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Control of chaos in permanent magnet synchronous motor by using optimal Lyapunov exponents placement

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ABSTRACT

Permanent Magnet Synchronous Motor (PMSM) experiences chaotic behavior for a certain range of its parameters. In this case, since the performance of the PMSM degrades, the chaos should be eliminated. In this Letter, the control of the undesirable chaos in PMSM using Lyapunov exponents (LEs) placement is proposed that is also improved by choosing optimal locations of the LEs in the sense of predefined cost function. Moreover, in order to provide the physical realization of the method, nonlinear parameter estimator for the system is suggested. Finally, to show the effectiveness of the proposed methodology, the simulation results for applying this control strategy are provided.

1. Introduction

Chaotic behavior has been extensively analyzed in many fields such as engineering, medicine, ecology, biology, and economy. As a matter of fact, chaos may occur in many natural processes. In many practical situations, in order to suppress chaos, it is required to control the related chaotic system.

One scientific field, in which chaos phenomenon has been appeared, is the motor drive system [1]. The Permanent Magnet Synchronous Motor (PMSM) that is under study in this Letter, exhibits chaotic behavior for a certain range of its parameters [2]. When the PMSM enters to these operational conditions, chaos control can be accomplished. Therefore, chaos control in PMSM has become an active research in the field of nonlinear control of electric motors.

There are some methods for controlling chaos. The OGY method is a basic methodology for controlling chaos [3]. Ref. [4] includes a comprehensive survey in the variety of the methods and their applications on this matter. Although a few of these methods have been used for controlling chaos in PMSM, some others are not quite appropriate for this aim. For instance, in OGY method, finding an adjustable parameter is not often simple. Also, control of chaos via the Time-Delay Feedback Control (TDFC) method though is used for PMSM [5]. However, it encounters with some problems as the control objective must be the equilibrium or the Unstable Periodic Orbit (UPO); moreover, determining the time delay for TDFC method is difficult. Some of the classical methods of chaos control also have been used to control the undesirable chaos in PMSM. An adaptive Dynamic Surface Control (DSC) law has been proposed in [6]. Also, for this purpose a nonlinear feedback control method is suggested in PMSM [7].

On the other hand, the Lyapunov exponents have been conventionally used in order to quantitatively characterize the exponential divergence of initially nearby trajectories and there are several methods for calculating them [8–10]. Although the Lyapunov exponents are known as the chaotic behavior indicators, they can potentially be used in the purpose of chaos control as well. In this regard, control of chaos by Lyapunov exponents placement has been proposed in [11,12] so that the Lyapunov exponents of the closed-loop system become negative and consequently the chaos is eliminated in the related system.

In this research, the control of chaos in PMSM through optimal placement of the Lyapunov exponents is proposed. For this purpose, the Jacobian approach is used continuously to calculate the Largest Lyapunov Exponent (LLE). At first, a control law is presented which assigns the Lyapunov exponents in the desired locations. Then, searching for the optimal Lyapunov exponents, subject to a defined cost function, will be the main goal. Guarantee of stability of the closed-loop system with the proposed control law is proved in Lemma 3.

The Letter is organized as follows: After introduction, modelling of the permanent magnet synchronous motor is briefly reviewed in Section 2. This model is fit for carrying on chaotic behavior analysis. Computation of the largest Lyapunov exponent is presented in Section 3. Then, the proposed control algorithm using optimal Lyapunov exponents' placement is suggested in Section 4.

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The simulation results of implementing the proposed control law are provided in Section 5 and finally some conclusion remarks are discussed in Section 6.

2. Modeling of the permanent magnet synchronous motor

The model of the permanent magnet synchronous motor with smooth air gap is described as follows [13,14]:

$$\begin{cases} \dfrac{d\tilde{\omega}}{d\tilde{t}} = (n_p \psi_r \tilde{i}_q - \beta\tilde{\omega} - \tilde{T}_L)/j, \\ \dfrac{d\tilde{i}_q}{d\tilde{t}} = (-R\tilde{i}_q - L\tilde{\omega}\tilde{i}_d - \psi_r\tilde{\omega} + \tilde{u}_q)/L, \\ \dfrac{d\tilde{i}_d}{d\tilde{t}} = (-R\tilde{i}_d + L\tilde{\omega}\tilde{i}_q + \tilde{u}_d)/L, \end{cases} \tag{1}$$

where, $\tilde{\omega}$, $\tilde{i}_q$ and $\tilde{i}_d$ are the state variables, which represent motor angular frequency (rad/s) and quadrature-axis and direct-axis currents (A), respectively; $\tilde{u}_q$ and $\tilde{u}_d$ are the quadrature-axis and direct-axis stator voltage components (V), respectively; $\tilde{t}$ is the time (s), $\tilde{T}_L$ denotes the load torque (N m), L is the winding stator inductance (H), R stands for the stator winding resistance (Ω), ψ_r is the permanent magnet flux (Wb), β is the viscous damping coefficient (N rad^{-1} s), j is the polar moment of inertia (kg m^2), and n_p denotes the number of pole pairs of the motor.

