Physics Letters A 380 (2016) 2582–2587

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Electromagnetic wave propagation in time-dependent media with antisymmetric magnetoelectric coupling

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A R T I C L E   I N F O

Article history:
Received 1 February 2016
Received in revised form 24 May 2016
Accepted 24 May 2016
Available online 30 May 2016
Communicated by V.A. Markel

Keywords:
Time-dependent media
Magnetoelectric effect
Linear birefringence
Moving media
Toroidal moment

A B S T R A C T

This paper deals with electromagnetic wave propagation in time-dependent media with an antisymmetric magnetoelectric coupling and an isotropic time-dependent permittivity. We identify a new mechanism of linear birefringence, originated from the combined action of the time-dependent permittivity and the antisymmetric magnetoelectric coupling. Permittivity with linear and exponential temporal variations exemplifies the creation and control of these two distinct types of linear birefringent modes. As a novel nonlinear optical effect, a scheme utilizing optical Kerr effect in moving media is proposed for the realization of the predicted birefringence.

1. Introduction

Wave propagation in time-dependent media is an intriguing topic. Once introducing time-varying parameters, dynamical properties of macroscopic Maxwell equations dramatically change [1,2], which deepens our understanding of interactions between light and matter. Indeed, it has been shown that the existence of time-varying parameters of the media can give rise to extraordinary effects such as spectral shifts of radiation [2], velocity modulation of electromagnetic waves [3], time refraction and reflection at the time discontinuous interface [4], frequency-periodic band structure and wave-vector forbidden band gaps in temporal photonic crystal [5], and various particular quantum effects in time-dependent media [6–9]. More specially, it has been demonstrated that the time-dependence of bi-isotropic magnetoelectric coupling will lead to time circular birefringence and time Faraday effect [10].

The previous work on time-dependent media mainly focused on the linear isotropic or bi-isotropic cases. Media with time-dependent and anisotropic parameters, however, have not been fully accounted for. Here, we investigate electromagnetic waves in a particular type of time-dependent magnetoelectric media, which is described by an isotropic time-dependent permittivity, a constant isotropic permeability, and a nonreciprocal antisymmetric magnetoelectric coupling. The nonreciprocal antisymmetric magnetoelectric coupling can be achieved in the moving media [11,12], or through the toroidal moment in toroidalized materials [11,13], such as the polar ferrimagnetic phase of $Ga_{2-x}Fe_xO_3$ [14,15], the spin-flop phase of antiferromagnetic Cr_2O_3 [16], and the magnetic field-induced weakly ferromagnetic phase of antiferromagnetic-incommensurate and ferroelectric $BiFeO_3$ [17]. On the other hand, it may be fabricated in artificial materials including nonreciprocal magnetoelectric coupling response. Such artificial materials have been investigated extensively theoretically [18–23]. Besides, as a practical realization, static and periodically loaded transmission lines have been recognized to emulate moving media [24]. By solving macroscopic Maxwell equations in this time-dependent system, we find some remarkable properties, especially a novel linear birefringent effect, caused by the combined action of the time-dependent permittivity and the magnetoelectric coupling term.

In the following section, we begin with our proposed constitutive relations and obtain the explicit equations governing the dynamics of the electromagnetic waves in such time-dependent media. In Sec. 3, we briefly review the dispersion relation in the corresponding time-independent media and find that there is no birefringence in this case despite the anisotropic toroidal coupling. In Sec. 4, we proceed to time-dependent case and discover an extraordinary linear birefringent phenomenon. Sec. 5 is devoted to the simulations of these birefringent effects. Two specific cases exemplify the creation and control of these two distinct types of linearly polarized modes. In Sec. 6, we show that this type of time-

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http://dx.doi.org/10.1016/j.physleta.2016.05.050

dependent magnetoelectric media can be realized through moving media and optical Kerr effect.

