Physics Letters A 380 (2016) 2136–2141

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Rational solitary wave and rogue wave solutions in coupled defocusing Hirota equation

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ARTICLE INFO

Article history:
Received 16 November 2015
Received in revised form 22 March 2016
Accepted 19 April 2016
Available online 22 April 2016
Communicated by A.P. Fordy

Keywords:
Rational solitary wave
Rogue wave
Coupled defocusing Hirota equation
Modulational instability

ABSTRACT

We derive and study a general rational solution of a coupled defocusing Hirota equation which can be used to describe evolution of light in a two-mode fiber with defocusing Kerr effect and some certain high-order effects. We find some new excitation patterns in the model, such as M-shaped soliton, W-shaped soliton, anti-eye-shaped rogue wave and four-petaled flower rogue wave. The results are compared with the solutions obtained in other coupled systems like vector nonlinear Schrödinger equation, coupled focusing Hirota and Sasa–Satsuma equations. We explain the new characters by modulational instability properties. This further indicates that rational solution does not necessarily correspond to rogue wave excitation dynamics and the quantitative relation between nonlinear excitations and modulational instability should exist.

1. Introduction

Rational solutions of nonlinear Schrödinger equation (NLSE) have been paid much attention, since rational solution of NLSE can be used to describe rogue wave (RW) dynamics in many different physical systems [1–7]. Recently, universal properties for RW arising in nonlinear models were discussed [8–10]. It is demonstrated that RWs exist in unstable systems, but not all unstable systems allow the existence of RWs. The link between RW and baseband modulational instability (MI) was firstly uncovered in [8]. It was proved theoretically that RW came from MI under "resonance" perturbations for which both dominant frequency and propagation constant are equal to the ones of plane wave background [9]. Baseband MI was suggested to be seen as the origin of RW [10]. Those results provide some useful tools to analyze whether RW can exist in a nonlinear system. Furthermore, it has been demonstrated that quantitative relation between nonlinear excitations and MI can be clarified based on dominant perturbation frequency or wave vector [9].

Among these different nonlinear systems, optical fiber plays an important role in experimental observations for its well-developed intensity and phase modulation techniques. The experimental studies in nonlinear fiber showed that the simplified NLSE can well describe the dynamics of localized waves, which only contains the group velocity dispersion (GVD) and its counterpart, namely, self-phase modulation (SPM). But for ultrashort pulses whose duration is shorter than 100 fs, which is tempting and desirable to improve the capacity of high-bit-rate transmission systems, the nonlinear susceptibility will produce higher-order nonlinear effects like the Kerr dispersion (i.e., self-steepening) and the delayed nonlinear response except for SPM, and even the third-order dispersion (TOD). These are the most general terms that have to be taken into account when extending the applicability of the NLSE [11–13]. Many efforts have been made to study rational solution of Sasa–Satsuma equation [14–17], and Hirota equation [18,19]. Based on the linearized stability analysis of Sasa–Satsuma equation, rational W-shaped soliton was presented on modulational stability (MS) regime [16]. A rational solution was obtained on critical boundary lines between MI and MS regimes, which described an autonomous transition from MI to MS regime [17]. The results suggested that not all rational solutions of nonlinear partial equation corresponded to RW dynamics. This was also demonstrated in the Hirota equation [19]. As done in coupled NLSE [20–24], recent studies were extended to coupled Sasa–Satsuma model [25] and coupled Hirota (CH) equation [26–28]. However, the coupled models are just considered as focusing case to derive rational solution and study on their dynamics. Less attention has been paid on the coupled model with defocusing case. Recently, RW of coupled defocusing NLSE was discussed [8]. Therefore, we intend to study coupled defocusing model with high-order effects.

In this paper, we study on rational solution of a CH with defocusing effects which can be used to describe evolution of light in a two-mode defocusing fiber with some certain high-order effects. We find there are mainly four types of nonlinear localized waves for the CH model, M-shaped soliton, W-shaped soliton, anti-eye-

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http://dx.doi.org/10.1016/j.physleta.2016.04.033

shaped rogue wave and four-petaled flower rogue wave. The transition between them can be realized by varying the background frequencies. The superposition of localized waves in the two modes is a dark rogue wave or M shaped soliton here, which is different from the ones in coupled CH with focusing effects. We explain the characters by MI properties, based on that the quantitative relation between nonlinear excitations and MI should exist [9]. This further indicates that rational solution does not necessarily correspond to RW excitation dynamics.

