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Effective cosmological constant within the expanding axion universe

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ABSTRACT

We show that the value of an effective cosmological constant, Λ_{eff}, is influenced by the dimensionality of the space. Results were obtained in the framework of the axion model describing expansion of the inhomogeneous universe. Λ_{eff} determines the tension of the space (i.e. elasticity), and is relaxed when extra dimensions are accessible. We demonstrate that the effective value of the cosmological constant may be tuned to be consistent with experimental observation. Inhomogeneities considered are representative of temperature fluctuations observed within the cosmic microwave background radiation.

1. Introduction

Ever since 1929 when universal expansion was first observed, physicists have wrestled to develop an explanation as to precisely why. From *steady-state* universe, to *big-bang* and *inflationary* scenarios; with evermore enticing observational evidence at our disposal, we are beginning to converge upon its precise nature.

Today's state of cosmology is epitomised by the accelerative expansion of the universe, with the so-called *dark energy* being responsible for this elusive driving force. Numerous experiments, of which include the WMAP [1] and COBE [2] satellite projects, and further via detailed analyses of Type Ia supernovae [3,4], have proven to exhibit conclusive evidence that our universe is subject to accelerated growth. In particular, both COBE and WMAP have provided detailed maps of the distribution for the cosmic microwave background (CMB) radiation within the universe. Such distributions or patterns are characteristic snapshots of the energy-density structure of the universe taken at the moment when matter was decoupled from radiation. The conclusion of these seminal recent experiments, was that the principal constituents of the universe take the form of dark matter and dark energy, of which there is believed to be a constant battle between the two [5,6]. One pushes the universe to expand, while the other – to collapse. Both dark matter and dark energy co-exist in a universe whereby billions of stars are self-organised into spiralling galaxies, which group into larger, more stable galactic clusters. These structures extend further into enormous superclusters that thread throughout the universe, all of which are moving apart. These observed structures which are entirely compatible with inflation theory and big-bang, show that an understanding of dark energy and its role to play in structure formation, is one the most fundamental problems in modern cosmology.

More recently, the Planck satellite sought to map the CMB in greater detail than ever before. The mission which was carried out by the European Space Agency (ESA), certainly held true to its word, and after four years of gathering data, Planck delivered its spectacular results [7]. A key feature which has further motivated this paper, is the apparent lensing of CMB photons due to all intermediary matter. Furthermore, the patterns of localised distortions show no random characteristic, with hot or cold spots moving coherently in a single direction [8]. This implies that structure formation in the universe has been influenced by a well-defined external influence. We shall seek to explain how one may obtain such a scenario. Furthermore, Einstein's remarkable theory of general relativity tells us that matter and energy distort space–time. On a cosmic scale, the net matter/energy-density of the universe determines its overall space–time curvature. This, in turn, determines the geometry of the universe (i.e., either open, closed or flat). Most believe that the answers reside within the CMB; the primordial light from some half a million years after the big-bang. The geometry of space affects the observed size of hot and cold spots within the CMB – measurements of these variations have indicated that our observable universe is flat to within an error of one percent [1].

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2. An approach to unifying dark matter and dark energy

2.1. Dark matter

Particle physicists have postulated WIMPs (Weakly Interacting Massive Particles) such as axions, dilatons or neutralinos as dark matter candidates [9,10], while the nature of dark energy is somewhat more elusive. Heterotic string theory even provides as the candidate, a very light universal axion, convenient to describe the nearly massless pseudoscalar field theory [11].

There is common belief that dark matter and dark energy have nothing to do with each other. However, it has been shown that both may arise from some kind of scalar field [12,13]. Both may account, on different scales, for inflation [14,15], dark matter halos of galaxies [16,17], or even dark matter condensations (the so-called boson stars) [18,19] as candidates for Massive Compact Halo Objects (MACHOs). Independently, the views of superstring theory [20] suggest an importance of the scalar field with as small a mass as $\sim 10^{-23}$ eV.

In this paper, we follow an approach similar to those of [21–23], where axion-like scalar models with periodic self-interaction have been studied. Additionally, the authors of [24] show that an axion Bose–Einstein condensate can provide a substantial contribution to the observed rotation curves of galaxies [24], and has probably been observed via gravitational lensing in merging clusters. Recent images captured by the Hubble Space Telescope (HST) reveal a mysterious clump of dark matter, thought to be the remnants of a massive galactic collision [25]. It seems that the soliton-type dark matter bullets described in [24] provide a natural explanation as to the formation of such dark matter clumps.

