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A theoretical study of the spheroidal droplet evaporation in forced convection

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ABSTRACT

In many applications, the shape of a droplet may be assumed to be an oblate spheroid. A theoretical study is conducted on the evaporation of an oblate spheroidal droplet under forced convection conditions. Closed-form analytical expressions of the mass evaporation rate for an oblate spheroid are derived, in the regime of controlled mass-transfer and heat-transfer, respectively. The variation of droplet size during the evaporation process is presented in the regime of shrinking dynamic model. Comparing with the droplets having the same surface area, an increase in the aspect ratio enhances the mass evaporation rate and prolongs the burnout time.

1. Introduction

Evaporation of liquid droplets has a wide range of industrial applications, such as spray combustion [1–3], liquid fuel preparation [4,5], and spray drying [6]. One of the simplifications in droplet evaporation simulation is that of droplet sphericity. Most of the current theoretical models were developed for a spherical droplet. However, a non-static droplet tends to deform especially at high value of Reynolds number [7,8]. Experiments and numerical studies show that the oblate spheroid is a proper shape-approximation for a droplet [9,10]. Since the spheroidal droplet cannot possess the unique point-symmetry of the spherical droplet, the surface area and the species mass flux distribution along the droplet surface are greatly influenced by the droplet shape.

A few of theoretical investigations have been carried out to develop the non-spherical droplet evaporation models. Ignoring the Stefan flow, Grow [11] proposed an integral expression of the surface mass flux for a prolate spheroid under steady-state conditions. Based on Grow's work [11], Gera et al. [12,13] defined a shape-influenced factor that accounts for the effect of non-sphericity on the mass evaporation rate. The factor was calculated by solving the ratio of the average vapor flux to that for the spherical droplet having the same surface area. In their studies, the diameter of a sphere with equal external surface area was chosen as the equivalent diameter. The shape-influenced factor was further developed by Yin et al. [14], in which a simple algebraic shape-influenced factor was proposed to replace the existing complex integral expression. Their investigations [14–16] treated the diameter of an equal-volume sphere as the equivalent diameter.

It can be seen that although the droplet evaporation models for a non-spherical droplet have been conducted and developed, the current models still have some problems. (a) The Stefan flow at the droplet surface is ignored. (b) The droplet evaporation model for a spheroidal droplet contains an integral term, which is rather complicated for engineering application. (c) The proposed models are only applicable to a prolate spheroid rather than an oblate spheroid, which cannot apply to the droplet evaporation. (d) The theoretical expressions are restricted to the case of a droplet under static conditions. Actually, the relative motion between the droplet and flow cannot be neglected in many areas of engineering science. Thus, efforts for mathematical improvements with detailed treatments are needed to obtain a more accurate evaporation model for an oblate spheroid droplet.

In this study, the influences of convective mass transfer and Stefan flow are taken in consideration. Fully algebraic solutions for the droplet evaporation rate in the spheroidal regime are obtained. To avoid the mathematical and computational complications, the following assumptions for a droplet evaporating are invoked. (a) The droplet shape is described as oblate spheroid along with the evaporation process. Dynamic evolution processes of droplet, such as breakup, collision, and coalescence are not included in this study. (b) A series of concentric spheroids constitute isothermal surface cluster and isoconcentration surface cluster of droplet. (c) The physical properties (density, thermal conductivity,

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Nomenclature

a, b, c	semi-axes of spheroid .... (m)
A	droplet surface area .... (m^2)
B_f	pre-exponential factor .... ($g/(m^2\,s\,Pa)$)
c_{pg}	specific heat of vapor .... ($J/(kg\,K)$)
d	diameter of a spherical droplet with the same volume .... (m)
d_0	initial diameter of a spherical droplet with the same volume .... (m)
D	diffusion coefficient of gas species .... (m^2/s)
E_f	activation energy .... (KJ/mol)
$\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$	Cartesian coordinate unit vectors .... (m)
g	mass flux .... ($kg/(m^2\,s)$)
G	mass flow rate .... (kg/s)
H	scale factor
k	mass transfer coefficient .... (m/s)
L_f	specific latent heat of vaporization .... (J/kg)
m_k	droplet mass .... (kg)
m_{k0}	initial droplet mass .... (kg)
M	droplet aspect ratio
M_0	initial droplet aspect ratio
n	unit vector outward normal to spheroid surface .. (m)
Nu	Nusselt number
Nu_0	Nusselt number in a stagnant environment
p	pressure .... (MPa)
Pr	Prandtl number
r	radius of a spherical droplet with the same surface area .... (m)
R	universal gas constant .... ($J/(kmol\,K)$)
Re_k	Reynolds number
Sh	Sherwood number
t	time .... (s)
T	temperature of concentric spheroid droplet .... (K)
T_b	droplet boiling point .... (K)
x, y, z	Cartesian coordinates .... (m)
Y	mass fraction

