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Quantum characteristics of occurrence scattering time in two-component non-ideal plasmas

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ABSTRACT

The quantum diffraction and plasma screening effects on the occurrence time for the collision process are investigated in two-component non-ideal plasmas. The micropotential model taking into account the quantum diffraction and screening with the eikonal analysis is employed to derive the occurrence time as functions of the collision energy, density parameter, Debye length, de Broglie wavelength, and scattering angle. It is shown that the occurrence time for forward scattering directions decreases the tendency of time-advance with increasing scattering angle and de Broglie wavelength. However, it is found that the occurrence time shows the oscillatory time-advance and time-retarded behaviors with increasing scattering angle. It is found that the plasma screening effect enhances the tendency of time-advance on the occurrence time for forward scattering regions. It is also shown the quantum diffraction effect suppresses the occurrence time advance for forward scattering angles. In addition, it is shown that the occurrence time advance decreases with an increase of the collision energy.

The atomic collision processes [1–11] such as the elastic collision, electron-impact excitation and ionization, and electron capture have received considerable interests in astrophysics, atomic physics, and plasma physics since the collision processes provide useful information on the collision mechanisms as well as the surrounding physical environments including the collision system. It is shown that the time-development of the atomic collision process has the physical characteristics of the energy dependence since the phase shift contains the physical information of the collision process [1,4]. In addition, the occurrence time characterizing the time of emergence of the moving wave packet has been investigated by Suzuki [12,13] for the quantum collision process. Recently, the angular and energy dependences [14–18] of the occurrence scattering time for the atomic collisions has been extensively explored in order to obtain the useful information on the plasma parameters as well as the scattering mechanism in plasmas since the occurrence time advance would be used for plasma diagnostics and the description of nonideal correlation effects in various plasmas. The atomic collision and radiation processes have been actively investigated by using the effective potentials based on the Yukawa-type Debye–Hückel model [6,19] and the ion-sphere model [20], respectively, in weakly coupled and strongly coupled plasmas. Recent years, the physical characteristics and properties of dense plasmas have been of great interests since the dense non-ideal plasma can be found in various astrophysical and laboratory environments such as astrophysical compact objects, intense laser-plasma experiments, nano-wires, quantum dots, and semiconductor devices [21–28]. In non-ideal plasmas [21–23], however, it is shown that the effective interaction potential and the electron pair-correlation function would not be properly represented by the Debye–Hückel theory owing to the influence of quantum and collective correlation effects. Hence, it is expected that the occurrence times for the elastic electron–ion collision in non-ideal plasmas can be quite different from those in weakly coupled plasmas due to the quantum diffraction [28] and plasma screening effects. However, the influence of quantum diffraction and plasma screening on the occurrence time in non-ideal plasmas has not been investigated as yet. Thus, in this paper, we research the quantum diffraction and plasma screening effects on the occurrence time for the collision process in two-component non-ideal plasmas.

The Hamilton–Jacobi equation [29] for the stationary nonrelativistic Schrödinger equation would be represented by

$$\frac{[\nabla S_{HJ}(\mathbf{r})]^2}{2\mu} - \frac{i\hbar}{2\mu}\nabla^2 S_{HJ}(\mathbf{r}) + V(\mathbf{r}) = E, \quad (1)$$

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since the eikonal wave function $\psi_{Eik}(\mathbf{r})$ can be written as $\psi_{Eik}(\mathbf{r}) \propto \exp[iS_{HJ}(\mathbf{r})/\hbar]$, where $S_{HJ}(\mathbf{r})$ is the Hamilton–Jacobi phase function, $\mathbf{r}$ is the position vector of the electron, μ is the reduced mass of the collision system, $V(\mathbf{r})$ is the interaction potential, E $(= \hbar^2 k_i^2/2\mu = \mu v_i^2/2)$ is the collision energy, $\hbar$ is the rationalized Planck constant, k is the wave number, and v_i is the relative collision velocity. In the cylindrical coordinate system with $\mathbf{r} = \boldsymbol{\xi} + \mathbf{z}$, i.e., the straight-line trajectory, where $\boldsymbol{\xi}$ is the impact parameter, $\mathbf{z}$ is the longitudinal variable vector normal to the momentum transfer $\mathbf{q}$ $(\equiv \mathbf{k}_i - \mathbf{k}_f)$, $\mathbf{k}_i$ and $\mathbf{k}_f$ are, respectively, the incident and final wave vectors, the normalized eikonal wave function solution $\psi_{Eik}(\mathbf{r})$ with the eikonal constraint [30] such as $|V(\mathbf{r})|/E < 1$ would be then given by

$$\psi_{Eik}(\mathbf{r}) \cong \frac{1}{(2\pi)^{3/2}} \exp\left[i\mathbf{k}_i \cdot \mathbf{r} - \frac{i\mu}{\hbar^2 k_i} \int_{-\infty}^{z} dz' V(\boldsymbol{\xi}, z') \right], \tag{2}$$

