Physics Letters A 378 (2014) 3021–3027

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Electromagnetic soliton propagation in an anisotropic Heisenberg helimagnet

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ARTICLE INFO

Article history:
Received 17 July 2014
Accepted 16 August 2014
Available online 19 August 2014
Communicated by R. Wu

Keywords:
Helimagnet
Reductive perturbation method
Nonlinear spin excitation
Kink soliton

ABSTRACT

We study the nonlinear spin dynamics of Heisenberg helimagnet under the effect of electromagnetic wave (EM) propagation. The basic dynamical equation of the spin evolution governed by Landau–Lifshitz equation resembles the director dynamics of the twist in a cholestric liquid crystal. With the use of reductive perturbation technique the perturbation is invoked for the spin magnetization and magnetic field components of the propagating electromagnetic wave. A steady-state solution is derived for the weakly nonlinear regime and for the next order, the components turn around s plane perpendicular to the propagation direction. It is found that as the electromagnetic wave propagates in the medium, both the magnetization and magnetic field modulate in the form of kink soliton modes by introducing amplitude fluctuation in the tail part of the same.

1. Introduction

The nonlinear evolution of Heisenberg ferromagnet offers several possible interesting dynamics when subject to different environments [1–3]. Especially few ferromagnets exhibit specific spin arrangement even in the absence of external fields, for instance the helimagnet [4]. Helimagnets are quite interesting because of their different orientations of spins in different lattices thereby leading to a helical structure when one moves along the helical axis. These spin arrangements quite resemble that of molecular orientation in cholestric liquid crystals and the arrangement of bases in DNA double helix chain. Thus the helimagnets are assigned to be one among the ordered magnetic spin systems and form interesting class of dynamical evolutions. The studies devoted to this peculiar arrangement of magnetic spin systems are quite numerous in different aspects. Many theoretical and experimental facts are established in the past. The transition from a helical to ferromagnetic phase is explored through the phase diagram studied by Rastelli and Tassi while performing quantum fluctuations and thermodynamical properties [4]. The low temperature properties of helimagnets show the usual spin wave excitations and the helicity is not affected by the temperature but depend on the nearest neighbour spin exchange [5]. Spin nematic formulation is witnessed when the usual helimagnet is subject to vector potential in the Hamiltonian and quantum fluctuations inducing an anisotropy in the helical structures [6]. In view of mean field theory calculations, the perturbation through the weak magnetic fields induces spin current in the helimagnets [7]. Further, application of spin polarized current induces the wave number and velocity of the helical magnetization that are analogous to those of moving domain walls [8]. The dynamics is explored for models of dissipative structures such Landau–Lifshitz and Gilbert system. Using the single-crystal neutron scattering experiments performed on the $LiCu_2O_2$ helimagnet show that exchange constant fluctuates the helical nature of the structure through the competition between the interacting nearest neighbor antiferromagnetic coupling and the next nearest neighbor ferromagnetic exchange [9]. Very recently, Beula et al. [10] in theoretical frame the nonlinear spin excitations of the Heisenberg helimagnet showed that in the presence of magnetic field applied parallel and perpendicular to the anisotropic axis exhibits the dynamics governed by soliton modes. A more rigorous approach is invoked on the Heisenberg helimagnet for the spin evolution in the classical continuum limit. The elementary and higher order spin excitations are the soliton solutions of the fourth order generalized nonlinear Schrödinger equation [11]. Most of the studies on the helimagnet show spin dynamics are more relevant for experimental support and the theoretical studies are very limited.

In this paper we investigate nonlinear dynamics of the Heisenberg helimagnet under the perturbation of electromagnetic (EM) wave propagation in the system. The paper is organized as follows. In Section 2, we construct the relevant dynamical model in the classical limit with Maxwell's equation. In Section 3, reductive perturbation method is employed on the coupled Maxwell–Landau–Lifshitz equation and derived the generalized derivative nonlinear

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http://dx.doi.org/10.1016/j.physleta.2014.08.020
0375-9601/

Schrödinger equation and the exact soliton solution for the same equation is systematically solved in Section 4. The results are summarized in Section 5.

