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Spontaneous emission and quantum discord: Comparison of Hilbert–Schmidt and trace distance discord

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ABSTRACT

Hilbert–Schmidt and trace norm geometric quantum discord are compared with regard to their behavior during local time evolution. We consider the system of independent two-level atoms with time evolution given by the dissipative process of spontaneous emission. It is explicitly shown that the Hilbert–Schmidt norm discord has nonphysical properties with respect to such local evolution and cannot serve as a reasonable measure of quantum correlations and the better choice is to use trace norm discord as such a measure.

1. Introduction

Characterizing the nature of correlations in composite quantum systems is one of the fundamental problems in quantum theory. When the system is prepared in a pure state, only entanglement is responsible for the presence of quantum correlations. On the other hand, once mixed states are taken into account, the problem becomes much more involved. Some features of separable mixed states are incompatible with a classical description of correlations. The most important among them is that a measurement on a part of composite system in some non-entangled states can induce disturbance on the state of complementary subsystem. Such "non-classical" behavior can be quantified by quantum discord – the most promising measure of bipartite quantum correlations beyond quantum entanglement [1]. For pure states discord coincides with entanglement, but in the case of mixed states discord and entanglement differ significantly. For example, it was shown that almost all quantum states have non-vanishing discord [2] and even local operations on the measured part can increase or create quantum discord [3,4].

In this paper we quantify non-classical correlations which may differ from entanglement by using geometric quantum discord. This quantity is defined in terms of minimal distance of the given state from the set of classically correlated states, so the proper choice of such a distance is crucial. The measure proposed in [5] uses a Hilbert–Schmidt norm to define a distance in the set of states. This choice has a technical advantage: the minimization process can be realized analytically for arbitrary two-qubit states. Despite of this feature, this measure has some unwanted properties. The most important problem is that it may increase under local operations performed on the unmeasured subsystem [7,8]. Fortunately, by using other norm in the set of states, this defect can be repaired: the best choice is to use Schatten 1-norm (or trace norm) to define quantum discord [9]. On the other hand, such defined measure is more difficult to compute. The closed formula for it is known only in the case of Bell-diagonal states or X-shaped two-qubit states [9,10].

The main scope of this paper is to reconsider the properties of those two measures of quantum discord in a concrete physical system where the quantum channel is given by the time evolution. As a compound system we take two independent two-level atoms not completely isolated from the environment. In this case the time evolution is given by a dissipative process of spontaneous emission. One-sided spontaneous emission in which only one atom emits photons and the other is isolated from the environment, gives the physical realization of local quantum channel. Although it was already established [7,8], in this framework we can explicitly show that Hilbert–Schmidt norm discord has nonphysical properties with respect to the local evolution and the better choice is to use trace norm. In particular we discuss the local creation of discord when the system is prepared in classical initial state [11–14]. In Ref. [15] we have studied time evolution of Hilbert–Schmidt quantum discord D_2, now we compare it with the behavior of trace norm quantum discord D_1. The results of our analysis show that when only the local creation of quantum discord in the

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classical initial state is considered, D_2 and D_1 provide the same information about the evolution of quantum correlations. This is no longer true when the initial states have non-zero discord. Local evolution can increase quantum discord and this phenomenon can be observed by using D_1 or D_2. On the other hand, there are initial states with decreasing quantum correlations quantified by D_1 whereas D_2 is increasing. The most spectacular manifestation of nonphysical properties of Hilbert–Schmidt norm discord is its behavior during the local evolution of the unmeasured subsystem. D_2 not only increases for a large class of initial discordant states (at the same time D_1 obviously decreases) but also it can increase even when the local evolution of the measured subsystem leads to decreasing D_2. This shows again that in contrast to trace norm discord, Hilbert–Schmidt norm discord cannot serve as a reasonable measure of quantum correlations.

2. Geometric measures of quantum discord

We start with the introduction of the standard notion of geometric quantum discord [5]. When a $d\otimes d$ bipartite system AB is prepared in a state ρ and we perform local measurement on the subsystem A, almost all states ρ will be disturbed due to such measurement. The (one-sided) geometric discord $D_2(\rho)$ can be defined as the minimal disturbance, measured by the squared Hilbert–Schmidt distance, induced by any projective measurement $\mathbb{P}^A$ on subsystem A, i.e.

