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Non-equilibrium dynamical phases of the two-atom Dicke model

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ABSTRACT

In this paper, we investigate the non-equilibrium dynamical phases of the two-atom Dicke model, which can be realized in a two species Bose–Einstein condensate interacting with a single light mode in an optical cavity. Apart from the usual non-equilibrium normal and inverted phases, a non-equilibrium mixed phase is possible which is a combination of normal and inverted phase. A new kind of dynamical phase transition is predicted from non-superradiant mixed phase to the superradiant phase which can be achieved by tuning the two different atom–photon couplings. We also show that a dynamical phase transition from the non-superradiant mixed phase to the superradiant phase is forbidden for certain values of the two atom–photon coupling strengths.

1. Introduction

The interaction of a collection of atoms with a radiation field has always been an important topic in quantum optics. The Dicke model (DM) which describes interaction of N identical two level atoms with a single radiation field mode, established the importance of collective effects of atom–field interaction, where the intensity of the spontaneously emitted light is proportional to N^2 rather than N [1]. The spatial dimensions of the ensemble of atoms are smaller than the wavelength of the radiation field. As a result, all the atoms experience the same field and this gives rise to the collective and cooperative interaction between light and matter. The DM exhibits a second-order quantum phase transition (QPT) from a non-superradiant normal phase to a superradiant phase when the atom–field coupling constant exceeds a certain critical value [2–5]. The Dicke model phase transition was observed recently in a trapped Bose–Einstein condensate (BEC) in an optical cavity [6–9]. In the BEC setup, the two spin states of the original DM are the two momentum states of the BEC which are controlled by the atomic recoil energy and Raman pumping schemes. This approach is similar to a novel scheme proposed by Dimer et al. [10]. An important aspect of these experimental developments is the possibility to explore exotic phases mediated by the cavity field. The superradiance phase transition in a BEC is accompanied by self-organization of the atoms into a checker board pattern [6–9,11].

Interesting equilibrium and non-equilibrium phases have been predicted in the DM with BEC [12,13], including crystallization and frustration [14], as well as spin glass phase [15–18]. Multimode DM has also been explored recently, revealing interesting physics such as Abelian and non-Abelian gauge potentials [19], spin–orbit induced anomalous Hall effect [20], and prediction of the Nambu–Goldstone mode [21]. An interesting extension of the BEC Dicke model is the optomechanical Dicke model which has been proposed for detection of weak forces [22,23]. In the present paper, we investigate the non-equilibrium properties of the two-atom Dicke model, which can be realized by a two species BEC in an optical cavity. Apart from the usual non-equilibrium normal and inverted phases, the dynamical phase diagram reveals a new kind of non-equilibrium mixed phase. This gives rise to a new dynamical phase transition from the mixed phase to the superradiant phase by manipulation of the two distinct atom–photon coupling strengths. In addition, we show that a dynamical phase transition from the non-superradiant mixed phase to the superradiant phase is not allowed for certain values of the atom–photon coupling strengths of the two sets of atoms.

2. The model

We consider two different ensembles of N_1 and N_2 atoms coupled simultaneously to the quantized field of an optical cavity mode (Fig. 1). The two sets of atoms have transition frequencies ω_1 and ω_2 while the frequency of the cavity mode is ω_c. The cavity is pumped by a transverse external laser with frequency ω_p. The light–matter coupling strengths for the two sets of atoms are λ_1 and λ_2. These coupling strengths λ_1 and λ_2 can be written as $\lambda_i = \lambda_{0i}\Omega_P/2(\omega_p - \omega_i)$ $(i = 1, 2)$, λ_{0i} is the single atom-cavity mode coupling while Ω_P is the transverse pump beam Rabi frequency. The detuning $(\omega_p - \omega_i)$ is considered to be large so as to

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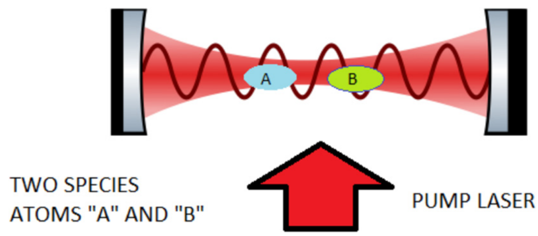

