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Geodesic mode spectrum modified by the energetic particles in tokamak plasmas

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ABSTRACT

Effect of a minor concentration of the energetic particles on GAM spectrum in a tokamak is analyzed by drift kinetic theory taking into the account the electron current and diamagnetic drift. A novel method of Jacobi functions is applied to solve the drift kinetic equation for the energetic bounce particles in the limit of high bounce frequency in comparison with the GAM frequency. Using the Q-asymptotic of Jacobi function, it is shown that the energetic minority ions can form the continuum minimum/maximum at the NB or ICR power deposition maximum where the geodesic eigenmode may be excited. In this case, the electron current modeled by shifted Maxwell distribution overcomes the ion Landau damping threshold thus resulting in the GAM instability.

1. Introduction

Typically, a minority concentration of energetic ions may appear during the neutral beam (NB) and/or ion cyclotron resonance (ICR) heating in all modern tokamaks, where discharges are frequently accompanied by the Geodesic Acoustic Mode (GAM) instability [1–4], which may have strong effect on plasma confinement [5]. Presence of the energetic particles is also expected in the future fusion reactor (ITER). Theoretically discovered GAM oscillations [6] have the $N = 0$ axisymmetric toroidal mode structures with $M = 0$, ± 1 poloidal mode numbers, which were confirmed by reflectometry measurements [3], and the $M = \pm 2$ mode structure is also visible in magnetic probe measurements [1,2,4]. The GAM spectrum is calculated in kinetic approximation in series of works [7–11] and the modes are subdivided into relatively high frequency mode $\omega_G^2 \approx (7T_i/2 + 2T_e)/R_0^2 m_i$ and ion-sound mode $\omega_{si}^2 \approx T_e/R_0^2 q^2 m_i$ [10,11], where R_0 is major radius, q is safety parameter, $T_{e,i}$ electron and ion temperatures. Recently, the semi-numerical theory of the GAM instability driven by energetic trapped beam ions [12] has been developed, that is later confirmed in numerical calculations [13] of the instability driven by a balanced beam. These instabilities are produced by perturbations of the ion equilibrium distribution in the velocity space. The theoretical analysis [14] demonstrates that the GAM spectrum does not changed for the frequency above the ion bounce resonance frequency $\omega_G^2 \gg \sqrt{\varepsilon T_e / 2R_0^2 q^2 m_i}$ of the main ions in the large aspect ratio tokamak $\epsilon = r/R_0 \ll 1$, but the modes practically disappear in the frequency band below the bounce frequency. It will be interesting to extend a theory of the bounce particle effect on the GAM spectrum in the case of the energetic ions minority.

Here, it is proposed to analyze the effect of the minor concentration of the energetic bounce particles on GAM spectrum in a tokamak by drift kinetic theory taking into the account the electron current and diamagnetic drifts. A novel method of Jacobi functions [15] adopted for the calculation of the electron wave dissipation [16] in tokamaks is applied to solve the drift kinetic equation for the energetic particles ($T_h \gg T_i$) in the limit of the higher bounce frequency in comparison with the GAM one, $\omega_G^2 \ll \sqrt{\varepsilon T_h / 2R_0^2 q^2 m_i}$. The dispersion relation for GAM type modes is obtained by averaging the divergence of Ampere equation over the magnetic surface and, as a result, the balance of the radial current $j_{rp} + j_{r0} = 0$ is used where j_{rp} is the polarization current and j_{r0} is the averaged value of the geodesic current.

2. Dispersion equation

To find the GAM dispersion for the toroidal mode number $N = 0$, the drift kinetic approach is employed in the quasi-toroidal coordinates (r, ϑ, ζ) for the large aspect ratio tokamak $\varepsilon = r/R_0 \ll 1$, where the circular surfaces ($R = R_0 + r \cdot \cos\theta$, $z = r \cdot \cos\theta$) are formed by the magnetic field with toroidal and

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poloidal components, $B_\zeta = B_0R_0/R$, $B_\vartheta = rB_\zeta/qR_0$ with $B_\theta \ll B_\xi$. A perturbed distribution function for each species (electron, cold and hot ions $\alpha = e, i, h$) driven by the radial $E_r = E_1 \sin\vartheta$ and parallel $E_3 = E_s \sin\vartheta + E_c \cos\vartheta$ oscillating electric fields $\propto \exp(-i\omega t)$ can be found from the drift kinetic equation where the dimensionless variables (u, λ) are used in the velocity space

$$w\frac{\partial f}{\partial\vartheta} - i\Omega_\alpha f = \frac{e_\alpha qR}{m_\alpha}F_\alpha\left[\frac{wE_3}{v_{T\alpha}^2} + \frac{2+\eta_\alpha(u^2-3)}{2v_{T\alpha}\omega_{c\alpha}d_r}E_2 - u^2\frac{(2-\lambda+\varepsilon(2+\lambda)\cos\vartheta)}{2R\omega_{c\alpha}v_{T\alpha}}E_1\sin\vartheta\right]. \quad (1)$$

