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Arbitrary amplitude solitary waves and double layers in an ultra-relativistic degenerate dense dusty plasma

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ABSTRACT

Arbitrary amplitude solitary waves (SWs) and double layers (DLs) in an ultra-relativistic degenerate dense dusty plasma (containing ultra-relativistic degenerate ultra-cold electron fluid, inertial ultra-cold ion fluid, and negatively charged static dust) have been investigated by the pseudo-potential approach. It has been found that for $\delta = 1$ (where δ is the ratio of nonlinear wave speed to linear wave phase speed) extremely large amplitude DLs with negative potential exist for $\mu = 0.537$ (where μ is the ratio of dust charge density to ion charge density) and SWs with negative potential exist for $1 > \mu > 0.537$. It is also shown that for $\delta > 1$ only SWs with positive potential exist for $0 \geqslant \mu < 0.537$, but SWs with positive potential coexist with SWs or DLs with a negative potential for $0.537 > \mu > 0.851$. The implications of our results in some compact astrophysical objects, particularly, in white dwarfs and neutron stars, have been briefly discussed.

Recently, there has been a great deal of interest in understanding the basic properties of matter under extreme conditions [1–5], which are found in some interstellar compact objects (e.g. white dwarfs, neutron stars, etc.), but are not found in terrestrial environments. One of these extreme conditions is the high density of degenerate matter in these compact objects, which are, in fact, 'relics of stars', and have ceased burning thermonuclear fuel, and thereby no longer generate thermal pressure. These interstellar compact objects are contracted significantly, and as a result the density of their interiors becomes extremely high to provide nonthermal pressure via degenerate fermion/electron pressure and particle–particle interactions. The observational evidence and theoretical analysis imply that these compact objects, which support themselves against gravitational collapse by cold degenerate fermion/electron pressure, are of two categories. The interior of the first category is close to a dense solid (ion lattice surrounded by degenerate electrons, and possibly other heavy particles or dust). One of the examples of this kind of stars is a white dwarf, which is supported by the pressure of degenerate electrons. On the other hand, the interior of the second category is close to a giant atomic nucleus (a mixture of interacting nucleons and electrons, and possibly other heavy elementary particles and condensates or dust). One of the examples of this kind of stars is a neutron star, which is supported by the pressure due to combination of nucleon degeneracy and nuclear interactions. These unique states (extreme conditions) of matter are occurred by significant compression of the interstellar medium. The degenerate electron number density in such a compact object is so high [e.g. the degenerate electron number density can be order of 10^{30} cm^{-3} even more in white dwarfs, and order of 10^{36} cm^{-3} even more in neutron stars] that the electron Fermi energy is comparable to the electron mass energy and the electron speed is comparable to the speed of light in vacuum [1–3].

The equation of state for degenerate electrons in such interstellar compact objects are mathematically explained by Chandrasekhar [4,5] for two limits, namely non-relativistic and ultra-relativistic limits. The degenerate electron equation of state of Chandrasekhar is $P_e \propto N_e^{5/3}$ for non-relativistic limit and $P_e \propto N_e^{4/3}$ for ultra-relativistic limit, where P_e is the degenerate electron pressure and N_e is the degenerate electron number density. We note that the degenerate electron pressure depends only on the electron number density, but not on the electron temperature. These interstellar compact objects, therefore, provide us cosmic laboratories for studying the properties of the medium (matter), as well as waves and instabilities [6–14] in such a medium at extremely high densities (degenerate state) for which quantum as well as relativistic effects become important [13,14]. The quantum effects on linear [8,10,12] and nonlinear [9,11] propagation of electrostatic and electro-magnetic waves have been investigated by using the quantum hydrodynamic (QHD) model [12,14], which is an extension of classical fluid model in a plasma, and by using the quantum magneto-hydrodynamic (QMHD) model [8–11], which involve spin-$\frac{1}{2}$ and one-fluid MHD equations.

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On the other hand, it has been established by observational evidence and theoretical analysis that the most space and laboratory plasmas contain massive solid particles or dust, which are not practically neutral, but are charged by absorbing plasma electrons and ions [15–23]. It has been shown that the presence charged dust significantly increase the phase speed of the electrostatic waves (particularly, ion-acoustic waves [24]) because of the inequality of electron and ion charge density, i.e. $n_{e0} \neq Z_i n_{i0}$, where n_{i0} (n_{e0}) is the ion (electron) number density at equilibrium and z_i represents the ion charge state.

Recently, a number of theoretical investigations have also been made of the nonlinear propagation of electrostatic waves in degenerate quantum plasma by a number of authors, e.g. Hass [26], Misra and Samanta [27], Mistra et al. [28] etc. However, these investigations are based on the electron equation of state $P_e \propto N_e^{5/3}$, and are valid for the non-relativistic limit and for a pure electron–ion plasma. To the best of our knowledge, no investigation has been made of the nonlinear propagation of the electrostatic waves in such a degenerate dense plasma based on the degenerate electron equation of state ($P_e \propto N_e^{4/3}$) which is valid for ultra-relativistic limit. Therefore, in our present work, we consider a degenerate dense dusty plasma containing negatively charged static dust. inertial ultra-cold ion fluid, and ultra-relativistic degenerate electron fluid following the equation of state $P_e \propto N_e^{4/3}$, and study the basic features of the arbitrary amplitude solitary waves (SWs) and double layers (DLs) in such an ultra-relativistic degenerate dense dusty plasma.

