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Deterministic mediated superdense coding with linear optics

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ABSTRACT

We present a scheme of deterministic mediated superdense coding of entangled photon states employing only linear-optics elements. Ideally, we are able to deterministically transfer four messages by manipulating just one of the photons. Two degrees of freedom, polarization and spatial, are used. A new kind of source of heralded down-converted photon pairs conditioned on detection of another pair with an efficiency of 92% is proposed. Realistic probabilistic experimental verification of the scheme with such a source of preselected pairs is feasible with today's technology. We obtain the channel capacity of 1.78 bits for a full-fledged implementation.

1. Introduction

Superdense coding (SC) [1] (sending up to two bits of information, i.e., four messages, by manipulating just one of two entangled subsystems of a quantum system) is considered to be a protocol that can give quantum computation yet another edge over a classical one.

So far the attempts to implement photon SC concentrated on the Bell states. The idea was to send four messages via four Bell states [see Eq. (1)] and herewith achieve a $\log_2 4 = 2$ bit transfer. To this aim, a recognition of all four Bell states was required. However, Vaidman's [2] and Lütkenhaus' [3] groups proved the following no-go result: Deterministic discrimination of all four Bell states with linear optics elements and only one degree of freedom (DOF) (e.g., polarization) is not possible. One can deterministically discriminate only three Bell states and they enable the so-called *dense coding* (channel capacity $\log_2 3 = 1.585$ bits) [4]. Fortunately, the no-go proof does allow a deterministic discrimination with two DOFs in a hyperentanglement setup. Such hyperentanglement experiments have been put forward and carried out [5–8].

Hyperentanglement of photon polarization and its orbital angular momentum recently served Barreiro, Wei, and Kwiat to beat the channel capacity of the dense coding [4] by a tight margin $1.63 > 1.585$ [8] in a postselection experiment. The result has been recognised as "breaking the communication barrier" and such a SC by means of a chosen primary DOF supported by another DOF has been referred as a *mediated SC* [9].

Another kind of hyperentanglement of photon polarization mediated by a time-spatial DOF has been proposed by Kwiat and Weinfurter [10] and carried out by Schuck, Huber, Kurtsiefer, and Weinfurter [5]. They make use of the spatial DOF in order to achieve a time delay.

The main feature of mediated SCs is that photons states are defined by one main DOF (e.g., polarization) and one ancillary DOF (e.g., a time-spatial, spatial, or photon angular momentum). The latter one enables a discrimination of the states of the former one. They require a sophisticated level of controlling qubit states, but at the same time in the existing designs we actually loose more information than in the dense coding. For instance, in the aforementioned hyperentanglement "each hyperentangled state is a unique superposition of four of the sixteen possible combinations of two-photon spin–orbit Bell states" [8].

On the other hand, it was shown that "more entanglement" does not necessarily imply "more computational power" [11] and therefore we considered it viable examining whether SC with mediated photons might be "less entangled." We make use of the so-called *mixed basis* states, two of which are mediated by a spatial DOF, to implement an ideally deterministic 2 bit transfer.

We proposed another mixed basis SC protocol previously in Ref. [12] but that one could not transfer more than 1.43 bits. The present protocol enables Alice to transfer $\log_2 4 = 2$ bits of information, via sending 4 messages to Bob, by manipulating only one photon—called a "travel" photon—from a pair of entangled photons in a Bell state $|\Psi^-\rangle$ generated by Bob. Bob keeps the other photon—called a "home" photon—delayed in a fibre spool. Alice encodes 2 of 4 messages by manipulating the travel photon so as

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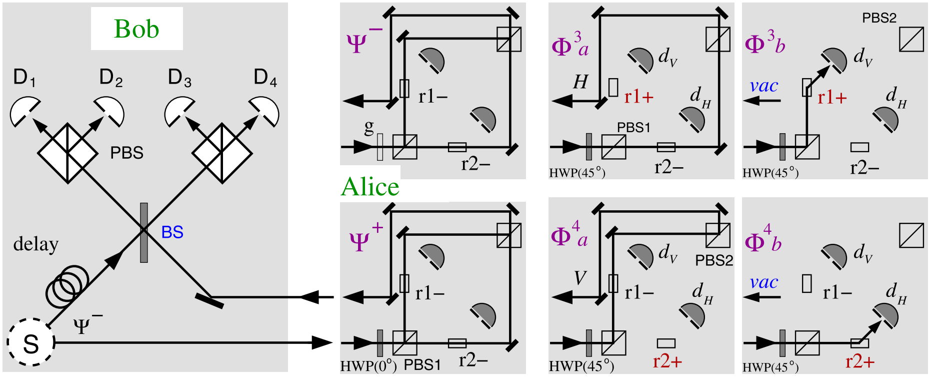