Here $w_\parallel = su\sqrt{1+\varepsilon\cos\vartheta-\lambda}$ is the parallel velocity with $s = \pm1$, $u = v/v_T$ is the module of the normalized velocity, as an equivalent to the energy variable, λ is the dimensionless magnetic moment, $v_\perp^2 B_0/v_T^2 B$; $\omega_{c\alpha} = eB/cm_\alpha$ and $\Omega_\alpha = \omega R_0 q/v_{T\alpha}$ are the cyclotron and normalized wave frequencies, $v_{T\alpha} = \sqrt{T_\alpha/m_\alpha}$ is thermal velocity, F_α is the Maxwell distribution of the α-specie, $\partial n_\alpha/\partial r = -n_\alpha/d_\alpha$, and is the density gradient of the ions. Potential approach is assumed for the parallel and binormal electric field that impose relation $E_2 = E_3/h_\vartheta$ where $h_\vartheta = B_\vartheta/B_0$ is the magnetic field inclination.

To obtain the GAM dispersion the oscillating density components are presented as the sum $n^{(\alpha)} = n_s^{(\alpha)}\sin\vartheta + n_c^{(\alpha)}\cos\vartheta$. Further, the sin or $\cos\vartheta$-density components and j_{r0}-current will be obtained integrating the solution of Eq. (1) in the velocity space (u, λ) and over ϑ

$$n_{s,c}^{(\alpha)} = \int_0^\infty v_{T\alpha}^3 u^2 du \oint \overbrace{\sin\vartheta}^{\{\cos\vartheta\}} d\vartheta \sum_{s=\pm1}\int_0^{1+\varepsilon\cos\vartheta}\frac{f_{s,\alpha}d\lambda}{\sqrt{1+\varepsilon\cos\vartheta-\lambda}};$$

$$j_r^\alpha = e_\alpha v_{T\alpha}^3\int_0^\infty u^2du\oint d\vartheta\sum_{s=\pm1}\int_0^{1+\varepsilon\cos\vartheta}\frac{V_r f_{s,\alpha}d\lambda}{2\sqrt{1+\varepsilon\cos\vartheta-\lambda}}$$

where integration should be performed over the trapped $1-\varepsilon \le \lambda \le +\varepsilon\cos\vartheta$ and untrapped $0 \le \lambda \le 1-\varepsilon$ regions separately. Finally, quasi-neutrality will be used for each component.

First, we begin to treat Eq. (1) changing the variable from λ to the new κ-variable for the untrapped $\kappa^2 = 2\varepsilon/(1+\varepsilon-\lambda)$ and trapped particles $\hat\kappa^2 = (1+\varepsilon-\lambda)/2\varepsilon$. In the untrapped equation, the Jacobi functions [15] $\sin\vartheta/2 = \mathrm{sn}(\kappa, x)$, $\cos\vartheta/2 = \mathrm{cn}(\kappa, x)$ with the Jacobi variable [16] $\vartheta/2 = \mathrm{am}(\kappa, x)$ are introduced that transforms Eq. (1) to the form

$$\frac{\partial f_{\rm un}}{\partial x} - \sqrt{2}is\frac{\Omega f_{\rm un}}{u\sqrt{\varepsilon}} + \frac{\sqrt{2}se_\alpha quF_\alpha(\kappa^2+2\varepsilon-H\kappa^2)}{\sqrt{\varepsilon}\omega_{c\alpha}m_\alpha v_{T\alpha}\kappa}\times E_1\,\mathrm{sn}(\kappa,x)\,\mathrm{cn}(\kappa,x) = \frac{4e_\alpha RqF_\alpha}{m_\alpha v_{T\alpha}^2}\left[E_s\,\mathrm{sn}(\kappa,x)\,\mathrm{cn}(\kappa,x) - E_c\big(\mathrm{sn}(\kappa,x)^2 - 1/2\big)\right]\times \mathrm{dn}(\kappa,x) \quad (2)$$

where $H = (2+u^2\eta-3\eta)R_0E_s/(u^2d_\alpha h_\vartheta E_1)$ and the drift term driven by the E_c-field is ignored. Expanding Jacobi functions in the Q-series [15] and using the periodic boundary conditions at $x = \pm K(\kappa)$, we get the solution of Eq. (2) induced by the E_1 and E_s-fields

$$f_{1,\rm un}^{(\alpha)} = i\frac{2\sqrt{2}\pi qe_\alpha F_\alpha E_1}{\sqrt{\varepsilon}\omega_{c\alpha}m_\alpha v_{T\alpha}}\sum_{p=1}^N\frac{(\kappa^2+2\varepsilon-H\kappa^2)Q^pu^2\breve\Omega_{\alpha,p}}{K(\kappa)\kappa^3(1+Q^{2p})(u^2-\breve\Omega_{\alpha p}^2)}\times\sin\left(\frac{\pi px}{K(\kappa)}\right) \quad (3a)$$

