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Multiple Quantum Well in static magnetic field and intense laser pulses

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ABSTRACT

The energy spectrum and dipole matrix elements of a multiple quantum well (MQW) system has been calculated numerically by solving the time-independent Schrödinger equation using finite difference method, in the presence of magnetic field. The effect of barrier width is also investigated. The energy difference between the levels of various minibands and the energy difference between various levels of the same miniband is calculated for different number of wells in the MQW. Finally, the dynamics of the system in short laser pulses has been calculated numerically by solving the time-dependent Schrödinger equation. The effect of magnetic field on the dynamics is clearly shown and explained.

1. Introduction

The fabrication of an artificial periodic structure consisting of alternate layers of two dissimilar semiconductor with layer thicknesses of the order of nanometer was proposed by Esaki and Tsu in 1970 [1] which was named as Superlattice. Superlattice (SL) and Multiple Quantum Well (MQW) are similar in construction, only difference being that the well separation in MQW is large enough preventing electrons tunneling from one well to another. The thin barrier width in a SL allows the electrons to tunnel through so that the electrons experience the alternating layers as a periodic potential in addition to the crystal potential.

There is currently impressive progress in the study of optical and electronic properties of SL because of their practical applications in various domains. Ahn et al. recently reported SL channel for producing more reliable thin film transistor (TFT) which is required for next generation displays [2]. The effect of external electric field on dielectric permittivity [3,4] has been recently reported. SLs are also used in laser technology. Huynh et. al. [5] have reported optical excitation and detection of terahertz acoustic waves with semiconductor SL. Many applications of SL in optoelectronic devices have already been reported by Ungan et al. [6]. Crystalline SLs have important application as quantum-well infrared photodetectors [7,8].

The intersubband transitions in semiconductor quantum wells have attracted significant interest because of its applications in ultrafast optoelectronic devices. Due to technological advances in the fabrication of novel heterostructures, a number of shapes of such quantum systems have been explored [9–18]. Application of external static electric and magnetic fields also affects the energy levels and wavefunctions in low dimensional structures [19–24].

By changing the various parameters, viz., the shape of confining potential, number of wells, concentration of atoms in barrier, well width, and impurity, the properties of quantum wells can be modified. Recently, it has been shown that in addition to these parameters, the properties of quantum well systems are also affected by varying the barrier width [25]. Hence barrier width becomes an important parameter in controlling the response of such systems to electromagnetic fields.

We have devoted our work to a theoretical SL or an MQW model and studied the electronic properties of $GaAs/Ga_{1-x}Al_xAs$ MQW under combined effect of a resonant laser field and a static magnetic field.

The work is organized as follows. We have taken a multiple quantum well structure of a fixed well width and a fixed barrier height but varying barrier widths, and studied the bound–bound and bound–continuum transitions in the minibands formed in the wells. We focused on the periodic structure consisting of five quantum wells ($N = 5$) and studied the energy spectrum, dipole matrix elements and dynamics of the MQW system. The change in the spectrum and dynamics is also studied by taking three different barrier widths.

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The eigenenergies and eigenfunctions of the system under the effect of external static magnetic field are solved by time-independent Schrödinger equation using finite difference method. These are then used to formulate the laser–MQW interaction using time-dependent Schrödinger equation which is solved with the help of fourth order Runge–Kutta method.

2. Theory

The Hamiltonian of the MQW system in static magnetic field along the growth direction, z, is given by (atomic units have been used, otherwise mentioned):

$$H(z) = H_0 + V(z) + \frac{B^2 z^2}{2m^*} \tag{1}$$

where H_0 is the unperturbed Hamiltonian, m^* is the effective mass, B is the magnetic field and $V(z)$ is the confining potential for the MWQ.

Let $d = \mathrm{ww} + \mathrm{ws}$, where ww and ws are the well width and barrier width respectively. The potential V(z) is given by:

$$V(z) = \sum_{l=-\infty}^{+\infty} V(z - ld) \tag{2}$$

in which

$$V(z - ld) = V_0 \quad \text{if } |z - ld| > \mathrm{ww}/2 \tag{3}$$

$$V(z - ld) = 0 \quad \text{if } |z - ld| \le \mathrm{ww}/2 \tag{4}$$

For a MQW structure, assuming that the many body interactions among electrons are negligible, the one electron Hamiltonian can be used to describe the motion of the electrons. Since the perturbing potentials are assumed not to effect the band structures of both the well material and barrier material, the effective-mass approximation can be applied to find the electron energy levels and envelope wavefunctions.

