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Hydrodynamic study of edge spin-vortex excitations of fractional quantum Hall fluid

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ABSTRACT

We undertake a theoretical study of edge spin-vortex excitations in fractional quantum Hall fluid. This is done in view of quantised Euler hydrodynamics theory. The dispersions of true excitations for fractions within $0 \le \nu \le 1$ are simulated which exhibit universal similarities and differences in behaviour. The differences arise from different edge smoothness and spin (pseudo-spin) polarisations, in addition to spin-charge competition. In particular, tuning the spin-charge factor causes coherent spin flipping associated with partial and total polarisations of edge spin-vortices. This observation is tipped as an ideal mechanism for realisation of functional spintronic devices.

1. Introduction

Two-dimensional electron gas system submitted to strong magnetic field usually forms strongly correlated quantum Hall fluids. The fluids are often incompressible [1] and characterised by dissipationless superfluid flow [2] and formation of quantised vortices [3]. Integer and fractional quantum Hall states are examples of quantum Hall fluids (QHFs). The distinction arises from an integer or fractional factor connecting the number of formed quantised vortices to a magnetic flux number associated with the applied field. The fractional factors present richer physics content than its integer cousin. These include the braiding statistics [1] and the recently conjectured double boundary layer [4]. In the same vein, the edges of fractional QHFs, under small Zeeman splitting, support spin textures [5]. The case of a QHF with the accompanying properties in graphene will be particularly interesting. Chiefly due to graphene unique magnetotransport properties [6–8], unusual are THz generation and amplification [9] and observation of anomalous quantum Hall states at room temperature [10–12]. Another important feature missed by conventional two-dimensional semiconductor heterostructures, caused by edge reconstruction, is the unusual edge charge and spin propagation [5]. In graphene fluid, there is a parameter window in which the edge reconstruction can be avoided [5]. In practice, this seems to have paved the way for the experimental observations of charge and spin excitations at the edges of fractional quantum Hall liquids [5,13,14].

On the other hand, quantum interpretations of experimental measurements based on quasiparticle theory revealed that edge spin (spin-flip) excitations are more energetically favourable compared to edge charge (conserved-spin) excitations and even spin waves in the bulk [15]. Ability to tune such spin excitations is very crucial in spintronics, which is witnessing an increasing interest through coherent spin dynamical properties. For instance, dissipationless spin superfluid transport realised on the platform of spintronics device could be very attractive, in terms of large data storage and information processing. Bulk spin superfluid transport has been reported [14]. The case of spin-polarised fractional quantum Hall states has also been studied for spin reversed excitations [16,17]. However, the main hindering factor towards realising a practical system will be the energy cost. Edge spin transport should be able to overcome the obstacles.

In this work, we study collective excitations concerning spin-vortex dynamics localised at Hall fluid edges. The fluid boundaries are modelled as smooth edges to capture the true double boundary layer property of fractional quantum Hall liquids. This is done in view of quantised Euler hydrodynamics theory for vorticity. The hydrodynamic approach is motivated by the fact that QHF is a manifestation of microscopic properties on the macroscopic scale. Hydrodynamic modes at finite frequencies emerge at long wavelength limit due to slow motion of quasiparticles. It is also based

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on its simplicity and ability to properly capture quasiparticles interactions. The new term arising from such interactions, called anomalous term [18,19] has non-trivial consequence on transport properties, which is missed by microscopic theories [5,13]. The hydrodynamic theory has been used to correctly yield exact Laughlin states [18] and bulk charge excitations [19].

Microscopically, graphene has fourfold spin $(\uparrow, \downarrow)$ and pseudo-spin (K, K') degeneracy. Within the macroscopic picture, the system possesses four fluid components though only one momentum flux is measured. Each component belongs to the space $\{K, K'\} \otimes \{\uparrow, \downarrow\}$.

The remaining of the paper is organised as follows. In Section 2, a solution for vortex flow will be derived from quantised Euler hydrodynamic equation. We will calculate the edge density and edge spin excitations for filling factors within $0 \le \nu \le 1$ together. The universal similarities and differences in behaviour will be discussed in Section 3. We will conclude in Section 4 highlighting possible applications of our results.

