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Absolute negative mobility in a one-dimensional overdamped system

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ABSTRACT

A one-dimensional overdamped system consisting of a symmetric periodic potential, a constant bias force and a trichotomous noise was investigated. In the frame of master equations, we derived analytical expression of its current. By means of numerical calculations, the results indicate that the current first increases, then decreases and finally increases with the bias force increasing, i.e., an absolute negative mobility (ANM) phenomenon. Our further investigations presented dependence of the ANM phenomenon on parameters of the noise. Its intrinsic physical mechanism was also open up, and a minimal model with ANM phenomenon is demonstrated.

1. Introduction

In recent years, wide application of Brownian motor to bioengineering and nanotechnology, such as ion pump Na–K–AT Pase [1], motor proteins [2], and nanomachine [3–6], and so on, has stimulated many investigators to study intensively statistical properties of Brownian particles in a periodic potential [7–13]. One of the most important parameters determining operating performance of Brownian motor is its current. It is well known that non-equilibrium fluctuations and symmetry breaking are the necessary conditions that a Brownian particle forms a directed motion. The non-equilibrium fluctuations are generally provided by noise [14,15], but the broken symmetry can come from a spatial potential [16–21], a temporal periodic signal, a noise [22–25], and an external force.

For a periodic system whose symmetry is broken by an external constant force, as its current increases with the external force increasing, the dynamical behavior is called a normal transport; contrariwise, an abnormal transport. In Ref. [26], the authors consider two coupled particles moving along a spatially symmetric periodic substrate potential driven by a thermal noise in the overdamped limit, and found that a sinusoidal external driving of appropriate amplitude and frequency may lead to spontaneous symmetry breaking, and in the action of a static bias force a dimer exhibits a motion opposite to that force, i.e., an absolute negative mobility (ANM) phenomenon. For a vibrational motor driven by a time-periodic signal, a wide range of the driving strength and angular frequency of the signal can also make the ANM phenomenon appear [27]. A Brownian particle subjected to an external time-oscillatory drive in thermal inertial ratchets moves oppositely to a constant bias force [28]. Of course, with the help of time-periodic driving, colored thermal fluctuations can also induce the ANM in spatially periodic symmetric systems [29]. In fact, the time-periodic driving is not always the key requirement for the ANM. The authors of Ref. [30] found that two kinds of pulsated perturbations can cause the ANM phenomenon for an overdamped Brownian particle on a symmetric periodic substrate. Experimentally, such a quite astonishing anomalous response behavior has so far been observed in bulk GaAs [31], semiconductor heterostructures [32], structured microfluidic systems [33], and so on. In what follows, it will be seen that without the help of external time-periodic signal, a symmetric additive trichotomous noise also induces movement of an overdamped Brownian particle opposite to a constant external force.

In this paper, we mainly focus on abnormal transport of a Brownian particle in a periodic potential and subjected to a trichotomous noise in the action of constant external force, and study effect of the trichotomous noise on the ANM phenomenon. The paper is constructed as follows. In Section 2, model and theoretical analysis are provided. Analytical expression of the system's current will be derived. In Section 3, results and discussions are presented. By means of numerical calculations from the analytical expression, effects of parameters of the trichotomous noise and symmetry of the periodic potential on the current will be analyzed in detail. In Section 4, conclusions are made.

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2. Model and theory

Here we consider an overdamped system subjected to a trichotomous noise and a constant bias force, which is described by the Langevin equation

$$\frac{dx}{dt} = -\partial V(x)/\partial x + F + \xi(t), \tag{1}$$

where x denotes the state variable of the system, $V(x)$ is the periodic potential, F is the constant bias force. And $\xi(t)$ represents the symmetric three-level random telegraph process. This is a random stationary Markovian process that consists of jumps between three values $a = a_0, 0, -a_0$. The jumps follow in time to a Poisson process, while the values occur with the stationary probabilities

$$P_s(a_0) = P_s(-a_0) = q, \qquad P_s(0) = 1 - 2q. \tag{2}$$

The trichotomous noise $\xi(t)$ satisfies the following statistical properties

$$\langle \xi(t) \rangle = 0, \qquad \langle \xi(t)\xi(t') \rangle = 2qa_0^2 e^{-\nu|t-t'|}, \tag{3}$$

with ν being the switching rate of noise. Its noise intensity is

$$D = 2\int_0^\infty \langle \xi(t)\xi(t+\tau) \rangle d\tau = 4qa_0^2/\nu. \tag{4}$$

The composite Fokker–Planck master equation corresponding to Eqs. (1) and (3) is

$$\frac{\partial P_n(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left\{ [-\frac{\partial V(x)}{\partial x} + F + a_n] P_n(x,t) \right\} + \sum_m \left[U_{nm} P_m(x,t) \right] \tag{5}$$

with $P_n(x,t)$ denoting the probability distribution function (PDF) for the combined process (x, a_n, t); $n, m = 1, 2, 3$; $a_1 = -a_0$, $a_2 = 0$, $a_3 = a_0$ and

$$U = \nu \begin{pmatrix} q-1 & q & q \\ 1-2q & -2q & 1-2q \\ q & q & q-1 \end{pmatrix}.$$