We consider the nonlinear propagation of electrostatic perturbation in an ultra-relativistic degenerate dense dusty plasma containing ultra-relativistic degenerate ultra-cold electron fluid, inertial ultra-cold ion fluid, and negatively charged static dust. Thus, at equilibrium we have $z_i n_{i0} = n_{e0} + Z_d n_d$, where n_d is the static dust number density, and Z_d is the number of electrons residing on the dust grain surface. The electron fluid is assumed to follow the equation of state $P_e \propto N_e^{4/3}$. The time scale of the electrostatic perturbation under consideration is much faster than the dust plasma period so that dust can be assumed to be stationary [24,25]. The nonlinear dynamics of the electrostatic waves propagating in such an ultra-relativistic degenerate dusty plasma is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial \phi}{\partial x} = \frac{3\beta}{4n_e} \frac{\partial n_e^{\frac{4}{3}}}{\partial x}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = (1-\mu)n_e - n_i + \mu, \tag{4}$$

where n_i (n_e) is the ion (electron) number density normalized by its equilibrium value n_{i0} (n_{e0}), u_i is the ion fluid speed normalized by $C_i = (z_i m_e c^2/m_i)^{1/2}$ with m_e (m_i) being the electron (ion) rest mass, and c being the speed of light in vacuum, ϕ is the electrostatic wave potential normalized by $z_i m_e c^2/e$ with e being the magnitude of the charge of an electron, $\mu = z_d n_d / z_i n_{i0}$, $\beta = K_0 \alpha$, $K_0 = (72\pi)^{-1/3} \equiv 0.164102$, $\alpha = \lambda_c n_0^{1/3}$, and $\lambda_c = h/m_e c = 2.425 \times 10^{-10}$ cm. The time variable (t) is normalized by ${\omega_{pi}}^{-1} = (m_i/4\pi n_{i0} z_i^2 e^2)^{1/2}$ and the space variables (x) is normalized by $\lambda_s = (m_e c^2/4\pi n_{e0} e^2)^{1/2}$.

To derive an energy integral [29,30] from (1)–(4), we first make all the dependent variables depend only on a single variable $\xi = x - Mt$, where M is the nonlinear wave speed normalized by C_i. We note that M is not the Mach number since it is normalized by C_i, which is not the ion-acoustic speed. Using the steady state condition, we can express (1)–(4) in terms of new variable ξ as

$$M \frac{dn_i}{d\xi} - \frac{\partial}{d\xi}(n_i u_i) = 0, \tag{5}$$

$$M \frac{du_i}{d\xi} - u_i \frac{du_i}{d\xi} = \frac{d\phi}{d\xi}, \tag{6}$$

$$\frac{d\phi}{\partial \xi} = 3\beta \frac{dn_e^{\frac{1}{3}}}{d\xi}, \tag{7}$$

$$\frac{d^2\phi}{\partial \xi^2} = (1-\mu)n_e - n_i + \mu. \tag{8}$$

Now, imposing the appropriate boundary conditions (namely $n_i \to 1$, $n_i \to 1$, $u_i \to 0$, $\phi \to 0$, and $d\phi/d\xi \to 0$ at $\xi \to -\infty$), one can solve (5) and (6), and can express n_e and n_i as

$$n_e = \left(1 + \frac{\phi}{3\beta}\right)^3, \tag{9}$$

$$n_i = \frac{1}{\sqrt{1 - \frac{2\phi}{M^2}}}. \tag{10}$$

Substituting (9) and (10) into (8), we obtain

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0. \tag{11}$$

Eq. (11) represents an energy integral [29,30] for an oscillating particle of unit mass, with pseudo-position ϕ, pseudo-time ξ, and pseudo-potential $V(\phi)$. The pseudo-potential $V(\phi)$ for our purposes reads

$$V(\phi) = C_1 - \mu\phi - \gamma\left(1 + \frac{\phi}{3\beta}\right)^4 - M^2\sqrt{1 - \frac{2\phi}{M^2}}, \tag{12}$$

where $\gamma = 3(1-\mu)\beta/4$, $C_1 = \gamma + M^2$ is the integration constant chosen in such a way that $V(\phi) = 0$ at $\phi = 0$. Eqs. (11) and (12) are valid for arbitrary amplitude stationary nonlinear waves in a degenerate dense dusty plasma under consideration.

To examine the possibility for the formation of SWs and DLs in a degenerate dusty plasma under consideration, let us first discuss the conditions for their existence. These conditions can be discussed as follows:

(i) $V(0) = dV(\phi)/d\phi|_{\phi=0} = 0$, which are already satisfied by the equilibrium charge neutrality condition, and by the boundary condition chosen for the integration constant.

(ii) $d^2V(\phi)/d\phi^2|_{\phi=0} \leqslant 0$, which will be satisfied if $\delta \geqslant 1$, where $\delta = M/M_c$ and

$$M_c = \sqrt{\frac{\beta}{1-\mu}} \tag{13}$$

represents the linear phase speed of the electrostatic waves under consideration. It is important to mention here that for $\delta > 1$ the point at the origin is unstable with respect to both sides (positive and negative ϕ-axis) and hence SWs or DLs can be formed in one or both sides (positive and negative ϕ-axes). However, for $\delta = 1$ the point at the origin is unstable with respect to one side (either positive or negative ϕ-axis), and thus SWs or DLs can be formed in either positive or negative ϕ-axis.

(iii) $V(\phi_m \neq 0) = 0$, which will be satisfied if

$$C_1 - \mu\phi_m - \gamma\left(1 + \frac{\phi_m}{3\beta}\right)^4 - M^2\sqrt{1 - \frac{2\phi_m}{M^2}} = 0, \tag{14}$$

where ϕ_m is the amplitude of the SWs or DLs.