractional TI case. In Sec. 3, we make discussions and concluding remarks.

2. Theory and discussion

In this section, we consider the point contact tunneling junction of 1D TSC and filling fraction $\nu = 1/m$ (m is an odd integer) fractional TI, as shown in Fig. 1. When $m = 1$, the fractional TI degenerates into 2D topological insulator. Next, we will use TI to denote the integer and fractional TI, except where confusion might result from these abbreviations.

The 1D TSC is characterized by the two Majorana fermions γ and γ' at end points, which can be obtained by a spin–orbit coupled quantum wire subjected to a magnetic field and proximate to an s-wave superconductor [7,8]. We assume all the important energy scales are smaller than the superconducting energy gap and the 1D TSC is sufficiently long so that Majorana fermion γ' do not couple to electrons in the TI. The fractional TI we analyze consists of two coupled fractional QH states, in which electrons of spin up form a Laughlin fractional QH states with filling fraction $\nu_\uparrow = 1/m$ and electrons of spin down form a Laughlin fractional QH states with filling fraction $\nu_\downarrow = -1/m$. The edge states of the TI are helical Luttinger liquid and the top edge of TI connects leads μ_1 and μ_2. Here, we label the right and left sides of junction by $x > 0$ and $x < 0$ respectively, and assume that the two leads are infinitely far away.

The Hamiltonian of the tunneling junction can be expressed as

$$H = H_0 + H_T \tag{1}$$

where H_0 is the Hamiltonian of TI edge theory and H_T tunneling Hamiltonian.

Firstly, we discuss the edge theories (helical Luttinger liquids) of integer TI [48,49] and fractional TI [39,40,50]. Here, we express these theories within a unified framework. When $m = 1$, these reduce to integer TI case. The edges of TI can be described by two chiral bosonic quantum fields ϕ_α and the density operators are

$$\rho_\alpha = \frac{1}{2\pi}\partial_x \phi_\alpha \tag{2}$$

where $\alpha = R$ (right-mover with spin up), L (left-mover with spin down).

The boson fields ϕ_α satisfy the Kac–Moody commutation relations

$$\left[\phi_\alpha(x), \phi_\beta(x')\right] = (\sigma_z)_{\alpha\beta}\frac{i\pi}{m} sgn(x - x') \tag{3}$$

Because of the time-reversal symmetry, the Hamiltonian of the edge of the TI is

$$H_0 = \int dx \left[\pi m v_F\left(\rho_R^2 + \rho_L^2\right) + 2g_2\rho_R\rho_L + g_4\left(\rho_R^2 + \rho_L^2\right)\right] \tag{4}$$

where g_2 and g_4 are the amplitudes for dispersion and forward scattering processes.

To simplify our derivation, we introduce the fields

$$\varphi = \frac{1}{2}(\phi_R + \phi_L), \theta = \frac{1}{2}(\phi_R - \phi_L) \tag{5}$$

According to the theory of Luttinger liquids [34,35], we can express the Hamiltonian as

$$H_0 = \frac{mu}{2\pi}\int dx \left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\varphi)^2\right] \tag{6}$$

with

$$K = \sqrt{\frac{\pi m v_F + g_4 - g_2}{\pi m v_F + g_4 + g_2}}$$

$$u = \sqrt{\left(1 + \frac{g_4}{\pi m v_F}\right)^2 - \left(\frac{g_2}{\pi m v_F}\right)^2}$$

where $K < 1$ ($K > 1$) for repulsive (attractive) edge interaction, and $K = 1$ corresponds to a noninteracting edge. For the noninteracting edge, the fractional TI can be substituted by a simple electron–hole bilayer where the two layers are in a Laughlin fractional QH states with filling fraction $\nu = \pm 1/m$.

The electron creation operators can be expressed as

$$\Psi_\alpha^\dagger(x) = \Gamma_\alpha e^{im(\sigma_z)_{\alpha\alpha}\phi_\alpha} \tag{7}$$

with Γ_α the Klein factor that is used to ensure the correct anti-commutation relations between different fermion species and obey the following commutation relations

$$\Gamma_\alpha^\dagger = \Gamma_\alpha, \left\{\Gamma_\alpha, \Gamma_\beta\right\} = 2\delta_{\alpha\beta}, \left\{\Gamma_\alpha, \gamma_\beta\right\} = 0 \tag{8}$$

From the first relation above, we can view Klein factors as additional Majorana fermions, which is important for studying related