So the current of the system is given by

$$J_n(x,t) = [-\frac{\partial V(x)}{\partial x} + F + a_n] P_n(x,t), \quad n = 1, 2, 3. \tag{6}$$

Obviously, Eq. (5) satisfies the following continuum equation of current

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x}, \tag{7}$$

where $P(x,t) = P_1(x,t) + P_2(x,t) + P_3(x,t)$ is the total PDF, and $J(x,t)$ is the total current

$$J(x,t) = J_1(x,t) + J_2(x,t) + J_3(x,t). \tag{8}$$

In what follows, in order to get analytical expression of the total current in the steady state, the period potential $V(x)$ is assumed to be a piecewise linear sawtooth-like potential

$$V(x) = \begin{cases} -(x-k-d)/d, & \text{when } k \le x \le k+d, \\ (x-k-d)/(1-d), & \text{when } k+d \le x \le k+1, \end{cases} \tag{9}$$

where k is an integer, and $d \in (0,1)$ is the symmetry parameter of the potential. As $d = 0.5$, the potential is symmetric, otherwise asymmetric. For convenience of calculations, the height and the period of the piecewise linear sawtooth-like potential in Eq. (9) are assumed to be equal to 1.

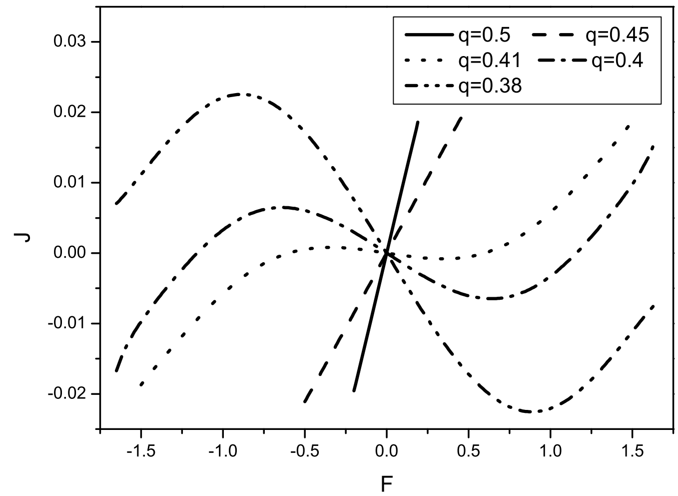

Fig. 1. The dependence of the current J on the bias force F at different values of stationary probability: $q = 0.38, 0.4, 0.41, 0.45, 0.5$. The other parameters are $a_0 = 9$, $\nu = 10$, and $d = 0.5$.

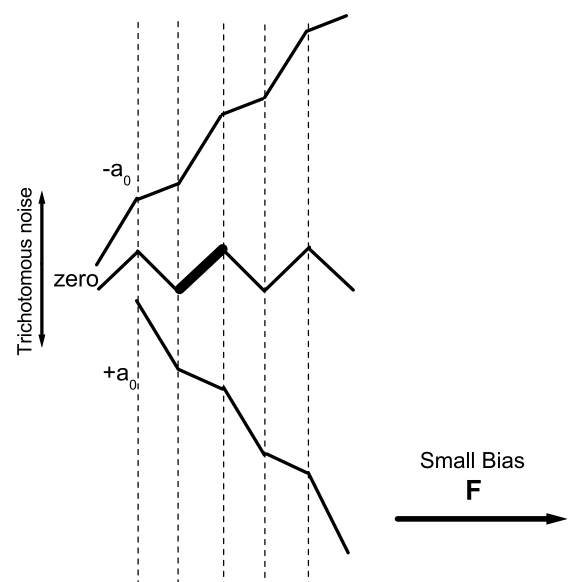

Fig. 2. Sketch map of the total potential as the system is in the three states ("zero"-state, the "$-a_0$"-state and the "$+a_0$"-state), with $F > 0$.

In the steady state, solving Eqs. (5), (6) and (8) then using the conditions of current continuum, periodic boundary and normalization, we can obtain the exact expressions of the current [34]

$$J = \frac{4\nu q\gamma\eta bc[A(b,c) - A(c,b)]}{A(c,b)B(c,b) + A(b,c)B(b,c) - 4\nu q\gamma\eta(b-c)[A(b,c) - A(c,b)]}, \tag{10}$$

where expressions of $A(b,c)$, $B(b,c)$, γ and η can be seen in Appendix A.

3. Results and discussions

By means of Eq. (10), we can calculate numerically the current of the system, and discuss effects of parameters of the trichotomous noise and symmetry of the periodic potential on it, and the results were plotted in Figs. 1–5.

Let's discuss the case of symmetric potential, i.e., $d = 0.5$, because it can allow us open up more clearly intrinsic physical mechanism of the ANM phenomenon in the system. Fig. 1 reflects the