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An integral control for synchronization of a class of unknown non-autonomous chaotic systems

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ABSTRACT

In this Letter, a high gain integral controller for synchronization of unknown non-autonomous chaotic systems is proposed. Perturbation theory is used to demonstrate that the synchronization error of the resulting feedback system converges to zero. Numerical simulations are presented to illustrate the effectiveness of the proposed method.

1. Introduction

Chaos synchronization has many applications in biological systems, secure communication, and laser dynamics [1–4]. Many techniques that use linear or nonlinear feedback control have been proposed to synchronize chaotic systems [5–16].

However, most physical chaotic systems include nonlinear components, which are commonly unknown; therefore modeling and analysis of such chaotic systems requires methods to quantify the effects of unknown nonlinearities, uncertain parameters, or both. To meet this requirement, Lyapunov's direct method [17] has been used to develop adaptive control schemes and parameter update rules for the synchronization of structurally equivalent autonomous chaotic systems. Y. Yu [18] proposed the chaos synchronization of a unified autonomous chaotic systems in the presence of unknown system parameters. Recently, adaptive approaches [19–21] were proposed for synchronization of different non-autonomous chaotic systems with unknown or uncertain parameters. Such systems are theoretically important and relevant for modeling the behavior of many engineering systems such as offshore platforms, buildings under earthquakes and orientation information and so on. However, these adaptive control approaches cannot be applied for synchronization of chaotic systems if the model of the systems includes unstructured uncertainties or if prior knowledge of the model is not available. The examples are the synchronization between neuron clusters [22], or β-cells in pancreas [23].

In this Letter, we present a high gain integral control for synchronization of different non-autonomous chaotic systems whose models are fully unknown. The error dynamics of resulting feedback systems can be considered to be two-time-scale nonlinear dynamic systems which have fast and slow dynamics. By Tikhonov's theorem [24], the slow error dynamic system shows quasi-steady-state properties if the fast error dynamic system has an isolated exponentially stable equilibrium point. Then, the proposed controller guarantees convergence of the error to the origin. The proposed method can provide complete synchronization and anti-synchronization by simply redefining the error signal. Numerical examples of synchronization and anti-synchronization are provided to demonstrate the effectiveness of the proposed method.

2. System description

We consider a class of non-autonomous chaotic drive and response systems as follows. The drive system has the form,

$$\dot{x}_d = Ax_d + Bf(t, x_d) \tag{1}$$

where $x_d \in \mathbb{R}^n$ is the state vector, $f(t, x_d) : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ is an unknown continuously differentiable nonlinear function,

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}. \tag{2}$$

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Similarly, the non-autonomous chaotic response system has the form

$$\dot{x}_r = Ax_r + B\big[g(t, x_r) + u\big] \tag{3}$$

where $x_r \in \mathbb{R}^n$ is the state vector, $g(t, x_r) : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ is an unknown continuously differentiable nonlinear function, and $u \in \mathbb{R}$ is the control input. Define the synchronization error as

$$e \triangleq x_d - x_r. \tag{4}$$

By subtracting (3) from (1), the synchronization error dynamics is

$$\begin{aligned}\dot{e} &= \dot{x}_d - \dot{x}_r \\ &= Ae + B\big[f(t, x_d) - g(t, x_r) - u\big].\end{aligned} \tag{5}$$

Then, the objective of the synchronization problem is to design a controller to make the synchronization error converges to the origin, i.e., $e(t) \to 0$ as $t \to \infty$.

3. Integral control for synchronization

In this section, we present a high gain integral control for synchronization of two unknown non-autonomous chaotic systems. The error dynamic system (5) is converted to a singularly perturbed system by applying high gain integral control; stability analysis is performed using the singular perturbation method.

The following theorem summarizes the main result.

Theorem 1. *Consider the unknown non-autonomous chaotic drive (1) and response (3) systems. Then, there exists $\epsilon^* > 0$ such that for $\epsilon \in (0, \epsilon^*)$ the synchronization error (4) converges to zero as $t \to \infty$ if the control law is given by*

$$u = \frac{\alpha_1}{\epsilon} e_n + u_s + \sigma, \tag{6}$$

$$\dot{\sigma} = \frac{\alpha_2}{\epsilon^2} e_n + \frac{1}{\epsilon} u_s, \tag{7}$$

$$u_s = Ke, \tag{8}$$

where e_n is the nth element of e; ϵ is a positive small constant; α_1 and α_2 are constants chosen such that the polynomial $s^2 + \alpha_1 s + \alpha_2$ is Hurwitz; and K is an $1 \times n$ gain matrix such that the transfer function $H(s) = (sI - (A - BK))^{-1}$ is Hurwitz.