2. Formulation of dynamical equation

We interested in estimating the dynamics of the Heisenberg helimagnet subject to external magnetic field applied via electromagnetic wave propagation. The helimagnetic spin system is modeled through the dynamical structure associated with that of molecular orientation of cholestric or twisted nematic liquid crystal first introduced by Daniel et al. [11]. The equation of motion for the liquid crystal system is deduced through the free energy represented by $[\mathbf{n}\cdot(\nabla\times\mathbf{n})+q_s]^2$, where $\mathbf{n}$ is the director of the liquid crystal system indicating the average molecular orientation and q_s is the pitch wave vector. In view of this helical arrangement, for the present system the classical Hamiltonian of the helimagnetic system can be written in the following form, $\{[\mathbf{S}\cdot(\nabla\times\mathbf{S})]^2-q_a^2\}^2$ with the dynamical variable $\mathbf{S}$ representing the average spin density function and q_a the pitch wave vector with magnitude $q_a=\frac{2\pi}{p}$, where p is the pitch of the helix. Thus the Heisenberg spin Hamiltonian for the helimagnetic spin system in the one-dimensional case can be written as

$$\mathcal{H}=-\sum_l\Big[J(\mathbf{S}_l\cdot\mathbf{S}_{l+1})+\tau_h\big\{\big[\hat{\mathbf{e}}_z\cdot(\mathbf{S}_l\times\mathbf{S}_{l+1})\big]^2-q_a^2\big\}^2 - B(\mathbf{S}_l\cdot\hat{\mathbf{e}}_z)^2+\mu(\mathbf{S}_l\cdot\mathbf{H})\Big], \tag{1}$$

where the terms J, B, τ_h are constant coefficients representing the strength of the various interactions such as nearest neighbour exchange, anisotropy and helicity involved in the system, and $\mu=g\mu_B$, where g is the gyromagnetic ratio and μ represents the Bohr magneton. The classical equation of motion for the above Hamiltonian can be deduced by employing the spin classical Poisson bracket, $\frac{\partial\mathbf{S}_l}{\partial t}=\{\mathbf{S}_l,H\}$. Further, in the continuum limit, the spin evolution equation can be written as [10]

$$\frac{\partial\mathbf{S}}{\partial t}=\mathbf{S}\times\left\{J\frac{\partial\mathbf{S}}{\partial z^2}-2B(S^z)\hat{\mathbf{e}}_z-\tau_h\mathcal{G}+\gamma\mathbf{H}\right\}, \tag{2}$$

where

$$\mathcal{G}=-4q_a^2\big[2(\mathbf{S}\times\mathbf{S}_z)^z(\mathbf{S}_z\times\hat{\mathbf{e}}_z)+(\mathbf{S}\times\mathbf{S}_{zz})^z(\mathbf{S}\times\hat{\mathbf{e}}_z)\big] +4\big[(\mathbf{S}\times\mathbf{S}_z)^z\big]^2 \times\big[2(\mathbf{S}\times\mathbf{S}_z)^z(\mathbf{S}_z\times\hat{\mathbf{e}}_z)+3(\mathbf{S}\times\mathbf{S}_{zz})^z(\mathbf{S}\times\hat{\mathbf{e}}_z)\big]. \tag{3}$$

The suffices represents the usual derivatives and the superfix z represent the z-component of the corresponding term, respectively. The above dynamical equation (2) satisfies the usual constrain $|\mathbf{S}|^2=0$, which take care of the time-independent nature of the length of the spin vector. Also it is interesting to note that Eq. (2) admits several integrable models when $\tau_h=0$ in the presence and in the absence of external magnetic field [12–14] with chaotic excitations depending on the direction of the applied magnetic field with respect to the easy axis of magnetization [15,16]. Nonlinear excitations of propagating electromagnetic wave (EMW) in ferromagnetic medium were is investigated for the past two decades [17–21]. The theory of developing the one-dimensional solitons in ferromagnets rely on the existence of nonlinear Schrödinger equation governing the propagating modes. Many effective investigations have been developed to represent the propagating electromagnetic wave in the form of this equation and some of its generalization as multiscale expansion techniques of Landau–Lifshitz model coupled to the Maxwell's equations [22–24]. In view of this the evolution of the magnetic field in the helimagnetic medium is governed by Maxwell equations which reduce to [25]

$$-\nabla(\nabla\cdot\mathbf{H})+\nabla^2\mathbf{H}=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\mathbf{H}+\mathbf{S}). \tag{4}$$

In Eq. (4), $c=1/\sqrt{\mu_0\varepsilon_0}$ is the velocity of the propagating EMW in the helimagnetic medium. Thus we have the coupled equations (2) and (4) that completely govern the dynamics of the propagation EMW and the spin excitations of the helimagnet.

