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Information theoretical methods as discerning quantifiers of the equations of state of neutron stars

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ABSTRACT

In this work we use the statistical measures of information entropy, disequilibrium and complexity to discriminate different approaches and parametrizations for different equations of state for quark stars. We confirm the usefulness of such quantities to quantify the role of interactions in such stars. We find that within this approach, a quark matter equation of state such as SU(2) NJL with vectorial coupling and phase transition is slightly favoured and deserves deeper studies.

1. Introduction

The thermodynamic macro-state of a physical system can be constructed from a statistical description of the system under consideration and is generally a function of its internal energy, volume and number of particles. Physicists have defined a mathematical function that encodes this macro-state and that is valid for all states of *equilibrium*. This function, called *entropy*, constitutes a *fundamental quantity* of the system and encodes all the thermodynamic knowledge regarding that system. By the mid-end of the 19th century, Gibbs and Boltzmann examined the meaning of this function and suggested a relation to the internal *order* of the physical systems. It must be said, however, that this concept of order was defined in terms of the number of ways a system can accommodate its energy given the internal structure. Hence, metals are more orderly than liquids because they have less ways to distribute the same amount of energy. See [1] for a review of the concept of entropy and its relation to energy and work and how entropy drives the evolution of a system.

Even if the above description seems already very wide, presently the word "entropy" is employed yet to encode another meaning, related to *information*, to be defined precisely later on. From this point of view we characterize bound structures in terms of "missing information", represented by the Shannon entropy [2]. Although the thermodynamic entropy and the Shannon entropy share some properties and mathematical relationships, each function has a different meaning. The Shannon information, formerly known as *cybernetic information* or *semiotic information*, is actually an abstract quantity related to messages, independently of the form a message is expressed, and is not obviously subject to the Second Law of thermodynamics. Actually, one cannot easily convert one into the other, at least in a general situation (but see [3]; see below).

Information theory has been applied to natural sciences, specially molecular biology (see, for example, the textbooks in [4,5]), with success to find correlations and patterns among a collection of systems of the same type or among different systems that show some common features. The quantity to be defined as the *information content* of a system can be related to a somewhat "more physical" quantity: the *complexity* of that system [6]. With the aid of this new concept, the hierarchy of white dwarf and neutron star types based on different compositions have been studied [7–9]. Here we intend to extend this vision, performing the calculation of these quantities for neutron stars of different compositions that

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include different interactions and phases of matter under that extreme conditions, inferring in this way a hierarchy for the equation of states based on how these quantifiers change when we change the composition of a neutron star.

2. Concepts and definitions

In mathematical terms, we can quantify information by means of the probability, p_i, of an event to occur. Guided by the mathematical properties that such a quantity must have, Shannon [2] defined information as $I = -\log_b(p_i)$ for some basis b. Later on, Shannon developed his information theory with the purpose of quantify the information that could be transmitted over a given line of communication.

According to these ideas, the average amount of (missing) information in a stream of N symbols is given by:

$$\frac{I}{N} = -K \sum_{i=0}^{N} p_i \log_b(p_i) \equiv H(p), \tag{1}$$

where K is a constant that together with the basis b gives us the units ($b = 2$ and $K = 1$ give us *bits*; $b = e$ and $K = 1$ give us *nats*, the units used in this work).

This quantity is defined as the *entropy* or, better, the *information entropy* of the stream of symbols. The generalization we seek should be valid for continuous systems admitting a probability distribution:

$$H = -K \int p(x) \log_b\bigl(p(x)\bigr) d\vec{x}. \tag{2}$$

In order to apply this concept to physical systems we have to define properly what quantity to use as a probability distribution. In condensed matter, the momentum and position distributions in the phase space or even the atomic number have been used [10–14] (these authors used the information entropy in order to study the *complexity* of a system, a concept to be defined below). In astrophysics, the first attempts were made to study white dwarf stars [7], neutron stars [8] and finally hadronic *vs* quark stars [9]. In all these works it was assumed that the probability distribution is proportional to the energy density (or the mass density in the non-relativistic case) profile of the star, $\epsilon(r)$, obtained when solving the equations of structure of the star (in the case of neutron stars, the Tolman–Oppenheimer–Volkoff equations, the relativistic version of hydrostatic equilibrium). This was justified by the statement that the energy/mass density is related to the probability of finding some particles at a given defined location inside the star. Even though it may be criticized, it proved difficult to suggest a better alternative from the available physical quantities.

