Physics Letters A 380 (2016) 869–873

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Formation of the weakly bound muonic molecule $(^4\mathrm{He}\mu t)^{2+}_{01}$ in the three-body $(t\mu)_{1s} + {}^4\mathrm{He} + {}^4\mathrm{He}$ collision

Wilhelm Czapliński *, Michał Rybski

AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. A. Mickiewicza 30, 30-059 Kraków, Poland

ARTICLE INFO

Article history:
Received 18 November 2015
Received in revised form 14 December 2015
Accepted 16 December 2015
Available online 21 December 2015
Communicated by V.A. Markel

Keywords:
Atomic physics
Exotic atoms
Muonic molecules
Scattering theory

ABSTRACT

Formation of the weakly bound muonic molecule $(^4\mathrm{He}\mu t)^{2+}_{01}$ in the excited rotational–vibrational state $(J, \nu) = (0, 1)$ due to the three-body collision $(t\mu)_{1s} + {}^4\mathrm{He} + {}^4\mathrm{He}$ is considered for the first time. It is assumed that the process occurs in T–^{4}He gaseous mixture in thermal equilibrium containing thermalized muonic tritium atoms. The corresponding reaction rate is calculated in the frame of the distorted wave Born approximation (DWBA) method using the dipole approximation for the interaction of $t\mu + {}^4\mathrm{He}$ system with the incoming helium atom. The obtained formation rate (normalized to helium density equal to the liquid hydrogen density) increases with temperature from $7.8 \cdot 10^6$ s^{-1} for 1000 K to $4.8 \cdot 10^7$ s^{-1} for 3000 K.

1. Introduction

Formation of exotic molecular ions (usually called muonic molecules) is one of the most important processes occurring in mixtures of light elements exposed on negative muons beam. Muon entering such a mixture initiates a complicated chain of atomic and molecular processes which generally are much faster than muon decay ($0.45 \cdot 10^6$ s^{-1}). The common designation of these processes is muon-catalyzed fusion (MCF) [1]. Muonic molecules such as $dt\mu$, $^3\mathrm{He}\mu d$, $^6\mathrm{Li}\mu p$, etc. (μ denotes negative muon) are unstable and their most spectacular decay channel is nuclear fusion [2]. Muon freed after the fusion may initiate another MCF chain. Muonic molecules provide an interesting tool for the investigation of nucleus–nucleus interaction at low energies, especially for the investigation of charge symmetry of strong interactions [3] and for the problem of the abundance of light nuclei in stars and in our galaxy [4]. At the same time, the processes in question (including spin flip in muonic atoms) form an unavoidable background in the experimental investigation of weak muon capture by hydrogen and helium nuclei [5], which provides information on such important issue as the structure of light nuclei [6]. Recently, a new and very promising method of controlling quantum states of $dd\mu$ and $dt\mu$ muonic molecules in super-intense laser fields was proposed in [7,8]. Especially, the reported fusion enhancement in these molecules is important for the practical application of the MCF for energy production. Furthermore, this method opens new possibilities for experimental investigation of the problems reported in [3–6].

Theoretical study of MCF comprises different theoretical methods of atomic and molecular physics, e.g. variational methods [9], adiabatic spherical [10] and hyper-spherical expansion method [11], quantum Monte Carlo [12], Faddeev equations [13], and Coupled Rearrangement Channel Method (CRC) [14]. Calculation of wave functions and binding energies of muonic molecules by the numerical solution of time dependent non-Born–Oppenheimer Schrödinger equation in the imaginary time was exploited in [7].

Muonic molecules composed of hydrogen isotope nuclei $(xx'\mu)^+_{J\nu}$, where x, $x' = p$, d, t, and J, $\nu = 0, 1$ are rotational and vibrational quantum numbers, are formed in collisions of muonic hydrogen atoms with hydrogen molecules. Generally, the process responsible for the formation is electron conversion however, the most efficient is the resonant formation of weakly bound states $(dd\mu)^+_{11}$ and $(dt\mu)^+_{11}$, in processes $(x\mu)_{1s} + \mathrm{D}_2 \to [(xd\mu)^+_{11}, d, 2e]_{K\upsilon}$, where $x = d, t$ (molecule D_2 can be replaced by DT). The corresponding binding energies, 1.97 eV and 0.66 eV, are small enough to excite rotational–vibrational states (K, υ) of the final complexes. These molecules are relatively large in comparison with other muonic systems and their size is about 0.05 of atomic unit (au) [9]. Consequently, the molecule $(dt\mu)^+_{11}$ has very much the character of a $t\mu + d$ system.