since the Hamilton–Jacobi phase function $S_{HJ}(z)$ can be represented by

$$S_{HJ}(z)/\hbar \cong kz - \frac{\mu}{\hbar^2 k_i} \int_{-\infty}^{z} dz' V(\boldsymbol{\xi}, z'), \tag{3}$$

when $|\hbar \nabla^2 S_{HJ}(\mathbf{r})| << |[\nabla S_{HJ}(\mathbf{r})]^2|$, where $|V(\mathbf{r})|$ is a typical strength of the interaction potential. The eikonal scattering amplitude $f_{Eik}(\mathbf{k}_i, \mathbf{k}_f)$ would be then expressed by

$$\begin{aligned} f_{Eik}(\mathbf{k}_i, \mathbf{k}_f) &= -ik_i \int_0^{\infty} d\xi \xi J_0(\xi q) \{ \exp[i\chi_{Eik}(\xi, k_i)] - 1 \} \\ &= |f_{Eik}(\mathbf{k}_i, \mathbf{k}_f)| \exp[i\eta_{Eik}(\mathbf{k}_i, \mathbf{k}_f)], \end{aligned} \tag{4}$$

where $J_0(\xi q)$ is the zeroth-order Bessel function, $\eta_{Eik}(\mathbf{k}_i, \mathbf{k}_f)$ is the argument of the scattering amplitude, and the total eikonal scattering phase shift $\chi_{Eik}(\xi, k_i)$ would be expressed by the following form based on the series expansion technique [31]:

$$\begin{aligned} \chi_{Eik}(\xi, k_i) = &-\sum_l \frac{1}{(l+1)!} \left(\frac{\mu}{\hbar^2}\right)^{l+1} \frac{1}{k} \left(\frac{\xi}{k^2} \frac{\partial}{\partial \xi} - \frac{\partial}{\partial k_i} \frac{1}{k_i} \right)^l \\ &\times \int_{-\infty}^{\infty} dz V^{l+1}(\xi, z). \end{aligned} \tag{5}$$

In atomic collisions, it is essential to obtain the scattering amplitude $f_{Eik}(\mathbf{k}_i, \mathbf{k}_f)$ in order to investigate the physical characteristics of the correlation phenomena in scattering dynamics since the scattering process can be reduced to the physical investigation of the scattering amplitude. It is shown that the occurrence time [12,13] $\tau_{OT}(\theta, k)$ for the elastic collision can be obtained by the first-derivative of the argument of the scattering amplitude $\eta_{Eik}(\mathbf{k}_i, \mathbf{k}_f)$ with respect to the initial wave number $|\mathbf{k}_i|$, when the collision center of the free wave packet of the projectile reaches the origin $\mathbf{r} = 0$ at $t = 0$,

$$\tau_{OT}(\theta, k) = \frac{\mu}{\hbar k} \left[\frac{\partial \eta_{Eik}(\mathbf{k}_i, \mathbf{k}_f)}{\partial |\mathbf{k}_i|} \right] \Bigg|_{|\mathbf{k}_i| = |\mathbf{k}_f| = k}, \tag{6}$$

where θ is the angle between the incident and final wave vectors and the elastic collision condition is obtained by $|\mathbf{k}_i| = |\mathbf{k}_f| \equiv k$.

Very recently, Ramazanov, Moldabekov, Gabdullin, and Ismagambetova [28] has obtained an extremely useful analytic expression of the effective interaction potential $V_{RMGI}(r)$ between plasma particles e_α and e_β in non-ideal plasmas taking into account the quantum diffraction, plasma screening and symmetric effects, where α and β are types of plasma particles. Based on the RMGI-micropotential model [28], the effective interaction potential $V_{RMGI}(r)$ between the electron (e) and the ion (i) with nuclear charge Ze in dense non-ideal plasmas including the quantum diffraction and plasma screening effects is repressed as

$$\begin{aligned} &V_{RMGI}(r) \\ &= -\frac{Ze^2}{r} \left[1 - \tanh\left(\frac{\sqrt{2}\lambda_{ei}^2}{a^2 + br^2} \right) \exp\left[-\tanh\left(\frac{\sqrt{2}\lambda_{ei}^2}{a^2 + br^2} \right) \right] \right] \\ &\quad \times [1 - \exp(-r/\lambda_{ei})] - \delta_{ie} k_B T \ln\left[1 - \frac{1}{2} \exp(-r^2/\lambda_{ee}^2) \right], \end{aligned} \tag{7}$$

