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Standing electromagnetic solitons in degenerate relativistic plasmas

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ABSTRACT

The existence of standing high frequency electromagnetic (EM) solitons in a fully degenerate overdense electron plasma is studied applying relativistic hydrodynamics and Maxwell equations. The stable soliton solutions are found in both relativistic and nonrelativistic degenerate plasmas.

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1. Introduction

A significant amount of recent publications describe electromagnetic (EM) waves in relativistic plasmas and majority of them discuss possible roles of these waves in different astrophysical phenomena. Highly relativistic plasmas are observed in the cores of white dwarfs [1], in magnetosphere of pulsars [2], in the MeV epoch of the early Universe [3] and additionally, they probably show up in the bipolar jets in Active Galactic Nuclei (AGN) [4]. Plasma can be relativistic in two following cases: either bulk velocities of fluid unite volume should be close to the speed of light, or the kinetic energy of particles should be greater then their rest energy. In compact objects, such as white dwarfs and magnetars, the number densities of electrons is believed to be roughly between 10^{26} cm^{-3} and 10^{34} cm^{-3} [5,6]. High density plasma can be produced in the laboratory as well, indeed contemporary petawatt laser systems have the focal intensities $I = 2\times 10^{22}$ W/cm^2 [7]. Moreover, pulses with higher than $I = 10^{26}$ W/cm^2 intensities are expected to be achieved soon [8]. Superdense plasmas might be formed with densities in the range of 10^{23} cm^{-3} and 10^{28} cm^{-3} [9], during the interaction of such EM pulses with solid or gaseous targets. Such plasma will be opaque for conventional laser systems operate at wavelengths $\lambda \sim 1$ μm. The Linac Coherent Light Source (LCLS) is an X-ray free-electron laser produce femtosecond powerful pulses of coherent soft and hard X-rays with wavelengths from 2.2 nm to 0.06 nm [10]. Exploiting the possibility to focus X-ray laser beams on a spot with down to laser wavelength, the focal intensities $I \simeq 7\times 10^{25}$ W/cm^2 are expected to be reached [11]. Successful operation of X-ray free-electron lasers in different centers world wide [12] opens up new perspectives to study the EM pulse penetration and its subsequent dynamics in super dense plasma in laboratory conditions.

Highly compressed plasma with an average interparticle distance smaller than their thermal de Broglie wavelength, can be considered as a degenerate Fermi gas. When plasma density increases, the more ideal it becomes and the interactions of its particles can be neglected [13].

EM solitons in classical relativistic plasma is being studied intensively [14], but existence and stability of solitary solutions in degenerate quantum relativistic plasma are investigated mostly for low frequencies (see [15] and references therein). Based on fully relativistic hydrodynamic model recently the high frequency solitary solutions have been examined in degenerate electron plasma for Langmuir [16] and in degenerate electron–positron plasmas for EM waves [17]. The publication goal is to consider existence of a standing, high frequency EM soliton in the relativistic degenerate electron plasma. Importance of the standing soliton solutions for overall dynamics of EM pulses is established theoretically [18–21] as well as experimentally for classical relativistic plasma [22]. These publications state, that during interaction of a circularly polarized strong laser pulse and a plasma, part of the laser energy is trapped in non-propagating soliton-like pulses. Similar dynamics is expected in the case of strong EM pulse interaction with degenerate electron plasma.

2. Basic equations

Plasma can be considered cold, if the thermal energy of its electrons is negligible compared to their Fermi energy. In this case temperature can be assumed to be zero, even though it is of the order of 10^9 K [23]. For the Fermi energy of electrons we have $\epsilon_F = m_e c^2 \left[\left(1+R^2\right)^{1/2} - 1\right]$, where $R = p_F/m_e c$ and p_F is the Fermi momentum. The latter is related to the proper density of

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electrons n by the following equation $p_F = m_e c\,(n/n_c)^{1/3}$, where $n_c = 5.9 \times 10^{29}$ cm^{-3} is the normalizing critical number-density. Therefore, when $n \geq n_c$, electrons move with relativistic momentum inside plasma unit volume and the plasma can be told as relativistic.

