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Electron–phonon interaction effect on persistent current in a one-dimensional quantum ring by using a simple model

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ABSTRACT

We use a simple model to study the electron–phonon interaction influences on persistent current in a one-dimensional quantum ring enclosed by a magnetic flux. With increasing the temperature, persistent current amplitude is reduced, especially in a quantum ring with two ions per primitive cell (diatomic ring) because of the participation of optical phonons. Furthermore, the periodicity of the Aharonov–Bohm oscillations changes to $\Phi_0/2$ (Φ_0 is magnetic flux quantum). In a diatomic ring, by increasing the difference between left and right nearest-neighbor hopping integrals at zero temperature, persistent current variations show a transition from metallic to insulator against distinctive behavior at nonzero temperature.

1. Introduction

In quantum rings that the loop circumferences are smaller than the electron's phase coherence length (l_φ), it is possible to induce currents by threading the center of the ring with a magnetic flux [1–13]. Scattering due to disorder, electron–electron and electron–phonon interactions changes l_φ and the related quantum interferences including persistent current (PC) and Aharonov–Bohm (AB) oscillations [14–30]. Sharvin et al. have shown that elastic scattering does not destroy electron coherence [31]. Landauer and Buttiker have found that inelastic scattering can cause fluctuations in the persistent current but it does not destroy it [32]. Yang et al. have studied the inelastic scattering in a one-dimensional mesoscopic ring of finite temperature [33]. Their study shows that inelastic scattering between electron and phonon reduces the amplitude of the PC, and with increasing temperature, it will destroy the AB phase coherence. According to Kulik's results, both the weak and strong electron–phonon coupling does not change the periodicity of the current oscillations in spite of the fact magnitude of the PC is suppressed [34]. Influence of the electron–phonon interaction on AB oscillation has been investigated by Vasilopoulos et al. [35]. They have shown that for elastic collisions, the period of oscillation is $\Phi_0/2$ and for inelastic collisions, the period of oscillation remains Φ_0. Bayindir and Kulik have studied electron–phonon interaction by using Frohlich model in tight-binding approximation [36]. According to their study, in both weak and strong coupling, the effective mass of electrons is increased and amplitude of PC is suppressed. Caro et al. have proposed the well-known classical stochastic Langevin equations that ion–electron interaction is taken into account with stochastic forces with random magnitude and orientation [37].

This study investigates the effects of electron–phonon interaction on persistent current in a one-dimensional quantum ring through a simple model. When a phonon travels and interacts with an electron, the random energy exchange between them. Because of the inelastic collision of electrons with phonons and phase randomization of electron wave function, the amplitude of PC is reduced and the periodicity of the oscillations changes to $\Phi_0/2$ at nonzero temperature. Furthermore, in a quantum ring with two ions per primitive cell (diatomic ring), persistent current via the difference between the left and right nearest-neighbor hopping integrals (Δt) shows that increasing Δt leads to a transition from metallic to insulator at zero temperature against the distinctive behavior at higher temperature. In addition, due to the participation of the optical phonons in collisions, the reduction of the persistent current is more than monatomic ring. Moreover, diagonal disorder decreases PC significantly and the presence of disorder maintains the periodicity of Φ_0 even at nonzero temperature.

The paper is organized as follows. The model and formalism are explained in the next section. We present the results and discussion in Section 3. Finally, we end the paper with a brief conclusion.

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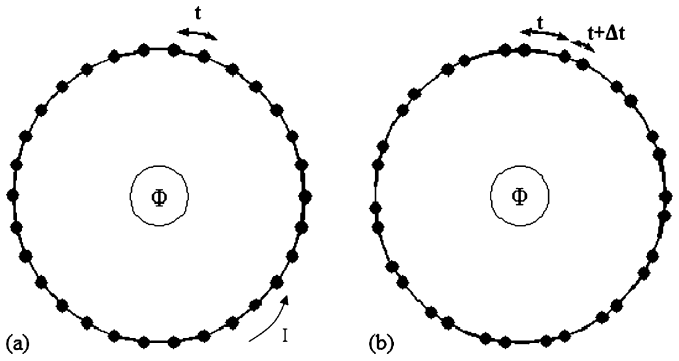

Fig. 1. (a) One-dimensional monatomic quantum ring threaded by a magnetic flux. (b) One-dimensional diatomic quantum ring threaded by a magnetic flux with the difference between the left and right nearest-neighbor hopping integrals Δt.

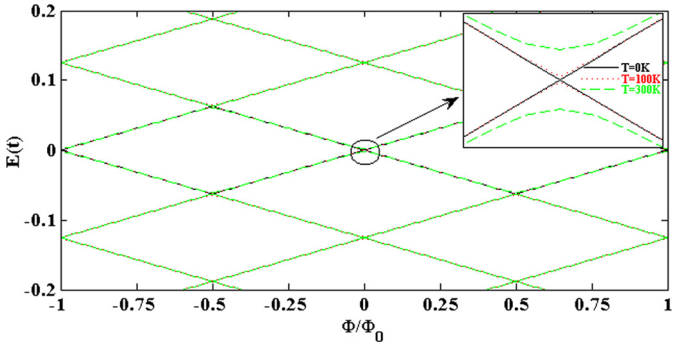

Fig. 2. Energy spectrum as a function of magnetic flux in a one-dimensional monatomic quantum ring with $N = 100$ at $T = 0$ K, 100 K and 300 K.