Actually, depending on the adopted system of *units*, the density profile can lead us to a "negative" information entropy since the density can be greater than one in the log factor. However, in principle this should not be surprising: even a normalized probability density function like the squared wave function of the quantum square potential well would give us a negative information entropy if calculated with the above expression (for some specific parameters of the well, of course). It should be noticed that in the works quoted above, the authors do not use directly the information content (the Shannon information) of the system but its exponential. This prescription, of the form $I \equiv \exp(H)$, is due to Sañudo and López-Ruiz in [12] who, in turn, adopted it from [15] (we will see that this is a more suitable quantity than H below). In the present work we keep the latter and use the following expression to compare the information entropy of the neutron stars:

$$H \equiv -K b_0 \int_0^R \epsilon(r) \ln\bigl(\epsilon(r)\bigr) 4\pi r^2 dr, \tag{3}$$

where $K = 1$, $b_0 = 8.89 \times 10^{-7}$ km^{-3}, R is the radius of the object in km and ln is the natural logarithm. The constant b_0 is a properly chosen quantity that makes the integral dimensionless and is related to the system of units used in the density profile. Thus, $b_0 \equiv \frac{\epsilon_0}{c^2 M_\odot}$, where $\epsilon_0 = 1$ MeV/fm^3, c is the speed of light and $M_\odot = 2 \times 10^{33}$ g is the mass of the Sun.

In Ref. [9], we have reviewed the complexity and the disequilibrium related to the intuitive behaviour of gases, liquids and solids. Briefly, simple systems should have low complexity (zero complexity if the system is *ideal*), defined as

$$C \equiv I \times D = \exp(H) \times D \tag{4}$$

where H is the information entropy and the disequilibrium, D, quantifies how far the system is from equiprobability, being expressed by the integral

$$D \equiv b_0 \int_0^R \bigl[\epsilon(r)\bigr]^2 4\pi r^2 dr. \tag{5}$$

Following the same reasoning of [6] we can link the information content of any system with the information content of two extremes of ideal systems with quite opposite descriptions and physical properties. These are the *ideal gas* and the *perfect crystal*: the former can be described by having all its accessible states as equally probable, while the latter has a privileged accessible state (in the limit of idealization, one accessible state with $p = 1$ and consequently $p = 0$ for the others).

The crystal has, by definition, minimum (information) entropy, while the gas has maximum information content.[1]

Disequilibrium is, in this approach, a quantity defined to represent some kind of distance to the equiprobability of the accessible states. In other words it represents the information energy [16]. Thus, the perfect crystal is at maximum distance of the equiprobability (highest possible disequilibrium) while the perfect gas is at equiprobability (disequilibrium equals zero).

If we measure the complexity of some system, and allow its proximity to a crystal or a gas to encode the degree of order of the system under consideration, then one can wonder what state of the system is preferable for Nature to realize. In particular, by calculating the information entropy of each star allowed by the chosen equation of state, we find a quite univocal behaviour of this quantity with the mass and radius of the stars in the stellar sequence.

3. The equations of state

To clarify the issue of the composition using these concepts, we have compared in Ref. [9] the information entropy, the disequilibrium and the complexity for two different sample cases: the quark stars constructed from a MIT Bag model equation of state[2] (an admixture of quarks up, down and strange in approximately equal amounts, see [17,18]) and hadronic neutron stars from SLy4 equation of state (neutron rich in the core) [19]. We found that the information entropy trend for both cases is pretty much the same when viewed as a function of the stellar mass, but differs a lot when plotted as a function of the stellar radius.

In that occasion,[3] we found that (having in mind what we stated in previous section):

[1] This is physically grounded to how energy can be distributed among the degrees of freedom of the system under consideration.

[2] For this model we employed the exact solution for the Einstein equations first developed by [20] and effectively calculated by us in [21].

[3] The conclusions of Ref. [9], i.e., the range of the masses, should be slightly corrected due to a numerical error later detected in our calculations; we present the corrected figures here.