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Recurrence quantification analysis of chimera states

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Chimera states, characterised by coexistence of coherence and incoherence in coupled dynamical systems, have been found in various physical systems, such as mechanical oscillator networks and Josephson-junction arrays. We used recurrence plots to provide graphical representations of recurrent patterns and identify chimera states. Moreover, we show that recurrence plots can be used as a diagnostic of chimera states and also to identify the chimera collapse.

1. Introduction

Network dynamical systems have been studied as models of spatiotemporal complexity. Among the spatiotemporal features recognised in coupled systems we can find chaos synchronisation [1], suppression [2,3], pattern formation [4], and multistability [5].

Dynamical systems may be modelled by coupled ordinary differential equations (CODE). A network of coupled differential equations has a continuous state variable and time, while the space is discrete. CODE present various applications to spatially extended systems in nonlinear dynamical systems [6]. For instance, production and transfer of energy and information in conservative systems [7], creation of hyperchaotic attractors in a system of coupled Chua circuits [8], and phase synchronisation between collective rhythms of coupled oscillator groups [9]. Moreover, biophysical complex systems may be modelled by coupled differential equations, such as tumour growth [10,11], and synchronisation of bursting neurons [12,13].

Here we focus on dynamical features in CODE such as coherence and incoherence states. When these states coexist the phenomenon is called a chimera state [14]. The network contains a coherent and phase locked domain, and an incoherent domain. The coexistence of coherence and incoherence was first observed by Kuramoto and Battogtokh in a non-locally coupled phase oscillators [15]. The existence of chimera states has also been verified in networks with symmetrically coupled identical oscillators [16].

Recently, it has been shown that chimera states can be seen in experimental studies. Hagerstrom and coworkers showed that these states can be realised in experiments using a liquid-crystal spatial light modulator [17]. Tinsley and coworkers reported experimental studies in which they observed chimera states in coupled Belousov–Zhabotinsky oscillators [18]. In addition, an experimental work about chimera states can be found in Ref. [19], where it was shown that chimeras could emerge coupled mechanical oscillators. The experimental setup was realised with metronomes coupled by means of adjustable springs. Swing and metronome displacements were measured by digital tracking of UV fluorescent spots located on the pendula and swings. Through simple mechanical oscillators, known as Huygen clock, Kapitaniak and collaborators [20] verified the existence of imperfect chimera states in pendula coupled on the ring by means of springs and dampers.

Our main result is to show that recurrence quantification analysis can be used as a diagnostic of chimera states. Recurrence analysis is a graphical method designed to locate hidden recurring patterns, structural and non-stationarity changes [21,22]. Recurrence quantification can be applied to scientific data. Marwan and collaborators [23] applied recurrence analysis of time series to a marine palaeo-climate record. They identified the subtle changes to the climate regime. Recurrence quantification was also considered by Zbilut and collaborators [24] as a tool for nonlinear exploration of non-stationary cardiac signals. Ding analysed the combination of three recurrence quantification analysis variables [25]. Local complex recurrence plot structures were explored and the

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results demonstrated that the combination improved nonlinear dynamic discriminant analysis.

With regard to recurrence quantification, we have calculated recurrence rate, determinism, and laminarity when the system exhibits chimera states. In this work, we have verified that the recurrence quantification is a good diagnostic for the determination of chimera states, as well as for identification of the collapse of a chimera state.

This paper is organised as follows. Section 2 introduces the model equations. In Section 3, the plot of recurrence is proposed as a diagnostic for the identification of chimera states. In the last section, we draw the conclusions.

2. Chimera states

We consider a spatially extended system formed by coupled ordinary differential equations, in which the space is discrete, while the state variable and the time are continuous. The network to be treated in this work is a set of Kuramoto oscillators that can exhibit coherent and incoherent behaviours, and it is given by

$$\dot{\Psi}_k(t) = \omega_k - \frac{1}{2R}\sum_{j=k-R}^{k+R} \sin[\Psi_k(t) - \Psi_j(t) + \alpha], \tag{1}$$

where the system is composed of N oscillators, each oscillator k ($1 \leq k \leq N$) with phase Ψ_k has an intrinsic natural frequency ω_k, R is the coupling range, and α is Sakaguchi's phase lag parameter [26]. Several nontrivial synchronisations can be observed for certain phase lags, such as decreasing synchronisation with increasing coupling strength, coexistence of stable incoherence with a partially synchronised state, and coexistence of two stable partially synchronised state [27]. In our simulations we consider $r = R/N$, $\omega_k = 0$, and the initial conditions are distributed in the interval $[-\pi, \pi]$ aiming to obtain chimera states. For $\omega_k = \omega$, Abrams and Strogatz [28] had obtained chimera states for nonlocal coupled oscillators. Rosin and collaborators [29] studied a nonlocally network of coupled electronic oscillators that approximately follows a Kuramoto-like model. They assumed identical oscillators to observe chimera states, namely the same intrinsic natural frequency for all oscillators. Laing and collaborators showed that similar patterns occur with nearly identical oscillators [30]. In this article, we considered a finite range coupling (1) that can exhibit chimera states. Moreover, this coupling presents a local (next-neighbour) coupling when $R = 1$, and a global (all-to-all) coupling when $R = (N/2) - 1$ [31].