2. Electrodynamics of time-dependent media

The constitutive relations hide all physics of media in macroscopic electromagnetics. Here, we consider one kind of magnetoelectric constitutive relations

$$\mathbf{D}=\epsilon(t)\mathbf{E}+\hat{\alpha}\mathbf{H}, \quad \mathbf{B}=\mu\mathbf{H}+\hat{\alpha}^T\mathbf{E}, \tag{1}$$

where $\epsilon(t)$ and μ are time-dependent permittivity and magnetic permeability respectively, and

$$\hat{\alpha}=\mathbf{g}\times\hat{I}=\begin{pmatrix} 0 & -g_3 & g_2 \\ g_3 & 0 & -g_1 \\ -g_2 & g_1 & 0 \end{pmatrix}.$$

The parameter $\mathbf{g}$ is the toroidal moment and $\hat{\alpha}$ denotes the antisymmetric magnetoelectric coupling tensor with $\hat{\alpha}^T$ representing its transpose. All the parameters are required to be real such that the medium is nondissipative. In writing the constitutive relations (1), we have assumed instantaneous response for the frequency range of interest, and hence, the medium is nondispersive. Noteworthily, the time dependence of $\epsilon(t)$ indicates the effective macroscopic property of the media changing with time. If ϵ is time-independent, Eqs. (1) may be conceived as effective constitutive relations of moving media. Since $\hat{\alpha}=-\hat{\alpha}^T$, according to Lorentz reciprocity theorem, this kind of medium is nonreciprocal [25].

The electrodynamics of the time-dependent media starts from macroscopic Maxwell equations [26]:

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t}, \quad \nabla\cdot\mathbf{D}=0, \quad \nabla\cdot\mathbf{B}=0. \tag{2}$$

We assume no free charge and free current inside the media.

For convenience, we restructure the constitutive relations into

$$\mathbf{E}=\hat{\varepsilon}\mathbf{D}+\hat{\chi}\mathbf{B}, \quad \mathbf{H}=\hat{\upsilon}\mathbf{B}+\hat{\chi}^T\mathbf{D}, \tag{3}$$

where $\hat{\varepsilon}=\left(\epsilon-\frac{1}{\mu}\hat{\alpha}\hat{\alpha}^T\right)^{-1}$, $\hat{\upsilon}=\left(\mu-\frac{1}{\epsilon}\hat{\alpha}^T\hat{\alpha}\right)^{-1}$, $\hat{\chi}=-\frac{1}{\mu}\hat{\varepsilon}\hat{\alpha}$, and $\hat{\chi}^T=-\frac{1}{\epsilon}\hat{\upsilon}\hat{\alpha}^T$, with superscript -1 representing inverse of a matrix. By substituting Eq. (3) into the first two equations of (2), we have

$$\nabla\times(\hat{\varepsilon}\mathbf{D}+\hat{\chi}\mathbf{B})=-\frac{\partial\mathbf{B}}{\partial t}, \tag{4}$$

and

$$\nabla\times(\hat{\upsilon}\mathbf{B}+\hat{\chi}^T\mathbf{D})=\frac{\partial\mathbf{D}}{\partial t}. \tag{5}$$

For the time-dependent media, we can seek for a class of solutions $\mathbf{D}(\mathbf{r},t)=\mathbf{D}(t)e^{i\mathbf{k}\cdot\mathbf{r}}$ with a constant wave vector $\mathbf{k}$. We deal with the problem in Cartesian coordinates and set $\mathbf{k}=k\hat{\mathbf{z}}$. With the ansatz $\mathbf{D}(t,k)=\mathbf{D}(t)e^{ikz}$, Maxwell equations imply that $D_z=0$ and $B_z=0$. Therefore, the Eqs. (4) and (5) transform to

$$\begin{aligned}&\hat{\mathbf{x}}:\ ik(\epsilon\mu-g_2^2)D_y-ikg_1g_2D_x-ik\epsilon g_3B_x=\\&\epsilon(\epsilon\mu-g^2)B_x',\end{aligned} \tag{6}$$

$$\begin{aligned}&\hat{\mathbf{y}}:\ -ik(\epsilon\mu-g_1^2)D_x+ikg_1g_2D_y-ik\epsilon g_3B_y=\\&\epsilon(\epsilon\mu-g^2)B_y',\end{aligned} \tag{7}$$

and

$$\begin{aligned}&\hat{\mathbf{x}}:\ ik(\epsilon\mu-g_2^2)B_y-ikg_1g_2B_x+ik\mu g_3D_x=\\&-\mu(\epsilon\mu-g^2)D_x',\end{aligned} \tag{8}$$

$$\begin{aligned}&\hat{\mathbf{y}}:\ -ik(\epsilon\mu-g_1^2)B_x+ikg_1g_2B_y+ik\mu g_3D_y=\\&-\mu(\epsilon\mu-g^2)D_y'.\end{aligned} \tag{9}$$

Here, $g^2=g_1^2+g_2^2+g_3^2$, and the primed letters denote time derivatives. The Eqs. (6)–(9) are coupled linear ordinary differential equations of $B_x(t)$, $B_y(t)$, $D_x(t)$ and $D_y(t)$ with variable coefficients. Obviously, if $\hat{\alpha}=0$, i.e., $g_1=0$, $g_2=0$ and $g_3=0$, then these equations will reduce to the cases of trivial isotropic media. For time-dependent magnetoelectric media, however, new effects will occur.