2. Theoretical model

2.1. Euler hydrodynamics of charge vortices

The dynamics of incompressible vortex fluid is governed by Euler and continuity equations,

$$D_t \rho^\alpha = 0 \quad \text{and} \quad D_t \mathbf{u}^\alpha + \nabla \mathrm{p}^\alpha = 0. \tag{1}$$

There the material derivative $D_t \equiv \partial_t + \mathbf{u} \cdot \nabla$ and $\mathbf{u}$ is macroscopic fluid velocity connected with the microscopic electron velocity, $\mathbf{v} = v_F \mathbf{k}/k$. p is the partial pressure per density and α is the fluid component index (K↑, K↓, K′↑, K′↓). Taking the curl of Eq. (1), we get

$$D_t \omega^\alpha = 0. \tag{2}$$

There the vorticity $\omega = \nabla \times \mathbf{u}$. The continuity equation for the vortex, $D_t \rho_{\mathrm{v}}^\alpha = 0$ also holds. The solution of Eq. (2) has been obtained by Helmholtz and reported in [20]. In a disk geometry having image vortices, the solution consists of point-like vortices of the form

$$\mathrm{u}^\alpha = \mathrm{i} \sum \frac{\Gamma^\alpha}{z^\alpha - z_i^\alpha(t)} + c.c + b.t, \tag{3}$$

where $c.c$ is the complex conjugate, $z = x + \mathrm{i}y$ and $u = u_x - \mathrm{i}u_y$. $b.t$ is the boundary term which includes drift velocity arising from a confining potential at fluid boundary. We have assumed a flow in which the strength of circulation, Γ_i^α $(= \Gamma^\alpha)$ is both minimal and chiral such that in the thermodynamic limit, rotation can be compensated by the large number of vortices. This means, one can have Bohr–Sommerfeld phase-space quantisation of the circulations, $m_{\mathrm{v}} \Gamma^\alpha = 2\pi \beta^\alpha \hbar$. There m_{v} is the vortex inertia and β^α is an integer or fraction. The equation of motion for vortices is such that

$$\mathrm{v}_i^\alpha(t) = \mathrm{i} \sum_{i \neq j} \frac{\Gamma^\alpha}{z_i^\alpha(t) - z_j^\alpha(t)} + c.c + b.t. \tag{4}$$

Writing v in terms of u using the identities $\pi\delta(r) \equiv \bar{\partial}(1/z)$ and $\sum_{i \neq j}[2/(z - z_i)(z_i - z_j)] \equiv [\sum 1/(z - z_i)]^2 - \sum[1/(z - z_i)]^2$, it is straightforward to show that [18] $\mathrm{v} = \mathrm{u} + (\mathrm{i}\Gamma/2)\partial_z \log\rho_{\mathrm{v}}$. We can recast the expression as

$$\rho_{\mathrm{v}} \mathrm{v} = \rho_{\mathrm{v}} \mathrm{u} + \frac{\Lambda_\nu}{4} \Delta \mathrm{u} + \rho_{\mathrm{v}} \mathrm{v}_D, \tag{5}$$

using the commutation relation $[\mathrm{u}, \rho] = \mathrm{i}(\hbar/m_{\mathrm{v}})\nabla\rho$. There v_D $(= e\ell_B^2 \nabla V/\hbar)$ is the drift velocity of vortices due to the boundary terms and $\Lambda_\nu = 1 - 1/\beta + 2m_{max}$, with m_{max} the angular momentum of vortices localised near the boundary.

2.2. Gilbert–Landau–Lifshitz equation of spin vortices

The equation of motion for spin-vortices having a unit magnetisation, $\mathbf{m}$ is best described by the Gilbert–Landau–Lifshitz equation (GLLE) [21,22]

$$\bar{D}_t \mathbf{m}_i + (\gamma/m_0)\varepsilon_{ijk} \mathbf{m}_j \partial_{\mathbf{m}_k} \mathcal{H}_s + \varepsilon_{ijk} \mathbf{m}_j \tilde{D}_t \mathbf{m}_k = 0. \tag{6}$$

