Physics Letters A 379 (2015) 2224–2229

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Hybrid thermal link-wise artificial compressibility method

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ARTICLE INFO

Article history:
Received 17 April 2015
Received in revised form 3 July 2015
Accepted 5 July 2015
Available online 10 July 2015
Communicated by F. Porcelli

Keywords:
Thermal flow
Numerical modeling
Artificial compressibility
Finite difference
Natural convection

ABSTRACT

Thermal flow prediction is a subject of interest from a scientific and engineering points of view. Our motivation is to develop an accurate, easy to implement and highly scalable method for convective flows simulation. To this end, we present an extension to the link-wise artificial compressibility method (LW-ACM) for thermal simulation of weakly compressible flows. The novel hybrid formulation uses second-order finite difference operators of the energy equation based on the same stencils as the LW-ACM. For validation purposes, the differentially heated cubic cavity was simulated. The simulations remained stable for Rayleigh numbers up to $\mathrm{Ra} = 10^8$. The Nusselt numbers at isothermal walls and dynamics quantities are in good agreement with reference values from the literature. Our results show that the hybrid thermal LW-ACM is an effective and easy-to-use solution to solve convective flows.

1. Introduction

Accuracy, consistency, stability and convergence are desirable properties when devising a numerical method. Moreover, if the method is easy to implement, then the Graal is close! Our study concerns the presentation and validation of a Hybrid Thermal Link-Wise Artificial Compressibility Method (HTLW-ACM) developed for the prediction of thermal flows. Of course, simulation of thermal flows is a subject of interest in many different areas of engineering applications as it is closely related to energy. Nowadays, applications are ranging from micro–nano scales to the city scale. The latest requires large computational cost. In the paper, we propose a easy-to-implement method with a high parallelism potential to compute incompressible thermal flows.

Recently, a novel formulation of the artificial compressibility method (ACM) [1], known as the link-wise artificial compressibility method (LW-ACM), was proposed [2]. The development lies in ideas similar to the Lattice Boltzmann Method (LBM), except that LW-ACM requires less degrees of freedom than LBM [3]. These recent developments allow to solve the incompressible Navier–Stokes equations. Besides other interesting features, the LW-ACM shares many similarities with the LBM and seems therefore well-suited for high-performance implementations [4–6].

Similarly to LBM, three approaches can be used to incorporate thermal effect in LW-ACM: the multi-speed, the hybrid and the double-population. The multi-speed approach consists in extending the distribution function in order to obtain the macroscopic temperature [7]. However, this approach requires much more computational effort because of the additional discrete speeds and suffers from numerical instabilities [8]. The hybrid approach consists in solving the temperature equation independently. The advection–diffusion equation for temperature is solved by a finite difference technique. The heat and flow are coupled by the buoyancy force. This approach enhances the numerical stability compared to the multi-speed approach [9]. In the double-population approach, two sets of distribution functions are used: one for the energy field and the other for the velocity field [10]. For LBM, it means that the number of discrete velocities is multiplied by a factor of 2. Of course, the two first approaches are preferable from a theoretical point of view.

A first double-population LW-ACM is presented in [2] and preliminary results are presented for the thermal Couette flow. To our knowledge, this is the only work dealing with thermal LW-ACM so far. As the hybrid approach is preferable from a theoretical point of view and allows to handle more complex physical phenomena (for example phase change), we decided to develop the HTLW-ACM and present it in this paper. The new model is implemented to simulate the Differentially Heated Cubic Cavity (DHCC) case. Results are compared to data from the literature and present a good agreement. We also discuss the implementation performance of the model.

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http://dx.doi.org/10.1016/j.physleta.2015.07.010

Nomenclature

c_s	speed of sound
$\boldsymbol{F}$	external force per unit mass
f_α	local density fractions
$f_\alpha^{(\mathrm{e})}$	local equilibrium functions
$f_\alpha^{(\mathrm{e,o})}$	odd parts of the equilibria
f_α^*	updated density fractions
Gr	Grasshoff number
N	cubic cavity size
p	pressure
Pr	Prandtl number
Q	number of discrete velocities
Ra	Rayleigh number
Re	Reynolds number
T	temperature
$\pm T_0$	imposed temperatures
t	time
$\boldsymbol{u}$	fluid velocity
w_α	stencil weights
$\boldsymbol{x}$	position

Greek symbols

δt	time step
δx	mesh size
ε	scaling coefficient
ζ	artificial compressibility
κ	thermal diffusivity
ν	kinematic viscosity
$\boldsymbol{\xi}_\alpha$	discrete velocities
ρ	artificial density
ω	relaxation frequency