Greek symbols

θ_{en}	shape-influenced factor
λ_g	thermal conductivity .... ($W/(m\,K)$)
ξ, η, ζ	ellipsoidal coordinates .... (m)
Π	surface of concentric spheroid
ρ	gas density .... (kg/m^3)
ρ_l	droplet density .... (kg/m^3)
ψ	droplet sphericity

Subscripts

f	fuel vapor
s	surface
sp	sphere
sup	equivalent film boundary
∞	surrounding environment

and specific heat) are assumed to be constant in evaporation. Although the physical properties at the droplet surface differ from those in the surrounding environment, this assumption allows a simple closed-form solution.

2. Droplet evaporation models

2.1. Mass-transfer controlled regime

2.1.1. Species conservation and total flow rate

With the assumption of quasi-steady conditions, the species conservation for the vapor can be written as

$$g_f = -\rho D \frac{\partial Y_f}{\partial n} + g_f Y_f, \tag{1}$$

where g_f, Y_f, n represent the vapor mass flux, the vapor mass fraction, and the outward unit normal vector, respectively. The first term on the right-side expression is the diffusional flux of vapor determined by Fick's law, and the second term is the mass flux of vapor associated with molecular diffusion.

The total mass flow rate, or the mass evaporation rate, is defined as the integral of the mass flux along the spheroid surface:

$$G = \iint_{\Pi} g_f ds, \tag{2}$$

where the integral surface Π denotes the spheroid surface.

Since the oblate spheroid is a possible approximation for the shape of non-spherical droplet in many applications, it is convenient to use the ellipsoidal coordinates. The equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ denotes an ellipsoid in a Cartesian coordinate system [17]. An oblate spheroid is an ellipsoid of revolution generated by rotating an ellipse about its minor axis. The aspect ratio M is defined as the ratio of the length projected on the symmetry axis to the maximum length normal to the axis. M is the ratio of axes for a spheroid, with $M < 1$ for an oblate spheroid. The ellipsoidal coordinates (ξ, η, ζ) are produced from the Cartesian coordinates (x, y, z) using the expressions as

$$x^2 = \frac{(a^2+\xi)(a^2+\eta)(a^2+\zeta)}{(a^2-b^2)(a^2-c^2)}, \tag{3a}$$

$$y^2 = \frac{(b^2+\xi)(b^2+\eta)(b^2+\zeta)}{(b^2-a^2)(b^2-c^2)}, \tag{3b}$$

$$z^2 = \frac{(c^2+\xi)(c^2+\eta)(c^2+\zeta)}{(c^2-a^2)(c^2-b^2)}. \tag{3c}$$

The formula $x^2/(a^2+\xi) + y^2/(b^2+\xi) + z^2/(c^2+\xi) = 1$ represents a series of concentric ellipsoids when $\xi =$ constant, and $\xi = 0$ corresponds to the ellipsoid surface.

With the aid of the ellipsoidal coordinates, the vapor mass fraction gradient along the coordinate ξ can be expressed as (see Appendix A)

$$\frac{\partial Y_f}{\partial n} = \frac{2}{H}\frac{\partial Y_f}{\partial \xi}, \tag{4}$$

where

$$H = \left[\left(\frac{x}{a^2+\xi}\right)^2 + \left(\frac{y}{b^2+\xi}\right)^2 + \left(\frac{z}{c^2+\xi}\right)^2\right]^{1/2}. \tag{5}$$

Thus the mass evaporation rate is then

$$G = \frac{8\pi\rho D}{Y_f - 1}\left(a^2+\xi\right)^{1/2}\left(b^2+\xi\right)^{1/2}\left(c^2+\xi\right)^{1/2}\frac{\partial Y_f}{\partial \xi}. \tag{6}$$

2.1.2. Film theory

In many industrial apparatus, droplets seldom experience the creeping flow. There is always relative velocity between the droplet and the free stream. The concept 'film theory', as an approximate approach in chemical engineering, has been extensively used to