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Expansion of lower-frequency locally resonant band gaps using a double-sided stubbed composite phononic crystals plate with composite stubs

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ABSTRACT

We studied the expansion of locally resonant complete band gaps in two-dimensional phononic crystals (PCs) using a double-sided stubbed composite PC plate with composite stubs. Results show that the introduction of the proposed structure gives rise to a significant expansion of the relative bandwidth by a factor of 1.5 and decreases the opening location of the first complete band gap by a factor of 3 compared to the classic double-sided stubbed PC plate with composite stubs. Furthermore, more band gaps appear in the lower-frequency range (0.006). These phenomena can be attributed to the strong coupling between the "analogous rigid mode" of the stub and the anti-symmetric Lamb modes of the plate. The "analogous rigid mode" of the stub is produced by strengthening the localized resonance effect of the composite plates through the double-sided stubs, and is further strengthened through the introduction of composite stubs. The "analogous rigid mode" of the stubs expands the out-of-plane band gap, which overlaps with in-plane band gap in the lower-frequency range. As a result, the complete band gap is expanded and more complete band gaps appear.

1. Introduction

In the last two decades, acoustic wave propagation in periodic composite materials—known as phononic crystals (PCs)—has attracted much attention due to its unique physical properties. For example, phononic band gaps exist where acoustic wave propagation is forbidden [1–7]. There are two mechanisms that lead to the formation of band gaps, known as Bragg scattering and local resonance. Band gaps are generated by Bragg scattering and their wavelengths are the same order as the periodic structure. The widths and positions of Bragg band gaps depend heavily on the difference between the elastic parameters of the scattering and the host material, as well as the geometric parameters and the shape of inclusions [8–11]. A large lattice constant is needed to obtain low frequency band gaps, which hinders the application of PCs in the audible frequency range. On the other hand, the associated wavelength of band gaps generated by local resonance—which was first proposed by Liu at el. [12]—is two orders of magnitude smaller than Bragg band gaps. A resonant band gap is related to the resonance frequency associated with scattering units, and depends less on the period and symmetry of the structure. This breaks the Bragg band gap limit and allows for low-frequency band gaps. As a result of this work, there has been much research on formation mechanisms of locally resonant band gaps [13–29]. However, little attention has focused on the expansion of these band gaps into the low-frequency range. Wu et al. [30] demonstrated that locally resonant band gaps can appear in two-dimensional PC plates composed of a square array of stubs on one side of a homogeneous plate. They demonstrated that low-frequency band gaps can be obtained when stub height was roughly three times the plate thickness. Based on their work, many studies have concentrated on the expansion of locally resonant band gaps using PCs based on a similar geometry. Bilalet al. [31] studied the band structure of a PCs formed by a periodic array of holes on one side of a stubbed plate. They demonstrated that a significant expansion of band gaps was obtained due to the trampoline effect. Li et al. [32] combined those ideas and proposed an original acoustic metamaterial plate composed of a square array of stubs on one side of a 2D locally resonant PC plate. The results indicated that the combination of these structures lead to a significant ex-

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pansion of the relative bandwidth in comparison to the classic one-sided stubbed plate (by a factor of 3). Nevertheless, all these studies focused on structures composed of a square array of stubs on one side of a finite thickness plate (either a homogeneous plate or a two-dimensional binary localized resonant PC plate). Assouar et al. [33] studied locally resonant band gaps in a double-sided stubbed PC plate composed of a square array of composite stubs on two sides of a homogeneous plate. In that study, the bandwidth was increased by a factor of 2 compared to the classic one sided stubbed plates. Based on their study, Zhao et al. [34] investigated the flexural vibration band gaps of this PC plate, and found that the bandwidth of the flexural vibration band gaps was obtained.

In this paper, we present an original structure composed of a square array of composite stubs on both sides of a composite plate. In this manner, the width and location limitations of the local resonance band gap can be overcome. A theoretical analysis and a numerical computation are used to investigate the physical behavior of the proposed PC plate, and to compare the behavior to the classic double-sided stubbed PC plates with composite stubs. We demonstrate that more band gaps appear, and that the relative bandwidth is expanded by a factor of 1.5. Furthermore, the opening location of the first band gap is reduced by a factor of 3.

2. Numerical results and discussion

The unit cell of the proposed structure is displayed in Fig. 1(c); it is formed by depositing composite stubs on both sides of a composite plate. The composite plate is fabricated by filling the drilled holes of a finite thickness epoxy plate with rubber filler, as shown in Fig. 1(a). The classic double-sided stubbed PC plate proposed by Assouar et al. [33] is shown in Fig. 1(b), and is composed of composite stubs deposited on a homogeneous plate. D, e, and a are the rubber filler diameter, the epoxy plate thickness, and the lattice constant, respectively. The height and diameter of the stub are denoted by h and d, respectively. Note that $h = h_R + h_S$, where h_R is the height of rubber stub, and h_S is the height of the steel stub.