Subscripts

α	stencil index
i, j, k	mesh indices
l	time index

Other symbols

$\widetilde{\partial}$	discrete partial derivative
$\widetilde{\nabla}^2$	discrete Laplacian

2. Hybrid thermal link-wise artificial compressibility method

2.1. Link-wise artificial compressibility method

First introduced by Chorin in 1967, the artificial compressibility method is an accurate numerical approach to solve the incompressible Navier–Stokes equations (INSE), which are expressed in dimensionless form as:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\partial_t \boldsymbol{u} + \mathrm{Re} \times \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \nabla^2 \boldsymbol{u} + \boldsymbol{F} \tag{2}$$

where $\boldsymbol{u}$ is the fluid velocity, p is the pressure, $\boldsymbol{F}$ is the external force per unit mass and Re is the Reynolds number.

Whereas most numerical methods eventually resort to a Poisson (or Helmholtz) solver to process the derived pressure equation, the ACM substitutes the artificial compressibility equation (ACE) to Eq. (1):

$$\zeta \times \partial_t p + \nabla \cdot \boldsymbol{u} = 0 \tag{3}$$

where ζ is defined as the artificial compressibility. The artificial density ρ is linked to the pressure by the artificial equation of state $p = \rho/\zeta$, yielding the artificial speed of sound: $c_s = 1/\sqrt{\zeta}$. The ACE leads to solutions of the INSE in the case of steady flows as well as for transient flows in the limit of vanishing Mach number [11].

The pressure time derivative term in the ACE enables explicit time-marching integration, making the ACM well-suited for parallel implementations. Attention has been drawn lately on the physical and numerical similarities between the ACM and the LBM [3], which is also an explicit time-marching method for solving the INSE. Operating on regular Cartesian meshes, the LBM has been successfully implemented on various high-performance computing (HPC) systems, especially on massively parallel accelerators such as graphics processing units [6].

The link-wise artificial compressibility method (LW-ACM) is a recently proposed formulation of the ACM which takes advantage of the analogies between ACM and LBM by following a similar integration scheme. As LBM, the LW-ACM operates on a *lattice*, i.e. a regular Cartesian grid of mesh size δx with a constant time step δt associated to a set of Q discrete velocities $\boldsymbol{\xi}_\alpha$. The discrete velocity set, hereafter referred to as the *stencil*, is generally chosen such

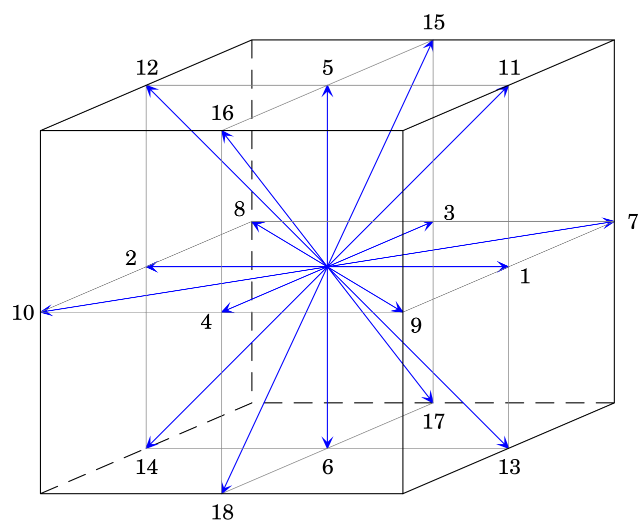

Fig. 1. The D3Q19 stencil — blue arrows represent the $\boldsymbol{\xi}_\alpha$ velocities. This stencil links any bulk node to 18 of its nearest neighbors. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

as to link neighboring nodes of the mesh within one time step. In the present work, we used the common three-dimensional D3Q19 stencil presented in Fig. 1, which consists of 19 velocities including the rest velocity $\boldsymbol{\xi}_0 = \boldsymbol{0}$. The symmetries of this stencil ensure the conservation of mass and momentum [12].

The fluid is represented by a set of Q dependent variables[1] $\{f_\alpha\}$ defined at the mesh points, and such that:

$$\rho = \sum_\alpha f_\alpha \tag{4}$$

$$\rho \boldsymbol{u} = \sum_\alpha f_\alpha \boldsymbol{\xi}_\alpha \tag{5}$$

[1] Within the LBM framework, the $\boldsymbol{\xi}_\alpha$ are usually referred to as *particular velocities* and the f_α as *particular densities* Unlike LBM, the LW-ACM is not derived from the Boltzmann equation and therefore does not share its mesoscopic point of view.