2.2. Dark energy

2.2.1. Cosmological constant

At first glance of the Friedmann equations, the phenomenon of dark energy may be described by the cosmological constant, for which many sub-candidates have been proposed (cf. [12,13] and references within). This was first introduced by Albert Einstein, in order to obtain static, stable solutions to the gravitational field equations. In effect, dark energy was used to prevent the gravitational collapse of the universe. Little was it known at the time, that should spatial inhomogeneities be present post-inflation, these could lead to an unstoppable expansion of the universe. Furthermore, the major crux here is an apparent screening of this parameter; the value predicted by experimental observation [3,4,26,27] remaining inconsistent with the energy scale predictions from particle physics [28–30]. The observed value of 7×10^{-30} g/cm^3 (or in natural units $\approx 10^{-35}$ s^{-2}), is more than 120 orders of magnitude smaller than the Planck density ($\approx 10^{93}$ g/cm^3) at the instant of the big-bang [21]. The value itself is merely representative of an overall averaging of the quantum vacuum fluctuations (the so-called *quantum foam*), and thus the characteristic energy-density associated with empty space [28].

2.2.2. Scalar fields and higher order curvature Lagrangians

A scalar field, minimally coupled to Einstein's general relativity is equivalent [12,13,31,32] to modified gravity in the relativistic framework of higher-order curvature Lagrangians. Such effective Lagrangians may also arise from the low-energy limit of superstrings (cf. for example [33]), which use a non-linear higher-order curvature Lagrangian to explain the present cosmic accelerated expansion. Our choice of Lagrangian will be outlined in Section 3.

Of all the proposed candidates for dark energy, perhaps the most elegant is the quintessence scalar field φ. The theory posits that some dynamic function (the scalar field), driven by an inherent universal potential $V(|\varphi|)$, constitutes the underlying mechanism for the observed expansion of the universe.

2.2.3. Axions

The existence of scalar fields is also predicted by the standard model of particle physics and quantum chromodynamics (QCD). However, QCD is afflicted with the issue of strong-CP symmetry breaking. Peccei–Quinn theory seeks to remedy this by adding a CP-violating term (the so-called φ parameter) [23,22] to the Yang–Mills Lagrangian. Not only does the theory predict that φ is representative of some dynamical field rather than a constant numerical value, but since quantum fields are synonymous with particles, the theory predicts the existence of a new particle also – the axion. This particle, as previously mentioned, is regarded by many as one of the best motivated candidates for cold dark matter (CDM) [34].

Although the cosmological constant will be able to describe the effects of dark energy, we are curious to consider the contribution of axions via scalar field fluctuations. An effective potential $V(|\varphi|)$ will arise from the chiral anomaly following integration of the gluon field, and is given as follows (cf. for details [22,35]),

$$V(|\varphi|) = \frac{m^4}{\lambda}\left[1 - \cos\left(\frac{\sqrt{\lambda}}{m}\varphi\right)\right]. \tag{1}$$

Each of the minima within this potential, are associated with different vacuum states, each possessing the same energy. The curvature of the potential at each minimum is related to the axion mass m. Due to the nature of the potential under consideration, a perfectly apt analogy may be associated with that of a pendulum.

3. Hybrid quintessence

Into our cosmological recipe, we wish to include all we have touched upon in the previous sections. Since the metric outlines the geometry of the space–time domain, this needs to be carefully defined.

3.1. Kaluza–Klein theory

An ideal starting point is the FRW metric for an expanding universe. Furthermore, our motivation for including electromagnetic fields will arise via Kaluza–Klein theory, for which the vector potential A_μ becomes an integral part of the metric [36,37]. As previously mentioned, the universe is flat to within an error of one percent [1] – we shall therefore assume the curvature k of the universe to be equal to zero. In Cartesian coordinates (denoted by $\tilde{x}^\mu = \{t, x, y, z, \chi\}$), we therefore commence as follows,

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} -1+\xi A_0 A_0 & \xi A_0 A_1 & \xi A_0 A_2 & \xi A_0 A_3 & \xi A_0 \\ \xi A_1 A_0 & a(t)^2 + \xi A_1 A_1 & \xi A_1 A_2 & \xi A_1 A_3 & \xi A_1 \\ \xi A_2 A_0 & \xi A_2 A_1 & a(t)^2 + \xi A_2 A_2 & \xi A_2 A_3 & \xi A_2 \\ \xi A_3 A_0 & \xi A_3 A_1 & \xi A_3 A_2 & a(t)^2 + \xi A_3 A_3 & \xi A_3 \\ \xi A_0 & \xi A_1 & \xi A_2 & \xi A_3 & \xi \end{pmatrix}. \tag{2}$$

Here we subscribe to the sign convention $(-,+,+,+,+)$ of Misner–Thorne–Wheeler (MTW) [38]. One will note the inclusion of an extra dimension χ. In principle, this alternate dimension (of which we have no experience) may be compactified via a periodic boundary condition to such small size, that it evades even the most powerful particle accelerators. The size of this dimension is determined by the constant ξ which appears in the metric.

A tilde is chosen to denote terms applicable to this higher dimensional metric $\tilde{g}_{\mu\nu}$. However, these terms can easily be expressed via the more standard metric, without vector potential A_μ included.

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a(t,r)^2 & 0 & 0 \\ 0 & 0 & a(t,r)^2 & 0 \\ 0 & 0 & 0 & a(t,r)^2 \end{pmatrix}. \tag{3}$$