3. Reduction to generalized derivative nonlinear Schrödinger equation

In many cases the treatment of the Landau–Lifshitz and Maxwell equations are rigorous to solve as such due to the highly vector and nonlinear nature of the governing equation [23,24]. Thus we are often intended to invoke asymptotic or perturbation technique to resolve the Maxwell–Landau (ML) model to a solvable scalar nonlinear equations. For reducing the ML system, in the long wave and small amplitude approximation the multiscale technique is introduced through a small parameter ϵ measuring the smallness and largeness of the fields. For this we expand the spin and magnetic fields as power series of ϵ in a nonuniform way as follows:

$$\mathcal{K}^z=\mathcal{K}_0+\epsilon\mathcal{K}_1^z+\epsilon^2\mathcal{K}_2^z+\ldots, \tag{5}$$

$$\mathcal{K}^\alpha=\sqrt{\epsilon}\big[\mathcal{K}_1^\alpha+\epsilon\mathcal{K}_2^\alpha+\ldots\big]. \tag{6}$$

The function $\mathcal{K}$ takes the spin $\mathbf{S}$, external magnetic field $\mathbf{H}$ variables and also functions of the slow variables $\zeta=\epsilon(z-Vt)$, $\tau=\epsilon^2 t$ and $\alpha=x,y$. The V being the velocity of the propagating pulse. We also assume that the spin exchange interaction and the helicity effects are more stronger which can be expressed by rescaling $J\to\epsilon^{-1}J$, $\tau_h\to\epsilon^{-2}\tau_h$ and $B\to\epsilon B$. The above expansions (5) and (6) are reported in Eqs. (2) and (4) and collects different orders of ϵ.

3.1. A steady-state solutions

The zeroth order of the perturbation normally represents the steady-state solutions which can be deduced from the corresponding terms in the Maxwell equation (4). The x and y component of the spin and magnetic fields admits linear variation and from Landau equation the zeroth order terms are identically satisfied by using the results obtained from Eq. (4). Thus we have at the $O(\epsilon^0)$: $H_1^x=r'S_1^x$, $H_1^y=r'S_1^y$, $H^0=-S^0$, where $r'=\frac{V^2}{(c^2-V^2)}\equiv\frac{H^0}{S^0}$.

3.2. Generalized derivative NLS equation

Having obtained the steady-state solution, the evolution of spin and magnetic fields turn around x–y plane at higher orders. Thus at $O(\epsilon^1)$:

$$\frac{\partial}{\partial\zeta}\big[H_2^x-r'S_2^x\big]=-\frac{\partial S_1^x}{\partial\tau}, \tag{7}$$

$$\frac{\partial}{\partial\zeta}\big[H_2^y-r'S_2^y\big]=-\frac{\partial S_1^y}{\partial\tau}. \tag{8}$$

In Eqs. (7) and (8), τ is rescaled as $\tau\to(2r'(1+r')/V)\tau$ and $H_1^z=-S_1^z$. The corresponding order in the Landau equation for the z-component is written as follows:

$$r'\mu S^0\frac{\partial}{\partial\tau}(-\mathrm{i}S_1^y)=V\frac{\partial^2}{\partial\zeta^2}(\mathrm{i}S_1^x) + JS^0\frac{\partial^3}{\partial\zeta^3}(-\mathrm{i}S_1^y)+BS^0\frac{\partial}{\partial\zeta}(-\mathrm{i}S_1^y) + 4\tau_h q_a^2 S^0\left\{4\left(S_1^x\frac{\partial^2 S_1^y}{\partial\zeta^2}-S_1^y\frac{\partial^2 S_1^x}{\partial\zeta^2}\right)\frac{\partial}{\partial\zeta}(\mathrm{i}S_1^x)\right.$$