Proof. Using the control law (6), the error dynamics (5) can be rewritten as

$$\dot{e} = Ae + B\left[d - \sigma - \frac{\alpha_1}{\epsilon} e_n - u_s\right], \tag{9}$$

where

$$d = f(t, x_d) - g(t, x_r). \tag{10}$$

To convert the error dynamical equation into singular perturbed form, we first define

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \triangleq \begin{bmatrix} \frac{e_n}{\epsilon} \\ d - \sigma \end{bmatrix}. \tag{11}$$

Differentiating (11) and multiplying both sides by ϵ, then using (9) yields

$$\begin{aligned}\epsilon \dot{\eta}_1 &= \dot{e}_n \\ &= -\frac{\alpha_1}{\epsilon} e_n - u_s + d - \sigma, \\ \epsilon \dot{\eta}_2 &= \epsilon[\dot{d} - \dot{\sigma}] \\ &= -\alpha_2 \eta_1 - u_s + \epsilon\big[\dot{f}(t, x_d) - \dot{g}(t, x_r)\big].\end{aligned} \tag{12}$$

Together, (9) and (12) constitutes the standard singularly perturbed model in which (9) is the slow subsystem and (12) is the fast subsystem.

The fast subsystem can be rewritten in state-space form:

$$\epsilon \dot{\eta} = \bar{A}\eta + \epsilon\left\{B_1\big[\dot{f}(t, x_d) - \dot{g}(t, x_r)\big] - B_2 \frac{u_s}{\epsilon}\right\}, \tag{13}$$

where

$$\bar{A} = \begin{bmatrix} -\alpha_1 & 1 \\ -\alpha_2 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{14}$$

Using (11), $\frac{1}{\epsilon} u_s(x_d, x_r, t)$ can be shown to be locally Lipschitz in its arguments because

$$\begin{aligned}\frac{u_s}{\epsilon} &= \frac{1}{\epsilon} K\left[\overset{(n)}{\int} e_n\,dt, \overset{(n-1)}{\int} e_n\,dt, \ldots, e_n\right] \\ &= K\left[\overset{(n)}{\int} \eta_1\,dt, \overset{(n-1)}{\int} \eta_1\,dt, \ldots, \eta_1\right].\end{aligned} \tag{15}$$

In addition, $\bar{A}$ is Hurwitz by design; this means that the origin is an exponentially stable equilibrium point of the boundary-layer model [24]. Therefore, from Tikhonov's theorem, $\eta = O(\epsilon)$ for $t \in [t_0, T(\epsilon)]$ where $\lim_{\epsilon \to 0} T(\epsilon) = 0$.

Hence, (9) can be rewritten as

$$\begin{aligned}\dot{e} &= Ae + B\big[u_s + O(\epsilon)\big] \\ &= (A - BK)e + O(\epsilon).\end{aligned} \tag{16}$$

This shows that $\|e\|$ is uniformly ultimately bounded and that the bound can be made arbitrarily small by choosing small ϵ. This means in turn that (e, η) will approach N_ϵ, a neighborhood of the origin, where N_ϵ can be made arbitrarily small by choosing small ϵ. Then, (9) and (13) can be rewritten as

$$\dot{e} = (A - BK)e + \delta_1(\eta), \tag{17}$$

$$\epsilon \dot{\eta} = \bar{A}\eta + \epsilon \delta_2(e, \eta) \tag{18}$$

in N_ϵ, where

$$\begin{aligned}\delta_1(\eta) &= B(\eta_2 - \alpha_1 \eta_1), \\ \delta_2(e, \eta) &= B_1\left(\dot{\eta_2} + \frac{\alpha_2}{\epsilon}\eta_1\right) + \frac{1}{\epsilon}(B_1 Ke - B_2 Ke).\end{aligned}$$

The functions δ_1 and δ_2 are locally Lipschitz in (e, η) and vanish at the origin. Then, in N_ϵ,

$$\begin{aligned}\big\|\delta_1(\eta)\big\| &= \big\|\delta_1(\eta) - \delta_1(0)\big\| \leqslant k_1\|\eta\|, \\ \big\|\delta_2(e, \eta)\big\| &= \big\|\delta_2(e, \eta) - \delta_2(0, \eta) + \delta_2(e, \eta) - \delta_2(e, 0)\big\| \\ &\leqslant \big\|\delta_2(e, \eta) - \delta_2(0, \eta)\big\| + \big\|\delta_2(e, \eta) - \delta_2(e, 0)\big\| \\ &\leqslant k_2\|e\| + k_3\|\eta\|\end{aligned} \tag{19}$$

with nonnegative constants k_1, k_2 and k_3 that are independent of ϵ.

If $\delta_1(t) = 0$ in (17), the origin is an exponentially stable and the existence of a Lyapunov function V_1 that satisfies

$$\frac{\partial V_1}{\partial e}(e)\big[(A - BK)e\big] \leqslant -k_4\|e\|^2, \qquad \left\|\frac{\partial V_1}{\partial e}\right\| \leqslant k_5\|e\|$$

is guaranteed by the converse Lyapunov theorem [24].

Choose the Lyapunov function candidate as $V = V_1(e) + \eta^T P\eta$. Then, in some neighborhood near the origin,