Muonic molecules composed of helium and hydrogen isotope nuclei $(^{3,4}\mathrm{He}\mu x)^{2+}_{J\nu}$, are Feshbach resonances in $(^{3,4}\mathrm{He}\mu)^{2+}_{1s} + x$

* Corresponding author.
E-mail addresses: gjczapli@cyf-kr.edu.pl (W. Czapliński), rybskimc@gmail.com (M. Rybski).

http://dx.doi.org/10.1016/j.physleta.2015.12.019
0375-9601/

collisions [15]. For $\nu = 0$ they are situated between 7.11 eV for $(^3\mathrm{He}\mu d)^{2+}_{20}$ and 81.34 eV for $(^4\mathrm{He}\mu t)^{2+}_{00}$ below the respective $(d\mu)_{1s}$ and $(t\mu)_{1s}$ thresholds [16]. The resonances have a reach experimental evidence [17] as intermediate states in muon transfer from hydrogen to helium [18] $(x\mu)_{1s} + {}^{3,4}\mathrm{He} \to [(^{3,4}\mathrm{He}\mu x)^{2+}_{J0}e]^+ + e$. According to theoretical calculations [16, 19] the only helium–hydrogen muonic molecule with $\nu = 1$ is $(^4\mathrm{He}\mu t)^{2+}_{01}$ (denoted as $(\alpha\mu t)^{2+}_{01}$ hereafter). The molecule was found in [16] using CRC method [14] as the resonance situated 0.134 eV below $(t\mu)_{1s}$ threshold.[1] The molecule was also found in [19] using much simpler (but less accurate) method, i.e. the one-channel adiabatic approximation for the $2p\sigma$ term. Namely, it was found as a solution of the bound-state eigenvalue problem and in addition, as the low-energy resonance in $(t\mu)_{1s} + \alpha$ elastic scattering.

Using the nuclear pseudo-wavefunction from Ref. [16] one can obtain an average $\alpha - t$ separation in $(\alpha\mu t)^{2+}_{01}$ molecule, which is about 0.08 au. This shows that $(\alpha\mu t)^{2+}_{01}$ is the largest molecule from all other muonic molecules composed of hydrogen as well as hydrogen and helium nuclei. At such large distances the two-center Coulomb function of the muon corresponding to the $2p\sigma$ state is practically centered on the triton. Therefore, this molecule has very much the character of $(t\mu)_{1s} + \alpha$ system.

Formation of this weakly bound muonic molecule may influence the kinematics of MCF, especially the cycle rate in D–^{3}He and D–T mixtures as α particles are produced in d–$^3\mathrm{He}^{++}$ and d–t fusion, respectively.

An interesting question is how this molecule can be formed. In this Letter we try to give an answer presenting the calculation of the reaction rate for the formation of the molecule due to the three-body collision

$$(t\mu)_{1s} + {}^4\mathrm{He} + {}^4\mathrm{He} \to \left[(\alpha\mu t)^{2+}_{01}, 2e\right] + {}^4\mathrm{He}, \tag{1}$$

occurring in T–^{4}He gas mixture in thermal equilibrium.

2. Method of calculation

The formation process (1) is the rearrangement reaction, which involves the three incoming (in the initial) and the two outgoing (in the final state) complex systems. The corresponding three-body reaction rate [20] adopted to the normalization of wave functions used in the present calculation is given by

$$R_{\mathrm{fi}} = (2\pi)^{-2}\delta(E_{\mathrm{f}} - E_{\mathrm{i}})|T_{\mathrm{fi}}|^2 n^2_{\mathrm{He}} n_{t\mu} \tag{2}$$