Fig. 1. Schematic of the protocol; Alice sends messages $\Psi^\mp, \Phi^{3,4}$; S is a source of photons in state $|\Psi^-\rangle$—see Subsec. 3.1; r1, r2 are routers (see text) which either let the photons through (*off* mode, r1−, r2−) or deflect them (*on* mode, r1+, r2+) into detectors d_V, d_H, respectively; D_{1-4} are photon number dissolving detectors; BS is a standard beam splitter; PBSs are polarizing beam splitters; g is a glass plate which preserves polarization and makes Ψ^--path length identical to the others; Alice sends Ψ^- by turning the routers off and Ψ^+ by keeping them off and sliding in HWP(0°); she sends Φ^3 (Φ^4) by sliding in HWP(45°) and turning r1 (r2) on and r2 (r1) off, indicated by r1+ (r2+) and r2− (r1−), respectively; photons randomly "choose" to exit PBS1 either in the H or V state—indicated as Φ^3a vs. Φ^3b and Φ^4a vs. Φ^4b options; in Φ^3b and Φ^4b vacuum (*vac*) is sent to Bob; d_V (d_H) is triggered [Φ^3b (Φ^4b)] or not [Φ^3a (Φ^4a)]; Bob receives Φ^3-*message* (Φ^4-*message*) as $|H\rangle_1|H\rangle_2$ ($|V\rangle_1|V\rangle_2$)—Φ^3a (Φ^4a), or as $|V\rangle_1|vac\rangle_2$ ($|H\rangle_1|vac\rangle_2$)—Φ^3b (Φ^4b).

to generate $|\Psi^\mp\rangle$ states and sends the travel photons to Bob who combines them with his home photons at a beam splitter (BS) and measures them. To send the other 2 messages Alice first generates a $|\Phi^-\rangle$ Bell state and then collapses it to 2 computational states mediated by a spatial DOF: two photon paths; one leads to Bob's BS and he measures the travel and home photons; the other leads to Alice's detector and Bob combines his home photon with the vacuum state at his BS.

As in the aforementioned experiments [8,5], we consider the SC protocol primarily as a computational resource. Thus, we only elaborate on the information transferred from Alice to Bob without Eve (eavesdropping) being involved although we briefly discuss a possible cryptographic implementation in Sec. 4.

The spatial DOF, Bob makes use of, when measuring the photons encoded by Alice, does not contain any information about the polarization states Alice imposes on photons taking different paths and therefore there is no classical information transfer involved in Alice's encoding. The classical information carried by photon spatial DOF is tantamount to the mediation of messages via these modes as in [5].

For our protocol to be feasible, a source of entangled photon pairs on demand or a very efficient source of heralded pairs are required because, for an equal efficiency of measuring both vs. only one of two photons, Bob cannot rely on a postselection as in a cryptography application where only detection of both photons are kept and those of single ones are discarded. None of the so far experimentally implemented candidates for such a source, even the most developed quantum dots, is sufficiently reliable and efficient. Therefore in this paper we come forward with a proposal for a very efficient source of heralded preselected entangled photon pair in a Bell state conditioned on a detection of another pair. The source can be implemented with today's technology so as to have a realistic efficiency of 92%.

An experiment in a postselection mode, similar to postselection experiments carried out in [5,6,8], can be carried out with today's technology as proposed in detail in Sec. 2. Actually, also a full-fledged experiment of the proposal can be carried out with today's technology with even higher efficiency, however, with high end versions of all components.

The paper is organised as follows. In Sec. 2 we give physical and technical details of our protocol and all the definitions of states, messages, and optical elements used in the paper. In Sec. 3 we describe the new source of preselected entangled photon pairs (Subsec. 3.1) and propose a postselection proof-of-principle experiment (Subsec. 3.3). At the end of the section we compare channel capacity of our proposal with previous experimentally obtained ones. In Sec. 4 we summarise and discuss the obtained results. At the end of the section we discuss (in)applicability of our SC protocol to quantum cryptography.

2. Protocol

The superdense coding (SC) is an encoding of four messages into the states of entangled pairs of qubits by means of an interaction with one of the qubits only.

We make use of the following three Bell states

$$|\Psi^\mp\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 \mp |V\rangle_1|H\rangle_2),$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 - |V\rangle_1|V\rangle_2), \tag{1}$$

and the following two states from the computational basis

$$|H\rangle_1|H\rangle_2, \qquad |V\rangle_1|V\rangle_2. \tag{2}$$

The Bell states $|\Psi^\mp\rangle$ given by Eq. (1) together with the states given by Eq. (2) form a basis called *mixed state basis* or simply *mixed basis*.

Bob prepares $|\Psi^-\rangle$ photon pairs ideally by using a source of entangled photon pairs on demand but realistically by making use of the source we propose in Subsec. 3.1 which can be realised with today's technology so as to have the efficiency of preselecting pairs of 92%. Bob then sends one photon from each pair to Alice who manipulates it so as to send four different messages to Bob. We call her photon a *travel* photon. The other (Bob's) photon from a pair we call a *home* photon. Alice ideally deterministically encodes the following four messages and sends them to Bob:

$$\Psi^+\text{-}message, \quad \Psi^-\text{-}message, \quad \Phi^3\text{-}message, \quad \Phi^4\text{-}message, \tag{3}$$

as shown in Fig. 1. We will discuss non-ideal realistic implementation of the protocol and take losses into account in Sec. 3. We also discuss a particular aspect of a realistic implementation at the end of this section.

To send a Ψ^--*message* Alice keeps both routers (r1, r2) off, meaning that they let photons through without affecting their states, indicated as r1− and r2− in Fig. 1. The routers make use of electro-optical modulators based on rubidium titanite phosphate [13]. When they are turned on, they can deflect incoming photons independently of their polarization unlike the standard optical switches like, e.g., Pockels cells, based on polarization selection.