Our investigation is based on the Maxwell equations and fluid model of relativistic electron plasma. The ions are considered to form a stationary neutralizing background. We begin with the manifestly covariant form of the fluid equations for electrons

$$\frac{\partial T^{\alpha\beta}}{\partial x^{\beta}} = -eF^{\alpha\beta}nU_{\beta} \tag{1}$$

Here $\partial_\alpha = \partial/\partial x^\alpha = \left(c^{-1}\partial/\partial t, \nabla\right)$; the Greek indexes take values from 0 to 3; $T^{\alpha\beta}$ is the energy–momentum tensor describing the plasma electrons with charge $-e$, mass m_e and the proper density n; the metric tensor is $g^{\alpha\beta} = diag\,(1, -1, -1, -1)$; $U^\alpha = (\gamma, \gamma\mathbf{V}/c)$ denotes the local four velocity, where $\gamma = \left(1 - V^2/c^2\right)^{-1/2}$; $\left(U^\alpha U_\alpha = 1\right)$. This equation implies the conservation of energy and momentum. The change of momentum through the collisions is neglected.

We assume, that the total number of electrons is conserved, thus the following continuity equation is held

$$\frac{\partial nU^{\alpha}}{\partial x^{\alpha}} = 0 \tag{2}$$

The EM field can be expressed through a tensor $F^{\alpha\beta} = [\mathbf{E}, \mathbf{B}]$. The Maxwell equations in these notations are $\partial_\beta F^{\alpha\beta} = -(4\pi/c)\,J^\alpha$ and $\epsilon^{\alpha\beta\gamma\delta}\partial_\beta F_{\gamma\delta} = 0$. Here $J^\alpha = (c\rho, \mathbf{J})$, where $\mathbf{J}$ is the current density and ρ is the total charge density of the plasma.

We use the energy momentum tensor of ideal isotropic fluid $T^{\alpha\beta} = wU^\alpha U^\beta - g^{\alpha\beta}P$, where $w = E + P$ is the enthalpy per unit volume, P is the pressure and E is density of the rest frame internal energy. If $nT/P << 1$, plasma can be treated as completely degenerate Fermi gas and the following equations are satisfied [24]

$$P = \frac{m_e^4 c^5}{3\pi^2\hbar^3} f\,(R) \tag{3}$$

$$E = \frac{m_e^4 c^5}{3\pi^2\hbar^3}\left[R^3\left(1 + R^2\right)^{1/2} - f\,(R)\right] \tag{4}$$

where

$$8\,f\,(R) = 3\sinh^{-1} R + R\left(1 + R^2\right)^{1/2}\left(2R^2 - 3\right) \tag{5}$$

The equation of state for the degenerate gas is $P \propto n^\Gamma$, with $\Gamma = 5/3$ for nonrelativistic case $(R << 1)$ and $\Gamma = 4/3$ for ultrarelativistic case $(R >> 1)$.

The model of plasma described above implies that the electron distribution function remains locally Juttner–Fermian. In case of zero temperature this results in thermodynamical quantities, depending only on density $E(n), P(n)$ and $w(n)$. Of course, these quantities are functions of x^α through the relation $n = N/\gamma$, where N is the electron density in the laboratory reference frame. The considered system is isentropic (furthermore, as temperature approaches zero, entropy tends to zero too). Hence, the following thermodynamic equality is held $d\,(w/n) = dP/n$ and taking into account this thermodynamic equality and making some standard manipulations (e.g. [25]), Eq. (1) can be represented in the form of the following system

$$\frac{\partial}{\partial t}\left(G\mathbf{p}\right) + m_e c^2 \nabla\left(G\gamma\right) = -e\mathbf{E} + \left[\mathbf{V} \times \mathbf{\Omega}\right] \tag{6}$$

$$\frac{\partial}{\partial t}\mathbf{\Omega} = \nabla \times \left[\mathbf{V} \times \mathbf{\Omega}\right] \tag{7}$$