3. Time-independent case

Before discussing the time-dependent magnetoelectric media, let us first consider the time-independent case as a comparison:

$$\mathbf{D}=\epsilon\mathbf{E}+\hat{\alpha}\mathbf{H}, \quad \mathbf{B}=\mu\mathbf{H}+\hat{\alpha}^T\mathbf{E}. \tag{10}$$

For such media, the plane wave solutions of the type $e^{i\mathbf{k}\cdot\mathbf{r}-\omega t}$ are admissible, then the Eqs. (4) and (5) reduce to

$$i\mathbf{k}\times(\hat{\varepsilon}\mathbf{D}+\hat{\chi}\mathbf{B})=i\omega\mathbf{B}, \quad i\mathbf{k}\times(\hat{\upsilon}\mathbf{B}+\hat{\chi}^T\mathbf{D})=-i\omega\mathbf{D}. \tag{11}$$

It is convenient to introduce the following antisymmetric matrix

$$\hat{k}\equiv\mathbf{k}\times\hat{I}=\begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix}.$$

Hence, we have

$$(\omega\hat{I}-\hat{k}\hat{\chi})\mathbf{B}=\hat{k}\hat{\varepsilon}\mathbf{D}, \quad (-\omega\hat{I}-\hat{k}\hat{\chi}^T)\mathbf{D}=\hat{k}\hat{\upsilon}\mathbf{B}. \tag{12}$$

Immediately, eigenequations for $\mathbf{D}$ can be written as

$$\left[(-\omega\hat{I}-\hat{k}\hat{\chi}^T)-\hat{k}\hat{\upsilon}(\omega\hat{I}-\hat{k}\hat{\chi})^{-1}\hat{k}\hat{\varepsilon}\right]\mathbf{D}=0. \tag{13}$$

The nontrivial solutions require the following characteristic equation:

$$\left|(-\omega\hat{I}-\hat{k}\hat{\chi}^T)-\hat{k}\hat{\upsilon}(\omega\hat{I}-\hat{k}\hat{\chi})^{-1}\hat{k}\hat{\varepsilon}\right|=0. \tag{14}$$

The solutions to the equations given above are

$$k_\pm=-\omega\left(g_3\pm\sqrt{\epsilon\mu-g_1^2-g_2^2}\right). \tag{15}$$

For a physical conclusion, only positive values of the solutions are acceptable. Hence, provided that $g^2<\epsilon\mu$, there will be only one eigenmode for the electromagnetic wave propagating in the time-independent magnetoelectric media.

4. Time-dependent case

Let's consider now the time-dependent case. From Eq. (15), it is easily recognized that g_1 and g_2 play the same role in the wave propagation when the wave vector is parallel to z direction. Consequently, we can always choose an appropriate coordinate frame such that $g_2=0$, then Eqs. (6)-(9) yield

$$\hat{\mathbf{x}}:\ ik\mu D_y-ik\epsilon g_3B_x=(\epsilon\mu-g_1^2-g_3^2)B_x', \tag{16}$$

$$\hat{\mathbf{y}}:\ -ik(\epsilon\mu-g_1^2)D_x-ik\epsilon g_3B_y=\epsilon(\epsilon\mu-g_1^2-g_3^2)B_y', \tag{17}$$

and

$$\hat{\mathbf{x}}:\ -ik\epsilon B_y-ik\mu g_3D_x=(\epsilon\mu-g_1^2-g_3^2)D_x', \tag{18}$$

$$\hat{\mathbf{y}}:\ ik(\epsilon\mu-g_1^2)B_x-ik\mu g_3D_y=\mu(\epsilon\mu-g_1^2-g_3^2)D_y'. \tag{19}$$