To show the existence of chaos in the PMSM, a new change of variables is used. Assume that $\tau = L/R$, $t = \tilde{t}/\tau$ and $\kappa = \beta/(n_p \tau \psi_r)$. The scaled state variables ω, i_q and i_d are defined as follows:

$$i_d = \frac{\tilde{i}_d}{\kappa}, \qquad i_q = \frac{\tilde{i}_q}{\kappa}, \qquad \omega = \tau\tilde{\omega}, \tag{2}$$

in which ω, i_q and i_d are the scaled motor angular frequency and the scaled quadrature-axis and direct-axis currents, respectively. Thus, the new scaled model for the PMSM becomes:

$$\begin{cases} \dfrac{d\omega}{dt} = \sigma(i_q - \omega) - T_L, \\ \dfrac{di_q}{dt} = -i_q - \omega i_d + \gamma\omega + u_q, \\ \dfrac{di_d}{dt} = -i_d + \omega i_q + u_d, \end{cases} \tag{3}$$

where

$$\gamma = -\psi_r/(\kappa L), \qquad \sigma = \beta\tau/j, \qquad T_L = \tau^2\tilde{T}_L/j,$$
$$u_q = \tilde{u}_q/(\kappa R), \qquad u_d = \tilde{u}_d/(\kappa R).$$

In Eqs. (3), u_q and u_d are the scaled quadrature-axis and direct-axis stator voltage components respectively, T_L is the scaled load torque, and σ and γ are previously defined system parameters.

Assume the desired state vector is (ω^*, i_q^*, i_d^*). Therefore, the errors are defined as $e = (e_\omega, e_q, e_d)^T = (\omega - \omega^*, i_q - i_q^*, i_d - i_d^*)^T$. The model of the PMSM in new coordinate can be described as:

$$\begin{cases} \dot{e}_\omega = \sigma(e_q - e_\omega) + \sigma\left(i_q^* - \omega^*\right) - T_L, \\ \dot{e}_q = -e_q - e_\omega e_d - \omega^* e_d - i_d^* e_\omega + \gamma e_\omega - i_q^* - \omega^* i_d^* \\ \qquad + \gamma\omega^* - \dot{i}_q^* + u_q, \\ \dot{e}_d = -e_d + e_\omega e_q + \omega^* e_q + i_q^* e_\omega - i_d^* + \omega^* i_q^* + u_d. \end{cases} \tag{4}$$

The PMSM is experiencing chaotic behavior for a certain range of its parameters. For example, for the case of $\sigma = 5.46$, $\gamma = 20$, $u_d = u_q = 0$, $T_L = 0$, $\omega(0) = -5$, $i_q(0) = 0.01$ and $i_d(0) = 20$ the system can exhibit chaotic behavior as shown in Fig. 1.

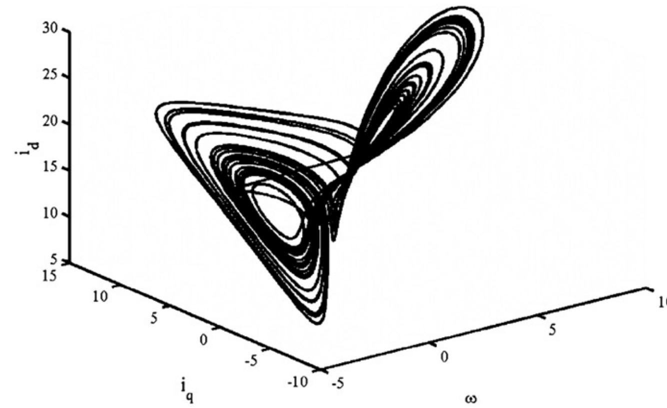

Fig. 1. The strange attractor in PMSM.

3. Calculation of the largest Lyapunov exponent (LLE)

Lyapunov exponents show the average rate of growing or shrinking of a small volume of initial conditions. They are used for characterizing several types of behaviors in nonlinear systems, especially the Lyapunov exponents that are the hallmark of chaos [15]. A positive value of the largest Lyapunov exponent indicates chaotic behavior. So computing the largest Lyapunov exponent has proven to be the most useful dynamical diagnostic for chaotic systems.

Consider a continuous system as follows:

$$\dot{\mathbf{x}} = \boldsymbol{f}(\mathbf{x}) + \mathbf{u}, \tag{5}$$

where $\mathbf{x} \in R^n$ is the state vector and $\boldsymbol{f}(\cdot)$ is a continuously differentiable smooth function in R^n and $\mathbf{u} \in R^n$ is the control input vector.

The largest Lyapunov exponent can be computed as follows [16]:

$$\lambda_m = \lim_{t\to\infty} \frac{1}{t} \int_0^t \frac{\boldsymbol{\xi}(t)^T J_{cL} \boldsymbol{\xi}(t)}{\|\boldsymbol{\xi}(t)\|^2}\, dt, \tag{6}$$

where J_{cL} is the Jacobian matrix for the closed-loop system and $\boldsymbol{\xi}(t) \in R^n$ is a normalization perturbation vector. $\boldsymbol{\xi}(t)$ is computed from differentiating with respect to time:

$$\dot{\boldsymbol{\xi}}(t) = \left(I_{n\times n} - \frac{\boldsymbol{\xi}(t)\boldsymbol{\xi}(t)^T}{\|\boldsymbol{\xi}(t)\|^2}\right) J_{cL}\boldsymbol{\xi}(t), \tag{7}$$

where $\boldsymbol{\xi}(0) \neq \mathbf{0}$ and $I_{n\times n}$ is the identity matrix with dimension $n \times n$.

4. The proposed controller design methodology

Any system containing at least one positive Lyapunov exponent is defined to be chaotic. If we place suitable negative Lyapunov exponent instead of the largest Lyapunov exponent for closed-loop system, then chaotic behavior of the system will be eliminated. For this purpose the following Lemma 1 is presented.

Lemma 1. *The largest Lyapunov exponent of the closed-loop system (5) is assigned in the desired negative location if the control is selected such that*:

$$\mathbf{u} = \left(-J_{oL} + \lambda_m^* I_{n\times n}\right)\mathbf{x}, \tag{8}$$

where J_{oL} is the Jacobian matrix for the open-loop system and λ_m^* is the desired negative largest Lyapunov exponent.