Here, for the generalized vorticity we have $\mathbf{\Omega} = -(e/c)\,\mathbf{B} + \nabla \times (G\mathbf{p})$, where $\mathbf{p} = \gamma m_e \mathbf{V}$ denotes the hydrodynamic momentum; $G = G(n)$ can be called the density dependent "effective mass" factor of electrons $G = w/m_e nc^2 = \left(1 + R^2\right)^{1/2}$. Now dynamics of the degenerate plasma can be completely described by Eqs. (6)–(7) together with Continuity and Maxwell equations. In other words, the set of equations is complete. The analogous set of equations is derived in Ref. [25] for classical relativistic plasma obeying Maxwell–Juttner statistics, where G is a function of temperature $G = G(T)$. In contrast, for degenerate plasma $w/m_e nc^2 = \left(1 + R^2\right)^{1/2}$ and as a result the effective mass factor of electrons depends just on their proper density. The corresponding relation $G = \left[1 + (n/n_c)^{2/3}\right]^{1/2}$ holds for any ratio n/n_c, thus for any strength of relativity [17,26].

We make use of the expressions for fields $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t - \nabla\varphi$ where $\mathbf{A}$ and φ are vector and scalar potentials respectively. Applying the Coulomb gauge condition $\nabla \cdot \mathbf{A} = 0$, the Maxwell equations take the following form:

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} - c^2\Delta\mathbf{A} + c\frac{\partial}{\partial t}\left(\nabla\varphi\right) - 4\pi c\mathbf{J} = 0 \tag{8}$$

and

$$\Delta\varphi = -4\pi\rho \tag{9}$$

where $\mathbf{J} = -e\gamma n\mathbf{V}$, is the current density and $\rho = e(n_0 - \gamma n)$ is the charge density, with n_0 denoting electron (ion) equilibrium density. We use equations (6) and (7) to describe wave dynamics in unmagnetized plasma. Eq. (7) makes clear that if generalized vorticity $\mathbf{\Omega}$ was zero everywhere once, it will stay zero always. Therefore Eq. (6) will reduce to

$$\frac{\partial}{\partial t}\left(G\mathbf{p} - \frac{e}{c}\mathbf{A}\right) + \nabla\left(m_e c^2 G\gamma - e\varphi\right) = 0 \tag{10}$$

Our goal is to find one dimensional localized solutions for equations (8)–(10). Let us assume, that every variable depends on nothing but coordinate z and time t. As transverse component of gradient is zero, Eq. (10) easily gives $\mathbf{p}_\perp = e\mathbf{A}_\perp/(cG)$. Integration constant is zero, because $\mathbf{p}$ should be zero at infinity, where fields vanish. Coulomb gauge condition requires $A_z = 0$, thus the longitudinal motion of the plasma is driven just by the "ponderomotive" pressure $\left(\sim \mathbf{p}_\perp^2\right)$ acting via the relativistic γ factor in Eq. (10) $\left(\gamma = \left[1 + \left(\mathbf{p}_\perp^2 + p_z^2\right)/m_e^2c^2\right]^{1/2}\right)$. The EM pressure forces electrons to move in z direction, the plasma density changes and charge separation occurs. Longitudinal motion of the plasma is described by the following set of equations:

$$\frac{\partial}{\partial t}Gp_z + \frac{\partial}{\partial z}\left(m_e c^2 G\gamma - e\varphi\right) = 0 \tag{11}$$

while the continuity (2) and Poisson's equations (9) become

$$\frac{\partial}{\partial t}\gamma n + \frac{\partial}{\partial z}\left(n\gamma V_z\right) = 0 \tag{12}$$

$$\frac{\partial^2\varphi}{\partial z^2} = 4\pi e(n\gamma - n_0) \tag{13}$$

The transverse component of the current density is $\mathbf{J}_\perp = \left(ne^2/cG\right)\mathbf{A}_\perp$ and substituting it into Eq. (8), we get

$$\frac{\partial^2 \mathbf{A}_\perp}{\partial t^2} - c^2\frac{\partial^2 \mathbf{A}_\perp}{\partial z^2} + \Omega_e^2\left(\frac{n}{n_0}\frac{G_0}{G}\right)\mathbf{A}_\perp = 0 \tag{14}$$

where n_0 is electron (ion) equilibrium density and $\Omega_e = \left(4\pi e^2 n_0/m_e^*\right)^{1/2}$ is the Langmuir frequency of the electron plasma. In this expression m_e^* denotes effective mass of electron $m_e^* = m_e G_0$, where $G_0 = \left[1 + R_0^2\right]^{1/2}$ and $R_0 = (n_0/n_c)^{1/3}$.

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