2. Rational solutions for the coupled defocusing Hirota equations

We consider ultrashort pulses propagation in the femtosecond regime for a two-mode nonlinear fiber, where the higher-order effects, such as third-order dispersion, self-steepening, and delayed nonlinear response must be taken into account. In this case, the characteristics of the vector optical rogue waves can be described by the completely integrable CH model [29], which involves the higher-order perturbation effects above. In dimensionless form, the CH model reads:

$$\begin{aligned} &\mathrm{i}q_{1,t}+\alpha(q_{1,xx}-2(|q_1|^2+|q_2|^2)q_1)\\ &\quad+\mathrm{i}\beta\left[q_{1,xxx}-(6|q_1|^2+3|q_2|^2)q_{1,x}-3q_1q_2^*q_{2,x}\right]=0,\\ &\mathrm{i}q_{2,t}+\alpha(q_{2,xx}-2(|q_1|^2+|q_2|^2)q_2)\\ &\quad+\mathrm{i}\beta\left[q_{2,xxx}-(3|q_1|^2+6|q_2|^2)q_{2,x}-3q_2q_1^*q_{1,x}\right]=0. \end{aligned} \tag{1}$$

Here, an arbitrary real parameter β scales the integrable perturbations of the NLS equation. When $\beta=0$, Eq. (1) and (2) reduces to the standard coupled NLS equations which have only the terms describing lowest order dispersion and self-phase modulation. It should be pointed that the nonlinearity coefficients here are defocusing type, for which the rational solution has not been studied before, in contrast to the ones of CH with focusing effects [26–28]. By exploiting the standard Darboux transformation procedure, we obtain the fundamental (first-order) rational solutions of Eqs. (1),

$$\begin{aligned} q_1&=a_1\left[1+\frac{1}{(\xi_1+e_2)^2+\xi_2^2}\frac{2\mathrm{i}\,[(\xi_1+e_2)X_2+\xi_2X_1]-1}{X_1^2+X_2^2+\frac{1}{4\xi_2^2}}\right]\mathrm{e}^{\mathrm{i}\theta_1},\\ q_2&=a_2\left[1+\frac{1}{(\xi_1-e_2)^2+\xi_2^2}\frac{2\mathrm{i}\,[(\xi_1-e_2)X_2+\xi_2X_1]-1}{X_1^2+X_2^2+\frac{1}{4\xi_2^2}}\right]\mathrm{e}^{\mathrm{i}\theta_2} \end{aligned} \tag{2}$$

where

$$\begin{aligned} \theta_1&=(e_1+e_2)x-\Big[(2a_1^2+2a_2^2+(e_1+e_2)^2)\alpha\\ &\quad+(6a_1^2(e_1+e_2)+6a_2^2e_1+(e_1+e_2)^3)\beta\Big]t,\\ \theta_2&=(e_1-e_2)x-\Big[(2a_1^2+2a_2^2+(e_1-e_2)^2)\alpha\\ &\quad+(6a_2^2(e_1-e_2)+6a_1^2e_1+(e_1-e_2)^3)\beta\Big]t,\\ X_1&=-2\,[\alpha\xi_2+\beta(3e_1\xi_2-2\kappa_2\xi_1-2\kappa_1\xi_2)]\,t,\\ X_2&=x+\Big[2\alpha(\xi_1-e_1)+\beta(-2(a_1^2+a_2^2)-3e_1^2+6e_1\xi_1-e_2^2\\ &\quad+4(\kappa_2\xi_2-\kappa_1\xi_1))\Big]t, \end{aligned}$$

a_1, a_2 and e_2 are real nonzero parameters which determine the background amplitudes and relative frequency respectively. They satisfy the following condition (5) for the rational solution. e_1 is a free real parameter which determines the background frequencies of the two modes. $\kappa=\kappa_1+\mathrm{i}\kappa_2$, $\xi=\xi_1+\mathrm{i}\xi_2$ ($\kappa_1,\kappa_2,\xi_1,\xi_2\in\mathbf{R}$) can be obtained by solving the following cubic equation with respect to the parameter ξ (3) and quantic equation with respect to the parameter κ (4),