When such a system is exposed to short laser pulses, the Schrödinger equation to be solved becomes as follows:

$$i\frac{\partial \psi}{\partial t} = H(z,t)\psi(z,t) \tag{5}$$

where $H(z,t)$ is the Hamiltonian of the system in the presence of time-dependent laser field.

$$H(z,t) = H(z) + V_{int}(z,t) \tag{6}$$

$V_{int}(z,t)$ be the interaction potential energy of the electron with the laser field given by:

$$V_{int}(z,t) = -\mu(z) \cdot E(t) \tag{7}$$

where $\mu(z)$ is the dipole moment of the MQW and $E(t)$ is laser field defined below.

The electric field of the laser is:

$$E(t) = E_0 f(t) \cos(\omega_0 t) \tag{8}$$

where $f(t)$ is the Gaussian envelope of the laser used.

$$f(t) = \exp\bigl(-(t - t_0)/t_p\bigr)^2 \tag{9}$$

where t_0 is the centre of the pulse and t_p defines the width of the pulse.

The wavefunction of the system can be expanded in terms of the basis states as:

$$\psi(z,t) = \sum_q c_q(t)\phi_q(z)\exp(-iE_q t) \tag{10}$$

where C's are the coefficients whose square defines the probabilities, q stands for the n, l of the MQW, where n is number of miniband and l is the number of energy level within the miniband. For example, in case of number of wells equal to five at $B = 0$ T, we have three minibands each consisting of five energy levels. So $E_{1,1}$ stands for energy of first level in the first miniband, $E_{3,1}$ stands for energy of first level in the third miniband, etc. Using the expression for the wavefunction $\psi(z,t)$ and the orthogonality of the eigenstate in time-dependent Schrödinger equation, we get a set of coupled first order differential equations as

$$i\frac{\partial c_p}{\partial t} = \sum_q c_q(t)\exp\bigl[-i(E_q - E_p)t\bigr]V_{pq} \tag{11}$$

where

$$V_{pq} = \langle \phi_p | \mu(z)E(t) | \phi_q \rangle \tag{12}$$

denotes the dipole matrix elements for the MWQ interacting with the laser.

Solving Eq. (11), we can find the transition probability for the transition $p \longrightarrow q$:

$$P_{p\to q} = P_{nl\to n'l'} = |c_{p\to q}|^2 = |c_{nl\to n'l'}|^2 \tag{13}$$

3. Results and discussion

Consider a multiple quantum well composed of GaAs/$Ga_{1-x}Al_xAs$ with concentration of Al, $x = 0.25$, fixed well width, $\mathrm{ww} = 2.83 \times 35$ Å and a barrier height $V_0 = 0.3$ eV, as has been taken by others [26]. In calculations, we have taken three barrier widths, viz., (i) ws1 = ww, (ii) ws2 = 2.83 × 22 Å and (iii) ws3 = 2.83 × 11 Å. In addition, to study the behavior of the system in short pulses, the laser field used for the interaction is Gaussian as described by Eq. (8) and (9) with intensity of laser, E_0. In all our calculations, the center of the pulse is taken as $t_0 = 60$ a.u. and the width of the pulse $t_p = 20$ a.u. Barring Fig. 3, the results are presented for five well system ($N = 5$). Although the results presented are for five well system but can be easily generalized to any number of quantum well system.

In order to find out the energy spectrum of the MQW system in the presence of magnetic field, we have solved Eq. (1) with nine point finite difference method [27]. We found the energy levels of various minibands of the MQW considered under study and the dipole matrix elements $\mu_{pq} = \langle \varphi_p | z | \varphi_q \rangle = \langle \varphi_{nl} | z | \varphi_{n'l'} \rangle$ where φ_p and φ_q and are the wavefunctions (solutions of Eq. (1)), indices n, l have been defined earlier in the text. Having obtained the eigenenergies and the wavefunctions, the time-dependent Schrödinger equation (5) is solved using fourth order Runge–Kutta method.

As mentioned earlier, we have considered a five well system ($N = 5$) and found that there are series of five-fold degenerate states both in the bound as well as continuum region in the MQW. For a five well system it has been observed that with the present potential profile and potential height of 0.3 eV, a total of thirteen states ($E_{1,1}$ to $E_{3,3}$) act as bound states and the rest are continuum states for the magnetic field $B = 5$ T. In the absence of external field fifteen states ($E_{1,1}$ to $E_{3,5}$) are bound and rest are in the continuum state. For $B = 0$ T, $E_{1,1} - E_{1,5}$ states are degenerate, similarly $E_{2,1} - E_{2,5}$, $E_{3,1} - E_{3,5}$, and so on are degenerate.

In Fig. 1a, the variation of energies (in meV) has been shown with static magnetic field B (in tesla) when ws = ws1. On application of an external static field, this degeneracy is lifted and the splitting in the bound states occur. In Figs. 1b and 1c are shown the results when ws = ws2 and ws = ws3 respectively. Further, it is found that with increase in magnetic field, the number of bound states decreases. At $B = 0$ T, it is fifteen and that in the case of $B = 5$ T, it is thirteen. The change in the number of states – bound