Through use of the finite element method [24], the systems described in Fig. 1 were studied theoretically. The band structures and displacement vector fields were computed according to the Bloch theorem. The single unit cell is determined by the periodicity of the structure. The following geometric parameters were used: $D = 8$ mm; $e = 1$ mm; $a = 10$ mm; $h = 5$ mm, $h_R = 2.5$ mm, $h_S = 2.5$ mm, and $d = 7$ mm. The material parameters used in the calculation are shown in Table 1.

The band structure of the proposed structure is illustrated in Fig. 2(a). Fifteen bands are contained in the normalized frequency range of 0–0.04, and can be classified into the traditional plate modes [32] or the flat modes. The traditional plate modes include the in-plane modes (mainly the symmetric Lamb modes, e.g. S_2 and S_4) and the out-of-plane modes (mainly the anti-symmetric Lamb modes, e.g. A_2). Moreover, the flat modes (e.g. S_1, S_3, A_1, 7, and 8) are the resonant modes of the composite stubs. The band gaps are a result of the coupling between the two kinds of modes mentioned above. To determine the number of band gaps, we first investigated the influence of the special flat bands on the band gaps. The mode displayed in Fig. 3 corresponds to the 7th flat band. The corresponding vibration is mainly due to stub rotation and non-coupling with the plate. This means that no reacting force was applied to the plate. Therefore, these special types of flat bands have no relationships to the band gaps.

The first out-of-plane band gap (green shaded regions shown in Fig. 2) is caused by the coupling between the out-of-plane mode (mode A_2) and the corresponding flat mode (mode A_1), and ranges from 0.006 to 0.033 (between the 6th and 15th bands). The relative band width, $\Delta\omega/\omega$, is defined as the ratio between the band gap width and the center frequency of the band. The relative bandwidth of the out-of-plane band gap is equal to 139%. There are two in-plane band gaps (blue shaded regions shown in Fig. 2), and they are caused by the coupling between the in-plane modes (modes S_2 and S_4) and the corresponding flat modes (modes S_1 and S_3). The first band gap ranges from 0.0054 to 0.011 (between the 4th and the 8th bands) and the second band gap ranges from 0.014 to 0.024 (between the 9th and the 14th bands). The relative bandwidths of the first and second in-plane band gaps are equal to 67% and 53%, respectively. Furthermore, there are two complete band gaps (red shaded regions shown in Fig. 2), and they are caused by overlap between both the first and the second in-plane band gaps and the first out-of-plane band gap. The first band gap ranges from 0.006 to 0.011 (between the 6th and the 8th bands), and the second band gap ranges from 0.014 to 0.024 (between the 9th and the 14th bands). The relative bandwidths of the first and second complete band gaps are equal to 59% and 53%, respectively.

As a comparison, we also calculated the band structure of the classic structure sans rubber filler, as shown in Fig. 4(a). There are two in-plane band gaps (the blue shaded regions shown in Fig. 4) and one out-of-plane band gap (the green shaded regions shown in Fig. 4). The relative bandwidths of the first and the second in-plane band gaps are equal to 66% and 37%, respectively. The relative bandwidth of the out-of-plane band gap is equal to 91%. The complete band gap (red shaded regions shown in Fig. 4) is due to the overlap between the first out-of-plane band gap and the second in-plane band gap. The relative bandwidth is equal to 37%.

From Table 2, it is clear that the introduction of the rubber filler into the proposed structure results in little change to the in-plane band gaps, but results in a large change to the out-of-plane band gap; particularly in comparison to the classic structure. The expanded out-of-plane gap overlaps the first in-plane band gap, thus generating a new complete band gap. As a result, the complete band gaps in the lower frequency ranges are expanded. The expanded complete band gap is mainly due to the expansion of the out-of-plane band gap. Moreover, the expansion of the out-of-plane band gap is caused by the rubber filler, which is capable of shifting the out-of-plane band gap into lower frequency ranges and maintaining the bandwidth.

2.1. Formation mechanism of the lower-frequency out-of-plane band gap

In order to study the physical mechanism for the expanded lower-frequency complete band gap in the proposed structure, sev-

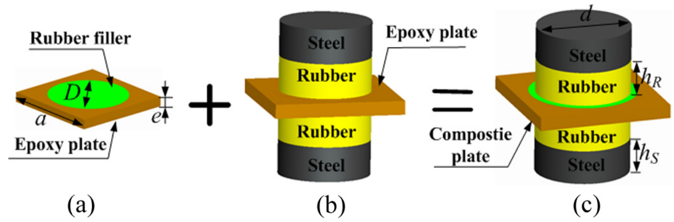

Fig. 1. Illustration of a unit cell of the proposed structure: (a) the composite plate; (b) the classic structure, a double-sided stubbed PC plate with composite stubs; and (c) the proposed structure, a double-sided stubbed composite PC plate with composite stubs.

Table 1
Material parameters in calculations.

Material	Mass density (kg/m^3)	Young's modulus (10^6 N/m^2)	Poisson's ratio
Epoxy	1180	4350	0.3679
Steel	7800	210000	0.29
Rubber	1300	0.1175	0.47