$$\xi^3-2\kappa\xi^2+(a_1^2+a_2^2-e_2^2)\xi+2e_2^2\kappa-a_1^2e_2+a_2^2e_2=0, \tag{3}$$

and

$$\begin{aligned} &64e_2^2\kappa^4-32e_2(a_1^2-a_2^2)\kappa^3\\ &-\left[4(-(a_1^2+a_2^2)^2+20e_2^2(a_1^2+a_2^2)+8e_2^4)\right]\kappa^2\\ &+36e_2(a_1^2-a_2^2)(a_1^2+a_2^2+2e_2^2)\kappa-4(a_1^2+a_2^2-e_2^2)^3\\ &-27e_2^2(a_1^2-a_2^2)^2=0. \end{aligned} \tag{4}$$

The discriminant for the quantic equation (4) is

$$\Delta=-4194304a_1^2a_2^2e_2^2[(a_1^2+a_2^2-4e_2^2)^3+27a_1^2a_2^2(4e_2^2)]^3.$$

The existence condition for the rational solution (2) is $\Delta<0$ i.e.

$$0<e_2^2<\frac{1}{4}(a_1^{2/3}+a_2^{2/3})^3. \tag{5}$$

Under the condition (5), we firstly solve the discriminant (4) to determine κ, and then solve the cubic equation (3) to determine the double root ξ. In principle, rational solution (2) can be determined uniquely. However, it is not possible to solve the above quintic equation (4) and cubic equation (3) in direct way. Even though we could obtain the analytical solution expressions through the Kadan formula, the expressions are so complex that we could not do any analysis on the properties of localized waves.

To solve this problem, we use the analysis method provided in Ref. [30]. We solve the quintic equation (4) and cubic equation (3) about a_1^2 and a_2^2. It follows that we can obtain two groups of solutions. Using one of them, to let the solutions a_1^2 and a_2^2 are valid, we must set $\mathrm{Im}(a_1^2)=\mathrm{Im}(a_2^2)=0$ and $\mathrm{Re}(a_1^2)>0, \mathrm{Re}(a_2^2)>0$. Then we introduce the following parameters transformation:

$$\begin{aligned} &\xi_1=e_2r\cos(\theta),\ \xi_2=-e_2r\sin(\theta),\\ &\kappa_1=\frac{e_2r\left(2\cos^2(\theta)r^2+r^2-3\right)\cos(\theta)}{2(r^2-1)},\ \kappa_2=\frac{e_2r^3\sin^3(\theta)}{1-r^2},\\ &a_1=\pm\sqrt{\frac{1-\cos(\theta)r}{2-2r^2}}e_2\left(r^2+2\cos(\theta)r+1\right),\\ &a_2=\pm\sqrt{\frac{\cos(\theta)r+1}{2-2r^2}}e_2\left(r^2-2\cos(\theta)r+1\right), \end{aligned} \tag{6}$$

where $0<\theta<\pi$ and $0<r<1$. Then we can obtain the expressions of X_1 and X_2 explicitly

$$\begin{aligned} X_1&=-2\,(3\beta e_2r\cos(\theta)-3\beta e_1-\alpha)\,e_2r\sin(\theta)t,\\ X_2&=x+\Bigg[2\,(e_2r\cos(\theta)-e_1)\,\alpha\\ &\quad+\Bigg(\left(6r^2-18\cos^2(\theta)r^2-12\cos^2(\theta)+9\right)e_2^2\\ &\quad+6e_1e_2r\cos(\theta)-3e_1^2-12\,\frac{e_2^2\sin^2(\theta)}{1-r^2}\Bigg)\beta\Bigg]t. \end{aligned}$$

The rational solution above can be used to investigate the dynamics of rational nonlinear localized waves which exist in the two-mode fiber with the certain high-order effects. We find that there are mainly three types of fundamental nonlinear localized waves: W-shaped soliton, M-shaped soliton, and RW. The W-shaped soliton has been found in a focusing Sasa–Satsuma model