where $E_{\mathrm{i,f}}$ is the total energy of the system in the initial (i) and the final (f) state of (1); T_{fi} is the matrix element of the corresponding three-body transition operator; n_{He} and $n_{t\mu}$ is the concentration of helium and muonic tritium atoms, respectively. Calculation of T_{fi} can be significantly simplified if we notice characteristic features of the colliding systems. Namely, the muonic tritium atom, being small and neutral object, can almost freely penetrate the electron cloud of helium atom[2] approaching the nucleus (α particle) and starts to form the $(t\mu)_{1s} + \alpha$ scattering state. It can be assumed that this occurs at a distance R_0 at which the absolute value of the interaction potential [19] is comparable to collision energy. If the latter exceeds 0.1 eV then R_0 is less than 0.1 au, i.e. less than the size of resulting muonic molecule. At the same time, a minimal inter-nuclear separation of the two slowly colliding helium atoms is significantly greater than 0.1 au due to a strong He–He repulsion. This fact suggests that the initial $(t\mu)_{1s} + \alpha$ and He + He scattering states are formed independently of each other. Therefore, the initial scattering and the final bound state of the $\alpha - \mu - t$ system (hereafter called the molecular system) can be considered as the two internal states of the compound nucleus of the hypothetic helium atom $^4\tilde{\mathrm{He}}$, which collides with the ordinary helium atom. Consequently, the formation process (1) can be viewed as the deexcitation of the excited complex nucleus of $^4\tilde{\mathrm{He}}^*$ atom due to two-particle collision[3]

$$^4\tilde{\mathrm{He}}^* + {}^4\mathrm{He} \to {}^4\tilde{\mathrm{He}} + {}^4\mathrm{He}. \tag{3}$$

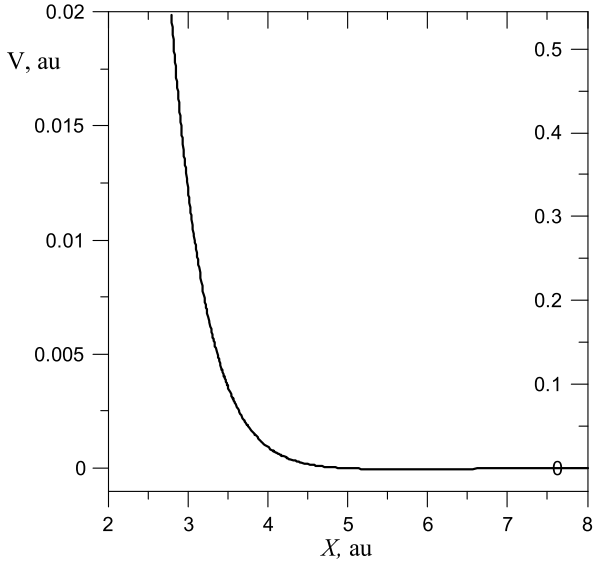

Fig. 1. Interaction potential of two helium atoms according to Ref. [23]. The right vertical axis shows the potential in the units of eV.

From among many theoretical and semi-empirical models of He–He interaction we have chosen the HFD-B(HE) potential presented in [23], which has especially simple analytical form: $V(X) = \varepsilon V^*(X/r_m)$, where X is the $\alpha - \alpha$ separation and

$$V^*(x) = A^* \exp\left(-\alpha^* x + \beta^* x^2\right) - F(x)\sum_{j=0}^{2} c_{2j+6}x^{2j+6};$$

$$F(x) = \begin{cases} \exp[-(D/x-1)^2], & \text{if } x < D \\ 1, & \text{if } x \geq D \end{cases}$$

Numerical values of parameters ε, D, r_m, A^*, α^*, β^*, c_6, c_8, and c_{10} are tabulated in Table 1 of Ref. [23].

The potential is shown in Fig. 1 as a function of inter-nuclear separation (the potential well responsible for the formation of helium dimmer [24] is hardly visible due to the large scale).

As is seen from the figure, the separation between nuclei of two colliding helium atoms at collision energies below 0.3 eV exceeds 3 au and is therefore more than 37 times greater than the size of $(\alpha\mu t)^{2+}_{01}$ molecule (0.08 au). It would therefore seem that the dipole approximation for the interaction of $(t\mu)_{1s} + \mathrm{He}$ system with the incoming helium atom (which generates the transition) is good enough. However, the $\alpha - t$ separation in the molecular system contributing to the matrix element of the molecular dipole moment (see below) receives even 1 au. Therefore, a possible contribution of higher multipole terms could also be significant. However, to achieve even a rough estimate of the molecule formation rate we restrict our calculation to the dipole approximation.

[1] Relativistic correction resulting from the vacuum polarization decreases the binding energy by 4 meV.

[2] According to Ref. [21] the interaction of $t\mu$ with the electrons can be practically neglected for collision energies above 4 meV.

[3] An analogous approach to the problem of three-body collisions was applied in [22] for non-radiative ^{8}B production in $e^- + p + {}^7\mathrm{Be}$ scattering.