2. Models and formalisms

To study the electron–phonon interaction contribution in persistent current, a simple model has been introduced in a one-dimensional (1D) quantum ring threaded by a magnetic flux depicted in Fig. 1. We have drawn attention to the Born–Oppenheimer approximation which suggests the separation of the electronic and nuclear motions. The Hamiltonian of the electrons in the absence of the electron–phonon interaction in a tight-binding model is considered as

$$H = \sum_{i=1}^{N} \varepsilon_i c_i^{\dagger} c_i + \sum_{\langle ij \rangle} t_{ij} c_i^{\dagger} c_j, \tag{1}$$

where N is the number of sites, and $c_i^{\dagger}$ and c_i are creation and annihilation operations of electrons at the site i. The on-site energy, ε_i, is considered zero in a perfect quantum ring and is randomly distributed in interval $[-W/2, W/2]$ in a diagonal disordered ring. The nearest-neighbors hopping integral energy, t_{ij}, is regarded $-t$ for odd i and even j, and $-t - \Delta t$ for even i and odd j. This means that $\Delta t \neq 0$ refers to ring with two atoms per primitive cell named as diatomic ring, Fig. 1(b), opposite to the monatomic ring with one atom per primitive cell shown in Fig. 1(a).

The number of phonons in any normal mode at temperature T is $n(q) = 1/(e^{\hbar\omega(q)/k_B T} - 1)$ that k_B is the Boltzmann constant. Phonon frequencies, $\omega(q)$, in a 1D monatomic ring with Born–Von Karman periodic boundary condition can be obtained by

$$\omega(q) = 2\sqrt{\frac{K}{M}}\left|\sin\left(\frac{1}{2}qa\right)\right|, \qquad q = \frac{2\pi}{a}\frac{n}{N},$$
$$n = -(N-1), (-N+2), \ldots, (N-2), (N-1) \tag{2}$$

In diatomic ring, connected with spring strengths K and G, ω vs. q leads to the two acoustic and optical branches. The $\sqrt{K/M}$ and $\sqrt{G/M}$ are regarded $\omega_{\max} = 10^{13}$ Hz and $1.1\omega_{\max}$, respectively.

It is supposed that the electron–phonon collision probability is P which is assumed 0.2. Only phonons with $\hbar\omega(q)$ comparable to or less than $k_B T$ can be absorbed or emitted by electrons. The electrons collide with phonons and after collision, move with different momentum and energy. This means that electron–phonon interaction is an inelastic interaction.

The probability for phonon absorption in scattering event, P_a, is proportional to $n(q)$ and a probability for emission of a phonon in a scattering event, P_e, is proportional to $n(q) + 1$ [38]. P_e and P_a versus transferred energy from phonons to electrons show symmetry about zero with exponential function. On the basis of this, the energy transferred to (from) electrons in collision with phonons put forward to be chosen randomly from a normal distribution with mean μ $(= 0)$ and variance σ $(= k_B T)$, and selected from the allowed phonon energies.

All corrections to the atomic potential required to produce the full periodic potential of the crystal are contained in an extra term in Hamiltonian and it can be held in the hopping integral term. In other words, the electron–phonon interaction can be included in hopping term as

$$H_{e\text{-}ph} = \sum_{\langle lk \rangle} \eta c_l^{\dagger} c_k \tag{3}$$

that η represents the exchange energy between electrons and phonons in electron–phonon collision. The indices l and k in Eq. (3) indicates $P \times N$ sites selected randomly from N sites. The eigenenergies are obtained by exact diagonalization of the Hamiltonian. At finite temperature T, the equilibrium persistent current, I, in the canonical ensemble with fixed electrons N_e can be expressed as

$$I = -\sum_n f_n \frac{\partial E_n}{\partial \Phi} \tag{4}$$

where $f_n = 1/[e^{\beta(E_n - \mu)} + 1]$ is the Fermi–Dirac distribution of electrons with chemical potential μ and $\beta = (k_B T)^{-1}$.

3. Results

By the model described in the previous section, the energy spectrum and the persistent current have been investigated in a one-dimensional quantum ring threaded by a magnetic flux at nonzero temperature. The energy, magnetic flux and persistent current are respectively in units of t, Φ_0 and t/Φ_0.

Energy spectrum as a function of magnetic flux is illustrated in Fig. 2 for a 1D monatomic ring with $N = 100$ at $T = 0$ K, 100 K and 300 K. It is realized that at nonzero temperature, the phonons are present and the thermal excitations interact with the electrons inelastically, the energy levels repel each other in points that the levels cross at zero temperature. Additionally, the energy gaps enhance as the temperature increases. The highly probable phonons which are taken part in collisions are the low frequencies phonons which have constant density of states. Therefore, the collision probability for different temperature is considered constant.

Fig. 3 illustrates energy spectrum as a function of magnetic flux in diatomic ring with $N = 100$ and $\Delta t = 0.05$ at $T = 0$ K. The diatomic basis can open up gap where anticrossing occurs in the monatomic basis as a result of breaking the effective time-reversal symmetry. As Δt increases, the energy gap enhances. In fact, quantum ring with two ions per primitive cell tends to insulator with increasing Δt.

The current carried out by an eigenstate is proportional to the slope of the energy versus the magnetic flux curve. Fig. 4 shows persistent current with respect to magnetic flux in a 1D