There the material derivatives are $\bar{D}_t = \partial_t + \mathbf{j} \cdot \nabla$ and $\tilde{D}_t = \alpha\partial_t + \beta\mathbf{j} \cdot \nabla$. $\mathbf{j}$ is the spin-vortex current and $\mathcal{H}_s$ is the spin Hamiltonian [21]. m_0 is the magnitude of the magnetic moment per area. α is the Gilbert damping constant, which quantifies the macroscopic response of spin angular momentum polarised along z-axis. It takes small values for temperatures far below room temperature [22]. γ is the gyromagnetic ratio and β is the non-adiabatic effect. The magnetisation is projected out-of-plane, for this reason one can map the in-plane coordinate $((r, \phi))$ onto a 3D unit sphere $(\Theta(r), \Phi(\phi))$. That is, $\mathbf{m} = (sin\Theta(r) sin\Phi(\phi), sin\Theta(r) cos\Phi(\phi), cos\Theta(r))$. We are concerned with the dynamics in the vicinity of an equilibrium fluid that may result in the deformation of the excitation spectrum. Equation (6) can be transformed into non-Newtonian force of interactions between vortices [22]

$$F_\mu = \varepsilon_{\mu\nu} N_0 (U_\nu - J_\nu) + \delta_{\mu\nu} D_0 (\alpha U_\nu - J_\nu), \tag{7}$$

governing the drift of the spin-vortices. U and J are the spin velocity and current, respectively. Assuming a vanishing dissipation dyadic, $D_0 = 0$, the number N_0 is expressed as

$$N_0 = m_0 \int \mathrm{d}^2 r \, [\nabla\Theta \times \nabla\Phi] \, sin\Theta. \tag{8}$$

Equation (8) is solved using the constraint $\Phi = \Gamma\phi + \pi/2$ together with the quantisation rule $[\Phi, \nabla\Theta] = \pm i\beta\hbar$. These yield $N_0 = \pm 2\hbar\beta(m_0 + \nabla\rho_{\mathrm{v}})$. Assuming for the moment that $J << U$, one obtains the spin velocity in a plane $U = F/N_0$ or in terms of energy of vortex–vortex interactions [22], $W = -2\pi(m_0\beta^2\hbar^2 J_s/\gamma) \times \sum_i \log|z - z_i|$.

$$U = \pm \frac{m_0 \beta J_s}{\gamma(m_0 \pm \nabla\rho_{\mathrm{v}})} \mathrm{u}. \tag{9}$$

For the intrinsic spin degree of freedom, one can redefine the charge and spin densities as $2\rho_c = \rho_\uparrow + \rho_\downarrow$, $2\rho_s = \rho_\uparrow - \rho_\downarrow$ and velocities as $\mathrm{u} = \mathrm{u}_\uparrow + \mathrm{u}_\downarrow$, $\mathrm{U} = \mathrm{u}_\uparrow - \mathrm{u}_\downarrow$. Equation (5) is transformed into the velocity field

$$\mathrm{v} = \mathrm{u} + \frac{\Lambda_\nu}{4(\rho_c^2 - \rho_s^2)} (\rho_c \Delta \mathrm{u} - \rho_s \Delta \mathrm{U}) + \mathrm{v}_{D,c}, \tag{10}$$

which can be simplified further in terms of u using Eq. (9).

2.3. Edge velocity and model of boundary term

In real space and at the boundary, Eq. (3) assumes the form

$$\mathrm{u}(r)\Big|_{\partial\Omega} = -\sum_i \frac{g\Gamma(r - r_i)}{|r - r_i|^2} + \sum_i \frac{g\Gamma(r - r_i')}{|r - r_i'|^2}$$

or in momentum space

$$\mathrm{u}(q)\Big|_{\partial\Omega} = -\sum_i g\Gamma \left(\frac{r^2 - r_i^2}{2\pi r}\right) \int_{r=R} \mathrm{d}^2 r \frac{e^{\mathrm{i}qr cos\theta}}{|r - r_i|^2}.$$

Following standard elementary calculations, this simplifies to give

$$\mathrm{u}(q)\Big|_{\partial\Omega} = -\sum_i g\Gamma \frac{\varepsilon_\ell \Gamma}{R} \left(\frac{r_i}{R}\right)^{|\ell|} \mathrm{i}^\ell J_\ell(qR) e^{i\ell\theta_q}. \tag{11}$$