Fig. 1. (Color online.) Experimental setup showing two sets of cold atoms (blue and green) in an optical cavity with transverse pumping. The two sets of atoms have different atom–photon coupling strengths which depend on their position in the cavity. On increasing the transverse pump intensity, one type of atoms can reach the critical point earlier.

avoid spontaneous emission. The effective Hamiltonian of the system takes the form of a two-atom Dicke model with

$$H = \hbar\omega_1 J_{1z} + \hbar\omega_2 J_{2z} + \hbar\omega_c a^\dagger a + \frac{\hbar\lambda_1}{\sqrt{N_1}}(J_{1+} + J_{1-})(a + a^\dagger) + \frac{\hbar\lambda_2}{\sqrt{N_2}}(J_{2+} + J_{2-})(a + a^\dagger), \tag{1}$$

where $\vec{J}_i = (J_{ix}, J_{iy}, J_{iz})$ is the effective collective spin of length $N_i/2$ for the two sets of atoms and $J_{i\pm} = J_{ix} \pm i J_{iy}$.

We now discuss the non-equilibrium dynamics arising from the above two-atom Dicke model. The semi-classical equations of motion for the system are given by

$$\dot{J}_{1z} = \frac{i\lambda_1}{\sqrt{N_1}}(a^\dagger + a)(J_{1-} - J_{1+}), \tag{2}$$

$$\dot{J}_{2z} = \frac{i\lambda_2}{\sqrt{N_2}}(a^\dagger + a)(J_{2-} - J_{2+}), \tag{3}$$

$$\dot{J}_{1-} = -i\omega_1 J_{1-} + \frac{2i\lambda_1}{\sqrt{N_1}}(a^\dagger + a) J_{1z}, \tag{4}$$

$$\dot{J}_{2-} = -i\omega_2 J_{2-} + \frac{2i\lambda_2}{\sqrt{N_2}}(a^\dagger + a) J_{2z}, \tag{5}$$

$$\dot{a} = -(\kappa + i\omega_c)a - \frac{i\lambda_1}{\sqrt{N_1}}(J_{1+} + J_{1-}) - \frac{i\lambda_2}{\sqrt{N_2}}(J_{2+} + J_{2-}). \tag{6}$$

Here κ is the decay rate of the cavity photons. In addition the magnitude of pseudo-angular momentum is conserved,

$$J_{1z}^2 + |J_{1-}|^2 = \frac{N_1^2}{4}, \tag{7}$$

$$J_{2z}^2 + |J_{2-}|^2 = \frac{N_2^2}{4}. \tag{8}$$

The long time steady state solutions from the equations of motion can be studied with $\dot{\vec{J}}_i = 0$ $(i = 1, 2)$ and $\dot{a} = 0$. These fixed point solutions can be stable or unstable. Separating $a = a_1 + i a_2$, $J_i^\pm = J_{ix} \pm J_{iy}$ $(i = 1, 2)$, one obtains the steady state equations as

$$\kappa a_1 - \omega_c a = 0, \tag{9}$$

$$\kappa a_2 + \omega_c a_1 = -\frac{2\lambda_1}{\sqrt{N_1}} J_{1x} - \frac{2\lambda_2}{\sqrt{N_2}} J_{2x}, \tag{10}$$

$$\omega_1 J_{1y} = 0, \tag{11}$$

$$\omega_1 J_{1x} = \frac{4\lambda_1}{\sqrt{N_1}} a_1 J_{1z}, \tag{12}$$

$$\omega_2 J_{2y} = 0, \tag{13}$$

$$\omega_2 J_{2x} = \frac{4\lambda_2}{\sqrt{N_2}} a_1 J_{2z}. \tag{14}$$

An analysis of these equations leads us to four types of steady states, namely $(a = 0,\ J_{1z} = \pm N_1/2,\ J_{2z} = \pm N_2/2)$. The state $(a = 0,\ J_{1z} = -N_1/2,\ J_{2z} = -N_2/2)$ is the normal phase while $(a = 0,\ J_{1z} = N_1/2,\ J_{2z} = N_2/2)$ is the inverted phase. The states $(a = 0,\ J_{1z} = -N_1/2,\ J_{2z} = N_2/2)$ and $(a = 0,\ J_{1z} = N_1/2,\ J_{2z} = -N_2/2)$ are called mixed phases. As we shall show later these mixed phases generate interesting non-equilibrium phase diagrams. The critical coupling strength corresponding to the onset of superradiance starting from the normal, inverted or mixed phase is obtained by putting $\vec{J}_i = (0, 0, \pm N_i/2)$ $(i = 1, 2)$.