and driven by E_c-parallel field where few necessary terms of the expansion may be kept

$$f_{\rm c,un}^{(\alpha)} = \frac{e_\alpha RqF_\alpha E_c}{m_\alpha v_{T\alpha}^2}\sum_{p=1}^4\frac{4u^2Q^p}{p(u^2-\breve\Omega_{\alpha,p}^2)}\left[\frac{1}{(1+Q^{2p})} + \frac{\pi^2(p^2-4Q-(2-p)Q^2-4p(4-p)Q^3)}{K(\kappa)^2\kappa^2}\right]\times\sin\left(\frac{\pi px}{K(\kappa)}\right) \quad (3b)$$

where $\breve\Omega_{p,\alpha} = \omega R_0 q\kappa K(\kappa)/p\pi\sqrt{2\varepsilon}v_{T\alpha}$, $Q = [1-(1-\kappa^2)^{1/4}]/2[1+(1-\kappa^2)^{1/4}]$, and $K(\kappa)$ is the first kind elliptic integral. The $\cos\vartheta$-part of the drift oscillations has no effect on the density or radial current oscillations due an antisymmetric distribution in the velocity space ($s = \pm1$), but the solution $f_{\rm s,un}^{(\alpha)}$ with drift term driven by the $E_s\sin\vartheta$-field component is important. To calculate the $n_{s,c}^{(\alpha)}$-density oscillation, integration of Eqs. (3a), (3b) have to be done in the velocity space (u, κ) and over ϑ following to procedure of Ref. [16]. It is easy to perform the integration over the u-variable accounting bounce resonances that produces expressions via the dispersion function $Z = \int_{-\infty}^{\infty}dt\frac{\exp(-t^2)}{(t-x)}$ where $x = \frac{\breve\Omega_{p,\alpha}}{\sqrt{2}}$ depends from $\kappa K(\kappa)$, but the final κ-integration is only possible to perform numerically that involves specific tokamak parameters [12]. To proceed with analytical calculations, it is assumed that the hot ions have the bounce frequency larger than the GAM frequency and the main ions are cold. In this case, using expansion over parameter $\Omega_h/\sqrt{2\varepsilon} \ll 1$ and taking into account the order $O(1/\sqrt{\varepsilon})$ in Eq. (3) for the hot minority ions, we get the $\sin\vartheta$-component of the ion density oscillations and ϑ-averaged radial current after integration over velocity space (u, κ) and over ϑ

$$n_{\rm un}^{(h)} = \frac{e_hn_0r_hq}{m_hv_{Th}}\left(0.56\frac{i\Omega_hE_1}{\sqrt{\varepsilon}\omega_{ch}} + \frac{R_0E_c}{v_{Th}}\right)\sin\vartheta, \quad (4)$$

$$\langle j_{\rm un}^{(h)}\rangle = -\frac{e_h^2n_0r_hq}{m_h\omega_{ch}}\left\{\frac{\rho_h\Omega_h}{\sqrt{\varepsilon}R_0}\left[\left(0.4i - 0.25\frac{\Omega_h^5}{\varepsilon^{5/2}}\right)E_1 - \left(0.27i + 0.06(3\eta_h-2)\frac{\Omega_h^3}{\varepsilon^{3/2}}\right)\frac{R_0E_s}{d_hh_\vartheta}\right] + \left(1+0.23i\frac{\Omega_h^5}{\varepsilon^{5/2}}\right)E_c\right\} \quad (5)$$

where $\rho_h = v_{Th}/\omega_{ch}$ is Larmor radius, r_h is the relative density of the hot ions. Due to the used expansion, it is obvious that the dissipative (real) part of the current in Eq. (5) driven by E_1 and E_c-components is relatively small in comparison with the E_s-component that also has the small value $\propto 0.06\Omega_h^3/\varepsilon^{3/2} \ll 1$. It should be noted that the number of decimal digits in coefficients corresponds to the 2% accuracy in numerical integrations over κ in all calculations of the density and current. The hot density in Eq. (4), as well for trapped ions, produces the small effect on the final GAM dispersion to have the order of $O(r_h^2T_i/T_h)$, which is not taken into account, and the dissipative part is not shown to be similar to Eq. (5).

For the main ions, which the bounce frequency smaller then the GAM frequency $v_{Th}\sqrt{2\varepsilon} \ll \omega qR_0$, the fluid solution with the $\sqrt{\varepsilon}$-corrections is found to be similar [14]

$$n_{\rm un}^{(i)} = -\frac{e_in_0q}{m_iv_{Ti}}\left[\left(2+\frac{4}{\Omega_i^2} - 1.4\sqrt{\varepsilon}\right)\frac{iE_1}{\omega_{ci}\Omega_i} + \frac{R_0E_c}{v_{Ti}\Omega_i^2}\right]\sin\vartheta, \quad (6a)$$