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Distant-neighbor hopping induced the Dirac points and the high Chern number topological phase on π-flux square lattice

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ABSTRACT

For the two-band system, an important route to achieve the high Chern number (HCN) topological phase is to add the number of low-energy Dirac points. In this work, we try to study the possibility of realizing HCN phase in π-flux square lattice when the $(n-1)$-nearest-neighboring hoppings Nn are introduced. We investigate N4 and N6 intersublattice hopping which can generate new Dirac points, whose chiralities and mergings are analyzed. While for intrasublattice hopping which can gap the Dirac points, we studied N2 and N5 cases to obtain the HCN phase diagram. We further discuss the experimental detections of the HCN phase by the edge states excitations and the transverse Hall conductance response.

1. Introduction

Searching new topological states of matter marks one of the major challenges in condensed matter physics. In 1988, Haldane first proposed the prototypical Chern insulator in honeycomb lattice, in which the quantum Hall effect (QHE) can be realized but without the Landau levels [1], or called the quantum anomalous Hall effect (QAHE). The most important property of the Chern insulator is that the system can be characterized by the Chern number C, which is a topological index of the filled bands. The nonzero C implies the existence of chiral edge states, giving rise to the dissipationless transport along the system's boundary. The problem of Chern insulator has attracted a lot of studies recently. In Cr-doped $(Bi_{1-x}Sb_x)_2Te_3$ magnetic thin film, the Chern insulator phase was first reported to be observed experimentally [2], although the phenomena only occurs below about 30 mK.

Among the studies of the Chern insulator, an intriguing problem is to search the high Chern number (HCN) phase whose Chern number is larger than 1. To generate the HCN phase, different physical mechanisms are proposed in theory. If the Rashba spin–orbit coupling is included in Haldane model, the system can be regarded as two copies of Haldane model [3,4] and the Chern number may reach 2. When introducing the distant hoppings in Haldane model, the HCN phase can also be realized [5]. The essence of these works is to add the number of low-energy Dirac points. Recent studies suggest that the arbitrary Chern number insulator can be produced in a class of flat bands in multilayer systems [6,7], in which the band flatness ratio (band gap/bandwidth) will reach a larger value for the higher Chern number. In fact, if the Hamiltonian vector of two-band is represented in spherical coordinate, when the azimuthal angle is enlarged to be N times, the N Chern number phase may be achieved. In addition, there are also interesting proposals to generate HCN phase in honeycomb lattice by the time-periodic modulation, in which different Floquet subbands in the quasi-energy gap are mixed [8,9]. These new theoretical HCN phase discoveries in solid state materials and in ultracold atomic systems may help us deepen the understanding of the topological band structures of low-dimensional electron gas.

Fortunately, the honeycomb lattice is not a necessary condition for the occurrence of the Chern insulator, which can also be realized on the square lattices under certain conditions [7,10–14]. More importantly, the recent experimental achievements for ultracold atoms are mainly performed on the square lattice [15,16]. While the influence of distant hoppings on the topological properties of the square lattice is still lacking [17], in this work we try to investigate the possibility of realizing the HCN phase in π-flux spinless square lattice by this mechanism. When the distant intersublattice hoppings are included in the tight-binding system which can generate the new satellite band-touching points around the regular Dirac points, we analyze the chirality and merging of the Dirac points. On the other hand, when the intrasublattice hoppings are included which can gap the Dirac points, we solve the mass expressions and further obtain the Chern number phase diagram.

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In addition, we point out two feasible methods to detect the HCN topological phase in experiment. Our studies about the creation and control of the new valley degree of freedom may shed some insights for use in such applications as data storage and transmission which are integral to the new valleytronics devices.

2. Distant-neighbor hopping in π-flux square lattice

To model the spinless two-band, a two-sublattice or two-orbit system is needed at least. The Hamiltonian which contains the minimal ingredients is:

$$H(\mathbf{k}) = \mathbf{h}(\mathbf{k}) \cdot \boldsymbol{\sigma}, \tag{1}$$

where the Pauli matrices σ_μ $(\mu = 1, 2, 3)$ act on the pseudospin space and $\mathbf{k} = (k_x, k_y)$ runs over the Brillouin zone (BZ). Here we neglect σ_0 term as it only breaks the particle–hole symmetry and has negligible influence on the topological phase transition discussed in this work.

The topological phase can be well characterized by the Chern number which gives the appearance of quantized Hall conductance at the edges of the insulator [18]. In the gapped system, the Chern number is defined as the integration of the Berry curvature in the momentum space:

$$C = \frac{1}{4\pi} \int_{\mathrm{BZ}} d\mathbf{k} (\partial_{k_x} \hat{\mathbf{h}} \times \partial_{k_y} \hat{\mathbf{h}}) \cdot \hat{\mathbf{h}}, \tag{2}$$

where $\hat{\mathbf{h}} = \mathbf{h}/|\mathbf{h}|$ is the unit Hamiltonian vector. For a two-band system, the Chern number can be calculated by separately treating the chirality of the Dirac points and the sign of mass term h_3 as follows [5]:

$$C = \sum_{\alpha=1}^{n} C_\alpha, \quad C_\alpha = \frac{1}{2} \mathrm{sgn}(h_{3\alpha}) \chi_\alpha(\mathbf{k}), \tag{3}$$

where the summation is to the Dirac point (valley) index α and the corresponding chirality $\chi_\alpha(\mathbf{k}) = \mathrm{sgn}(\partial_{k_x} \mathbf{h}_\alpha \times \partial_{k_y} \mathbf{h}_\alpha)_z$. In Eq. (3), the Chern number has been expressed as the sum of the components in each valley which equals $\frac{1}{2}$ or $-\frac{1}{2}$. For the two-band electron system which owns $2N$ Dirac points, the Chern number may reach the highest (lowest) value N $(-N)$ when the component in each valley takes $\frac{1}{2}$ $(-\frac{1}{2})$. So a direct route to achieve the HCN phase is to add the number of Dirac points in the Brillouin zone. We will take π-flux square lattice as an example to illustrate how to generate the new Dirac points and obtain the HCN topological phases by distant-neighbor hoppings.