Fig. 1 displays the scenario of coherence and incoherence states. Space–time plots are showed in the left column, and snapshots in the right column for phase lag parameter equal to 1.57, 1.47, and 1.37. The dynamics is spatially incoherent in Fig. 1a and 1b for $\alpha = 1.57$. Decreasing the value of α for 1.47 we can observe chimera state (Fig. 1c), where the oscillators with indices from 5 to 30 are in an incoherent state, while the remaining oscillators are in a spatially coherent state (Fig. 1d). In Fig. 1e and 1f, for $\alpha = 1.37$, the dynamics is spatially coherent.

3. Recurrence quantification analysis

We have studied the recurrence plots as a diagnostic of chimera states. Recurrence plots was introduced by Eckmann and collaborators [32], and it is based on the visualisation of a square matrix. The matrix elements correspond to times at which a state recurs. In the case of time series, the recurrence plot shows when the time series visits the same region of the phase space. In our case, instead of time series we use the recurrence plot in spatial series, that is given by

$$\mathrm{RP}_{i,j} = \Theta(\varepsilon - \|\Psi_i - \Psi_j\|), \tag{2}$$

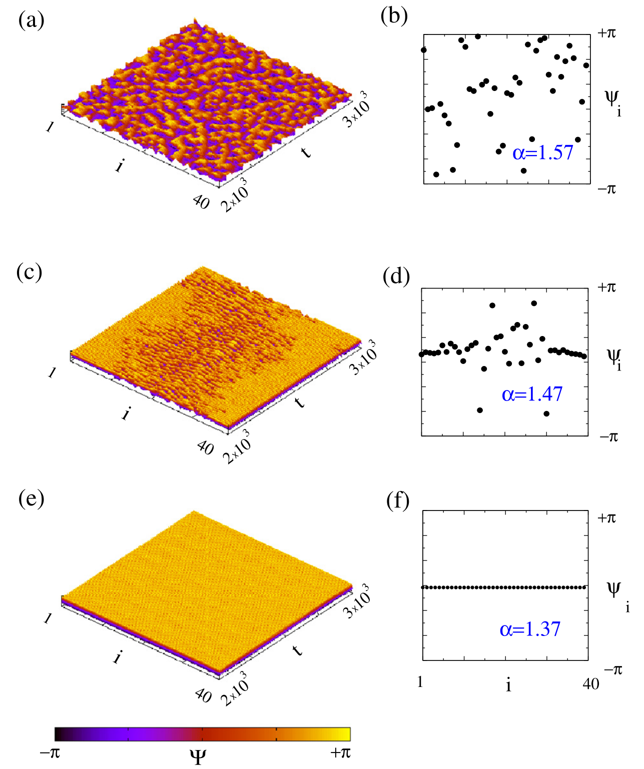

Fig. 1. (Colour online.) Space–time plots (left) and snapshots of the phases Ψ_i (right) for $r = 0.35$, $N = 40$, phase lag parameter equal to 1.57, 1.47, and 1.37. The chimera state in (c) and (d) results from a carefully chosen initial condition. The colour bar represents the values of Ψ_i.

where $\Psi_i \in \Re^m$ ($i, j = 1, \ldots, N$), N is the number of states Ψ_i, i and j in a m-dimensional space, ε is a threshold distance, $||.||$ stands for the Euclidean norm, and $\Theta(.)$ is the Heaviside function.

Fig. 2 shows recurrence plots for different values of the of the phase lag parameter and three different values of recurrence thresholds. In Figs. 2a, b, and c, we consider α equal to 1.57 for $\varepsilon = 0.01$, 0.1, and 0.3, respectively, the recurrence plot for the three cases shows one diagonal without large structures. When α is equal to 1.47 for a small ε value (Fig. 2d) there are few structures, and only some few sparse points. For an intermediate ε value the plot exhibits not only one diagonal line, but also large structures, as a result of coherent regions of a chimera state (Fig. 2e). The third case of $\alpha = 1.47$ (Fig. 2f) is for the biggest ε value. We observe a huge number of structures due the fact that an incoherent region is not anymore distinguished from coherent regions if we use an overestimated value of recurrence threshold. For α equal to 1.37 we can only see one grey region, that is independent of the ε value used (Figs. 2g, h, and i). The recurrence plot is completely grey due to regular spatial behaviour of the coupled oscillators. If we are interested in the quantification of the coherent regions observed in a chimera state, our results (Fig. 2) show that the intermediate value $\varepsilon = 0.1$ is optimal.

The recurrence quantification analysis can provide information about the system through the measures of complexity. A recurrence occurs whenever two states Ψ_i and Ψ_j visits roughly the same region in a m-dimensional space. For this reason, we have studied the chimera states that could be identified by means of the measures: recurrence rate (RR), determinism (DET), and laminarity (LAM) [33]. The recurrence rate (RR) is the density of recurrence point, given by

$$\mathrm{RR}(\varepsilon) = \frac{1}{N^2}\sum_{i,j=1}^{N} \mathrm{RP}_{i,j}(\varepsilon), \tag{3}$$