where $\lambda_{\alpha\beta}$ $[= \hbar/(4\pi \mu_{\alpha\beta} k_B T)^{1/2}]$ is the de Broglie wave length for the reduced mass $\mu_{\alpha\beta}$ $[= m_\alpha m_\beta/(m_\alpha + m_\beta)]$, m_α is the mass of the particle α, k_B is the Boltzmann constant, T is the plasma temperature, a is the average distance between particles, $b = 0.033$, and $\delta_{\alpha\beta}$ is the Kronecker delta. It would be understood that the symmetric term has no contribution to the electron–ion interaction in non-ideal plasmas since the term proportional to the Kronecker delta in Eq. (7) stands for the influence of symmetric case. If we set $\lambda_{\alpha\beta}^2/a^2 << 1$ [28] without the density effects in the quantum diffraction term in Eq. (6), the interaction potential would be then $V_D(r) = -(Ze^2/r)[1 - \exp(-r/\lambda_{ei})]$, which is the case of the general Deutch-potential [32] without the symmetric effect in non-ideal plasmas. From Eqs. (5) and (7), the scaled first-order eikonal scattering phase shift $\bar{\chi}_{Eik}(\bar{k}_i, \bar{\lambda}_D, \bar{\lambda}, \bar{\xi}, \beta)$ $(= \bar{k}_i \chi_{Eik}/2)$ for the elastic electron–ion collision in dense non-ideal plasmas including the quantum diffraction and plasma screening effects is given by the following integral with the upper-limit $\bar{\lambda}_D$:

$$\begin{aligned} &\bar{\chi}_{Eik}(\bar{k}_i, \bar{\lambda}_D, \bar{\lambda}, \bar{\xi}, \beta) \\ &= \int_0^{\bar{\lambda}_D} d\bar{z} \frac{1}{\sqrt{\bar{\xi}^2 + \bar{z}^2}} \left[1 - \tanh\left(\frac{\sqrt{2}\bar{\lambda}^2}{\kappa^2 + b(\bar{\xi}^2 + \bar{z}^2)} \right) \right. \\ &\quad \left. \times \exp\left[-\tanh\left(\frac{\sqrt{2}\bar{\lambda}^2}{\kappa^2 + b(\bar{\xi}^2 + \bar{z}^2)} \right) \right] \right] \\ &\quad \times [1 - \exp(-\sqrt{\bar{\xi}^2 + \bar{z}^2}/\bar{\lambda})], \end{aligned} \tag{8}$$

where $\bar{k}_i$ $(\equiv k_i a_Z)$ is the scaled wave number, a_Z $(= a_0/Z)$ is the Bohr radius of the hydrogenic ion with nuclear charge Ze, a_0 $(= \hbar^2/m_e e^2)$ is the Bohr radius of the hydrogen atom, m_e is the electron mass, $\bar{\xi}$ $(\equiv \xi/a_Z)$ is the scaled impact parameter, $\bar{\lambda}_D$ $(\equiv \lambda_D/a_Z)$ is the scaled Debye length in units of a_Z, $\bar{\lambda}$ $[\equiv \lambda_{ie}/a_Z = (\hbar/a_Z)/(4\pi \mu_{ie} k_B T)^{1/2}]$ is the scaled de Broglie wave length for the electron–ion system, κ $(\equiv a/a_Z)$ is the scaled density parameter, and $\bar{z} \equiv z/a_Z$. In the range of the eikonal analysis, the complex eikonal scattering amplitude $f_{Eik}(|\bar{\mathbf{k}}_i|, |\bar{\mathbf{k}}_f|, \theta)$ can be expanded as a power series in the potential strength:

$$f_{Eik}(|\bar{\mathbf{k}}_i|, |\bar{\mathbf{k}}_f|, \theta) = f_R(|\bar{\mathbf{k}}_i|, |\bar{\mathbf{k}}_f|, \theta) + if_I(|\bar{\mathbf{k}}_i|_i, |\bar{\mathbf{k}}_f|, \theta), \tag{9}$$

where $|\bar{\mathbf{k}}_i| \equiv |\mathbf{k}_i| a_Z$, $|\bar{\mathbf{k}}_f| \equiv |\mathbf{k}_f| a_Z$, $f_R(|\bar{\mathbf{k}}_i|, |\bar{\mathbf{k}}_f|, \theta)$ and $f_I(|\bar{\mathbf{k}}_i|_i, |\bar{\mathbf{k}}_f|, \theta)$ are the real and imaginary parts of the complex scattering amplitude, respectively,

$$f_R(|\bar{\mathbf{k}}_i|, |\bar{\mathbf{k}}_f|, \theta) = 2a_Z \int_0^{\bar{\lambda}_D} d\bar{\xi} \bar{\xi} J_0(\bar{\xi}\bar{q}) \bar{\chi}_{Eik}(\bar{k}_i, \bar{\lambda}_D, \bar{\lambda}, \bar{\xi}, \beta), \tag{10}$$

$$f_I(|\bar{\mathbf{k}}_i|, |\bar{\mathbf{k}}_f|, \theta) = \frac{2a_Z}{\bar{k}_i} \int_0^{\bar{\lambda}_D} d\bar{\xi} \bar{\xi} J_0(\bar{\xi}\bar{q}) \bar{\chi}_{Eik}^2(\bar{k}_i, \bar{\lambda}_D, \bar{\lambda}, \bar{\xi}, \beta), \tag{11}$$