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Phase synchronization in a two-mode solid state laser: Periodic modulations with the second relaxation oscillation frequency of the laser output

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ABSTRACT

Phase synchronization (PS) in a periodically pump-modulated two-mode solid state laser is investigated. Although PS in the laser system has been demonstrated in response to a periodic modulation with the main relaxation oscillation (RO) frequency of the free-running laser, little is known about the case of modulation with minor RO frequencies. In this Letter, the empirical mode decomposition (EMD) method is utilized to decompose the laser time series into a set of orthogonal modes and to examine the intrinsic PS near the frequency of the second RO. The degree of PS is quantified by means of a histogram of phase differences and the analysis of Shannon entropy.

1. Introduction

Synchronization in nonlinear systems has been extensively studied because of its critical importance in a wide variety of disciplines, including physics [1,2], mathematics [3], biology [4], physiology [5], ecology [6], chemistry [7], and others [8]. The phenomenon is caused by interactions between systems and can be classified into several categories, depending on the emerged correlations among the systems. The obvious one is complete synchronization (CS), in which interacting systems adjust their states and finally converge to *a single trajectory* [9]. Although the definition of CS is extremely simple, the requirements for achieving CS are relatively strict and therefore other synchronous phenomena, especially phase synchronization (PS) [10], are more common in both artificial and natural systems.

PS has been one of the most intriguing subjects in nonlinear science since the pioneering work done on it in 1996 [11]. PS is a type of synchronization that reflects rhythms identification of interacting systems, but the amplitudes of the systems remain uncorrelated. The mathematical definition of PS is $|n\phi_1 - m\phi_2| < c$, where ϕ_1 and ϕ_2 are instantaneous phases of interacting systems; n and m are integers, and c is a constant. Because of the universality of PS, it has been studied experimentally in various systems, such as plasma systems [12] and fluid systems [13]. PS has also been realized in a variety of lasers owing to its potential practical applications, examples of which include optically coupled Nd:YAG lasers [14], electronically coupled Nd:YAG lasers [15], the cw CO_2 laser that is driven by an intracavitary electro-optic modulator (EOM) [16], the Nd:YAG laser that is driven by an intracavitary acousto-optic modulator (AOM) with two frequencies [17], and others [18].

Recently, interest has been extended to PS in a periodically pump-modulated frequency-doubled Nd:YAG laser [19]. Ahlborn and Parlitz applied recurrence analysis and pseudo ensemble averaging to define and quantify PS in the system, with a focus on the emergent synchronous region, which is called the Arnold tongue. The Arnold tongue occurs close to the modulating frequency of $f \approx 1$ MHz, which is exactly the main relaxation oscillation (RO) frequency of the laser output in the absence of modulation. Subsequently, Lin et al. used a similar technique and realized PS in a two-mode microchip Nd:YVO_4 laser experimentally and theoretically [20]. These results are crucial to increasing the superimposed common output of several lasers that are driven by periodic pump modulation.

An interesting dynamic feature of a multi-mode laser is the presence of more than one frequency peak in the power spectrum of the laser output. Accordingly, the multi-mode laser essentially has several RO frequencies. The way in which the system responds to external periodic driving becomes a very interesting question in the study of PS. The power spectrum of the free-running two-mode microchip Nd:YVO_4 laser presents two distinct RO frequencies. Although PS has been demonstrated when the periodic pump is modulated to the first RO frequency of the free-running laser [20],

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according to standard analysis, this phenomenon is invisible close to the second RO frequency. In this Letter, the empirical mode decomposition (EMD) method [21] is used to elucidate the intrinsic PS in the vicinity of the second relaxation frequency. Hilbert transform is used to define the phases of, and the phase differences between, the decomposed mode of the laser output and the modulating signal. Further analysis reveals the existence of a robust Arnold tongue in the frequency region of interest.

This Letter is organized as follows. Section 2 introduces the two-mode solid state laser model and presents its power spectrum. In Section 3, the EMD process is used to decompose the laser output and the Hilbert transform is used to calculate its instantaneous phase. In Section 4, the appearance of PS is confirmed and PS is quantified by an analysis of phase difference. Finally, a brief conclusion is drawn.

2. Two-mode solid-state laser model

The scaled Tang–Statz–deMars (TSD) two-mode laser model has been demonstrated to be able to reproduce the dynamics of the periodically modulated microchip Nd:YVO_4 laser [20,22]. The TSD model with an externally driving term is

$$\frac{dn_0}{dt} = w_0\big[1+F(t)\big] - n_0 - \sum_{k=1}^{N}\gamma_k\left(n_0 - \frac{1}{2}n_k\right)s_k,$$
$$\frac{dn_m}{dt} = \gamma_m n_0 s_m - n_m\left(1+\sum_{k=1}^{N}\gamma_k s_k\right),$$
$$\frac{ds_m}{dt} = K\left[\gamma_m\left(n_0 - \frac{1}{2}n_m\right) - 1\right]s_m, \tag{1}$$

where $N = 2$ is the total number of lasing modes, and the subscript $m = 1, 2$ is the index of the corresponding mode. n_0 is the spatially averaged population inversion density with spatial hole burning, normalized by the threshold value; n_m is the first-order Fourier component of the population inversion density for the two modes, and s_m is the photo density, normalized by the steady-state value. The term w_0 is the pump power, normalized by threshold of the first lasing mode. The term $F(t) = \Delta w\cos(2\pi\tau\nu t)$ is a periodic pump-modulation, where Δw and ν are the amplitude and the frequency of the driving signal, respectively. γ_m is the gain ratio of the mth mode to the first lasing mode. $K = \tau/\tau_p$ is the lifetime ratio, where τ is the population lifetime of Nd:YVO_4 and τ_p is the lifetime in the laser cavity. The numerical time is scaled by the population lifetime, so $t = T/\tau$.