The π-flux square lattice is consisted of two sublattices A and B and there is a staggered magnetic flux π between the neighboring plaquettes. The parameter t_n corresponds to the Nn distant hopping which is $(n-1)$-nearest-neighboring as shown in Fig. 1. We choose the gauge that only the hoppings along the direction of bonds are affected by the magnetic flux, which means the Periels phase factor may affect the hoppings of N1, N3, N6, ... In Fig. 1, the gauged phase of N1 hopping is positive along the arrows.

The distant hoppings can be divided into two types: intrasublattice and intersublattice. The intrasublattice hoppings contribute to the diagonal terms which will induce the mass term, while the intersublattice hoppings contribute to the nondiagonal terms which may generate the additional Dirac points. We hope the physics of the model can be well simulated by cold atoms trapped in optical lattices, where Nn hopping integral can be controlled precisely [19]. In the following, t_1 will be set as the unit of energy.

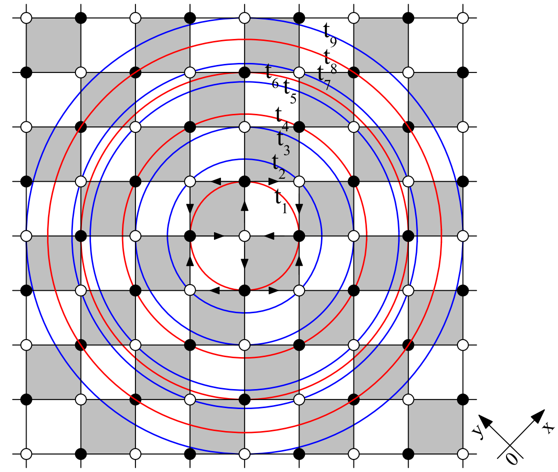

Fig. 1. (Color online.) The possible hoppings in π-flux square lattice model. The hopping integrals from the central A sublattice to the $(n-1)$ neighbor site placed on a concentric circle is denoted by t_n. The red (blue) circles show the intersublattice (intrasublattice) hoppings for the central sublattice. The darker (brighter) region represents the π $(-\pi)$ flux and the arrows show the gauged phase $\varphi_0 = \frac{\pi}{4}$ for N1 hopping.

3. Dirac points for N4 and N6 distant hoppings

We first consider the realization of additional Dirac points by the intersublattice hoppings. The eventual Dirac points should be gapped by the mass to yield the nontrivial topological phase. Here the 2×2 Hamiltonian $h(\mathbf{k})$ in the space spanned by two sublattices can be written as:

$$h(\mathbf{k}) = \begin{pmatrix} 0 & f(\mathbf{k}) \\ f^*(\mathbf{k}) & 0 \end{pmatrix}, \tag{4}$$

with the nondiagonal element $f(\mathbf{k}) = h_1(\mathbf{k}) - ih_2(\mathbf{k})$ or

$$f(\mathbf{k}) = \sum_n -t_n g_n(\mathbf{k}). \tag{5}$$

The properties of functions $g_n(\mathbf{k})$ up to N8 are summarized in Table 1, in which we characterize the properties of hoppings by the physical distance, the chemical distance, the number of neighbors and the specific expression.

When the system includes only N1 hopping that $f(\mathbf{k}) = -g_1(\mathbf{k})$. The two energy eigenvalues are given as $\epsilon_\pm = \pm|g_1(\mathbf{k})|$ and the bands touch at the regular Dirac points $\mathbf{K}_+ = \frac{\pi}{\sqrt{2}a}(1, 0)$ and $\mathbf{K}_- = \frac{\pi}{\sqrt{2}a}(0, 1)$ which correspond to the zeros of $f(\mathbf{k})$. At each of these points, there are two degenerate zero-energy eigenstates on sublattice A or B. Around $\mathbf{K}_\pm$, one can expand $f(\mathbf{k})$ in small momentum $\mathbf{q} = q(\cos\theta, \sin\theta)$. It follows that

$$\begin{aligned} f(\mathbf{K}_+ + \mathbf{q}) &= \frac{2\sqrt{2}a}{\hbar} q(\cos\varphi_0 \cos\theta - i \sin\varphi_0 \sin\theta), \\ f(\mathbf{K}_- + \mathbf{q}) &= \frac{2\sqrt{2}a}{\hbar} q(\cos\varphi_0 \sin\theta - i \sin\varphi_0 \cos\theta), \end{aligned} \tag{6}$$

where the linearity in $\mathbf{q}$ identifies the band touchings. Evidently, the existence of π-flux plays an essential role in forming the isotropic Dirac cone, where the corresponding gauged phase factor gives $\varphi_0 = \frac{\pi}{4}$ for N1 hopping.

If the band touching at the Dirac point has the form $f \propto (qe^{\mp i\theta})^n$, its chirality is $\pm n$. From the geometrical interpretation, when the two-dimensional vector can be written as $(h_1, h_2) \propto q^n(\cos(\pm n\theta), \sin(\pm n\theta))$, for θ sweeping once the interval $[0, 2\pi)$ it will rotate counterclockwise (clockwise) around the origin for n times [5,17]. Evidently, the chiralities at the Dirac points give $\chi(\mathbf{K}_\pm) = \pm 1$ which are opposite at different Dirac points.