This leads us to the following possible critical constants

$$J_{1z} = -\frac{N_1}{2}; \qquad J_{2z} = -\frac{N_2}{2} \quad (\textit{Normal Phase}):$$

$$\lambda_{1c} = \sqrt{\frac{(\kappa^2 + \omega^2)\omega_1}{4\omega} - \frac{\lambda_2^2\omega_1}{\omega_2}}, \tag{15}$$

$$\lambda_{2c} = \sqrt{\frac{(\kappa^2 + \omega^2)\omega_2}{4\omega} - \frac{\lambda_1^2\omega_2}{\omega_1}}, \tag{16}$$

$$J_{1z} = \frac{N_1}{2}; \qquad J_{2z} = \frac{N_2}{2} \quad (\textit{Inverted Phase}):$$

$$\lambda_{1c} = -\sqrt{\frac{(\kappa^2 + \omega^2)\omega_1}{4\omega} + \frac{\lambda_2^2\omega_1}{\omega_2}}, \tag{17}$$

$$\lambda_{2c} = -\sqrt{\frac{(\kappa^2 + \omega^2)\omega_2}{4\omega} + \frac{\lambda_1^2\omega_2}{\omega_1}}, \tag{18}$$

$$J_{1z} = -\frac{N_1}{2}; \qquad J_{2z} = \frac{N_2}{2} \quad (\textit{Mixed Phase 1}):$$

$$\lambda_{1c} = \sqrt{\frac{(\kappa^2 + \omega^2)\omega_1}{4\omega} + \frac{\lambda_2^2\omega_1}{\omega_2}}, \tag{19}$$

$$\lambda_{2c} = -\sqrt{\frac{(\kappa^2 + \omega^2)\omega_2}{4\omega} - \frac{\lambda_1^2\omega_2}{\omega_1}}, \tag{20}$$

$$J_{1z} = \frac{N_1}{2}; \qquad J_{2z} = -\frac{N_2}{2} \quad (\textit{Mixed Phase 2}):$$

$$\lambda_{1c} = -\sqrt{\frac{(\kappa^2 + \omega^2)\omega_1}{4\omega} - \frac{\lambda_2^2\omega_1}{\omega_2}}, \tag{21}$$

$$\lambda_{2c} = \sqrt{\frac{(\kappa^2 + \omega^2)\omega_2}{4\omega} + \frac{\lambda_1^2\omega_2}{\omega_1}}. \tag{22}$$

This set of expressions reveals one interesting point that the critical coupling strength for one set of atoms depends on the coupling strength of the other set of atoms. For a given set of J_{1z} and J_{2z}, Eqs. (15)–(22) determines the boundary between the nonsuperradiant (normal/inverted/mixed) and superradiant phase. A trivial manipulation of Eqs. (9)–(14) leads us to the following equation for $J_{1z} = -N_1/2$ and $J_{1x} = 0$,

$$J_{2x}\left(\omega_2(\kappa^2 + \omega_c^2) + \frac{8\lambda_2^2}{N_2}\omega_c J_{2z}\right) = 0. \tag{23}$$

Now there are two possibilities depending on whether $J_{2x} = 0$ and $J_{2z} = \pm N_2/2$ or $J_{2x} \neq 0$ and $J_{2z} = -N_2\omega_2(\kappa^2 + \omega_c^2)/8\lambda_2^2\omega_c$. The first condition implies that both the set of atoms are in the non-superradiant phase. The second solution corresponds to the case where the first set of atoms are in the non-superradiant normal phase while the second set of atoms are in the superradiant phase. Substituting the second expression for J_{2z} from above in the expression for $\lambda_{1c} = \sqrt{\frac{\omega_1(\kappa^2+\omega_c^2)}{4\omega_c} + \frac{2\lambda_2^2\omega_1}{N_2\omega_2} J_{2z}}$, one obtains $\lambda_{1c} = 0$. This implies that by keeping one coupling strength