To perform the numerical simulations, the parameters of the TSD model are chosen as follows: $w_0 = 12.5$, $\gamma_1 = 1$, $\gamma_2 = 0.595$, $\tau = 90$ µs, $\tau_p = 1.15$ ns, and $K = 78\,260$ [23]. The sampling interval is 9 ns. Fig. 1(a) shows the power spectrum of the numerically produced laser output $S(t) = s_1(t) + s_2(t)$ in the absence of modulation. The two peaks in the figure are located at the first RO frequency, $f_{r_1} = 1.644$ MHz, and the second RO frequency, $f_{r_2} = 410$ kHz, of the free-running two-mode solid-state laser. The dynamic feature of the pair of RO frequencies persists over a wide range of parameters. For example, Fig. 1(b) presents another power spectrum of the system with $w_0 = 12.5$, $\gamma_1 = 1$, and $\gamma_2 = 0.595$. The arrows in the figure indicate two different RO frequencies, $f_{r_1} = 1.455$ MHz and $f_{r_2} = 292$ kHz. This work focuses on the system with the parameters that are used in Fig. 1(a), and the results are confirmed using other parameters especially those in Fig. 1(b).

An earlier study presented evidence of PS when the frequency of the periodic pump-modulation approximates the first RO frequency of the free-running laser, $\nu \approx f_{r_1}$, and the modulating amplitude Δw exceeds some critical threshold [20]. However, no

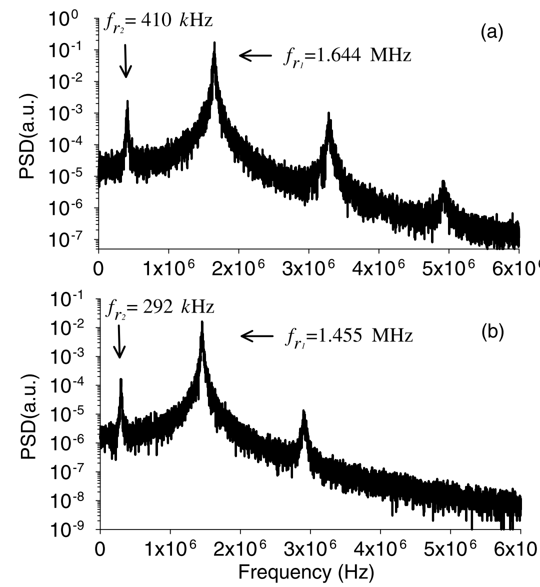

Fig. 1. The power spectrum of the laser output in the absence of modulations. The parameters are chosen as (a) $w_0 = 12.5$, $\gamma_1 = 1$, $\gamma_2 = 0.595$, and (b) $w_0 = 9.8$, $\gamma_1 = 1$, and $\gamma_2 = 0.590$.

evidence for PS has been obtained when ν approximates the second RO frequency, f_{r_2}. The purpose of this work is to identify the intrinsic PS under the condition $\nu \approx f_{r_2}$.

3. Empirical mode decomposition

Empirical mode decomposition (EMD) is adopted here to define explicitly the instantaneous phase and calculate the phase difference between the driver $F(t)$ and the laser output. Huang et al. proposed this method for analyzing non-stationary and nonlinear time series [21]. The method has since been applied to study the phase correlation and dynamic properties of financial data [24–26], cardiorespiratory synchronization [27], and human ventricular fibrillation [28]. The EMD method is based on the assumption that any time series consists of a finite number of intrinsic mode functions (IMFs), and each of which has its own characteristic time scale. Thus the EMD method aims to decompose the measured data into independent IMFs by a series of *sifting processes*. Since the decomposed IMFs are well-behaved intrinsic modes, the Hilbert transform can be used to calculate their phases directly.

The algorithm for obtaining IMFs in EMD follows.

Step 1: All local extremes of the laser output $S(t)$ are identified. Apply two cubic spline lines to connect the local maxima and the local minima respectively, and then construct the upper envelope $U(t)$ and the lower envelope $L(t)$ of the time series. Denote the mean of the two envelopes as $m_1(t) = (U(t) + L(t))/2$, and then the first component $h_1(t)$ is defined as the deviation of $S(t)$ from $m_1(t)$,

$$h_1(t) = S(t) - m_1(t). \tag{2}$$

Step 2: An IMF satisfies the following conditions. (a) The upper and lower envelopes are symmetric about zero. (b) The number of zero crossings equals or differs by one from the number of extremes. If $h_1(t)$ is already an IMF, then go to step 3. Otherwise a series of *sifting processes* must be performed to obtain an IMF from $h_1(t)$.

Treating $h_1(t)$ as the original laser output and denoting $m_1^{(1)}(t)$ as the mean of the upper envelope and the lower envelope of $h_1(t)$, it is easy to have $h_1^{(1)}(t) = h_1(t) - m_1^{(1)}(t)$. This process is the so-called sifting process. Perform the sifting process k times until the resulting $h_1^{(k)}(t)$ meets the requirements of an IMF. The first