$$D_2(\rho)=\frac{d}{d-1}\min_{\mathbb{P}^A}\left\|\rho-\mathbb{P}^A(\rho)\right\|_2^2, \tag{2.1}$$

where

$$\|a\|_2=\sqrt{\operatorname{tr} aa^*}. \tag{2.2}$$

Here we adopt normalized version of the geometric discord, introduced in Ref. [16]. In the case of two qubits, there is an explicit expression for D_2 [5]:

$$D_2(\rho)=\frac{1}{2}\left(\|\boldsymbol{x}\|^2+\|T\|_2^2-k_{\max}\right), \tag{2.3}$$

where the components of the vector $\boldsymbol{x}\in\mathbb{R}^3$ are given by

$$x_k=\operatorname{tr}(\rho\sigma_k\otimes\mathbb{1}), \tag{2.4}$$

the matrix T has elements

$$T_{jk}=\operatorname{tr}(\rho\sigma_j\otimes\sigma_k) \tag{2.5}$$

and $k_{\max}$ is the largest eigenvalue of the matrix $K=\boldsymbol{x}\boldsymbol{x}^T+TT^T$. As it was shown in Ref. [6], one can provide an explicit expression for $k_{\max}$ in terms of $\operatorname{tr}K$, $\operatorname{tr}K^2$ and $\operatorname{tr}K^3$. But the resulting function is complicated and in the study of time evolution of quantum discord, it is more convenient to use the formula (2.3). Despite of being easy to compute, the measure D_2 fails as a quantifier of quantum correlations, since it may increase under local operations on the unmeasured subsystem [7]. In the present paper we explicitly show that one-sided spontaneous emission of the unmeasured atom can create additional discord quantified by D_2 in the system of two independent atoms. Such defect of D_2 originates in the properties of Hilbert–Schmidt norm, which manifests also in the case of entanglement [17].

To repair this defect, one considers other norms in the set of quantum states. The best choice is to use the trace norm (or Schatten 1-norm) and define [9]

$$D_1(\rho)=\min_{\mathbb{P}^A}\left\|\rho-\mathbb{P}^A(\rho)\right\|_1, \tag{2.6}$$

where

$$\|a\|_1=\operatorname{tr}|a|. \tag{2.7}$$

D_1 has desired properties with respect to the local operations on unmeasured subsystem, but its computation is much more involved. Analytic expression for D_1 is known only for limited classes of two-qubits states, including Bell-diagonal [9] and X-shaped mixed states [10]. In the present paper, we consider X-shaped two-qubit states

$$\rho=\begin{pmatrix}\rho_{11}&0&0&\rho_{14}\\0&\rho_{22}&\rho_{23}&0\\0&\rho_{32}&\rho_{33}&0\\\rho_{41}&0&0&\rho_{44}\end{pmatrix}, \tag{2.8}$$

where all matrix elements are real and non-negative. The quantity D_1 for such states can be computed as follows. Let $x=2(\rho_{11}+\rho_{22})-1$ and

$$\alpha_1=2(\rho_{23}+\rho_{14}),\qquad \alpha_2=2(\rho_{23}-\rho_{14}),$$
$$\alpha_3=1-2(\rho_{22}+\rho_{33}). \tag{2.9}$$

Then [10]

$$D_1(\rho)=\sqrt{\frac{a\alpha_1^2-b\alpha_2^2}{a-b+\alpha_1^2-\alpha_2^2}}, \tag{2.10}$$

where

$$a=\max\left(\alpha_3^2,\alpha_2^2+x^2\right),\qquad b=\min\left(\alpha_3^2,\alpha_1^2\right). \tag{2.11}$$

Notice that we use normalized version of D_1 and the formula (2.10) is not valid in the case when $x=0$ and

$$|\alpha_1|=|\alpha_2|=|\alpha_3|. \tag{2.12}$$

In such a case, one can use general prescription how to compute D_1, also given in Ref. [10] (Eq. (65)).

In the case of pure states, D_1 as well as D_2 give the same information about quantum correlations as entanglement measured by negativity

$$N(\rho)=\left\|\rho^{\mathrm{PT}}\right\|_1-1, \tag{2.13}$$

where ρ^{PT} denotes partial transposition of ρ. In the case of mixed states, entanglement and discord significantly differ. For example, for two-qubit Bell-diagonal states one finds that [9]

$$D_1\geq\sqrt{D_2}\geq N. \tag{2.14}$$

The inequality $\sqrt{D_2}\geq N$ was proved to be valid for all two-qubit mixed states [16], and it is conjectured that (2.14) is also valid for all two-qubit states.

To show that inequalities in (2.14) can be sharp, consider the following family of states [18]

$$\rho_\theta=\begin{pmatrix}\frac{1}{2}\cos^2\theta&0&0&\frac{1}{4}\sin 2\theta\\0&0&0&0\\0&0&\frac{1}{2}&0\\\frac{1}{4}\sin 2\theta&0&0&\frac{1}{2}\sin^2\theta\end{pmatrix}, \tag{2.15}$$

where $\theta\in[0,\pi/2]$. By direct computation, one can check that

$$N(\rho_\theta)=\frac{\sqrt{6-2\cos 4\theta}-2}{4}, \tag{2.16}$$

whereas

$$D_2(\rho_\theta)=\min\left(\frac{1}{2}\sin^2\theta,\frac{1}{4}\sin^2 2\theta\right) \tag{2.17}$$

and

$$D_1(\rho_\theta)=\frac{1}{2}\sin 2\theta. \tag{2.18}$$