Physics Letters A 378 (2014) 3074–3078

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

On coherent radiation by relativistic electrons in ultrathin crystals

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ARTICLE INFO

Article history:
Received 23 August 2014
Accepted 28 August 2014
Available online 2 September 2014
Communicated by V.M. Agranovich

Keywords:
Coherent radiation
Ultrathin crystals
High energy
Eikonal approximation
TSF-effect

ABSTRACT

A quantitative theory of the radiation process by ultrarelativistic electrons in ultrathin crystals is proposed. The theory is based upon the factorization theorem of the radiation cross-section and upon the description of the scattering process on the basis of the eikonal approximation of quantum electrodynamics. The conditions are obtained, under which the effect of radiation suppression in ultrathin crystals must take place. It is shown that these conditions may be fulfilled at the interaction of electrons with the energy accessible on CERN accelerator with ultrathin silicon crystals. Since the last years one can produce such crystals for the experiments in high energy physics. This opens new possibilities in study of interaction of high energy particles with matter.

1. Introduction

The bremsstrahlung process by ultrarelativistic electron in matter develops in a large space range along the motion direction of the particle which quickly increases with the particle energy. This space range is called the coherence length of radiation process [1–3]. At high energies this length may succeed the longitudinal size of the target, with which the electron interacts. In this case different coherent and interference effects in radiation may become substantial, which may result in both increase and decrease of the radiation efficiency. In the case at which we can treat the electron radiation at collisions with different medium atoms as independent, the spectral density of radiation is proportional to the number of electron collisions with the medium atoms, and, consequently, it increases linearly with the target thickness L. Such situation takes place at passing of fast electrons through thin layer of amorphous medium (see Fig. 1a), in the case that the mean square value of the multiple scattering angle of the particle by target, $\overline{\vartheta_e^2}$, is small compared to the square of characteristic radiation angle by relativistic electron, $\vartheta_\gamma \sim m/\varepsilon$, where m and ε are its mass and energy. Then the spectral density of radiation is defined by the Bethe and Heitler formula and increases linearly with L [2]. At increase of the target thickness, nevertheless, the condition $\overline{\vartheta_e^2} < m^2/\varepsilon^2$ is broken. In this case, as is shown in [4–6], the linear dependence on L of the spectral density of radiation changes into a more weak logarithmic dependence. In other words, in this case an effect of suppression of the spectral density of radiation takes place, as compared to the corresponding result by Bethe and Heitler. The uncommonness of this effect consists in the fact that with the increase of target thickness the number of electron collisions with medium atoms increases, but, in the considered case, ($l_{\mathrm{coh}} \gg L$ and $\sqrt{\overline{\vartheta_e^2}(L)} \gg m/\varepsilon$), the radiation practically does not increase. This effect has the name of TSF-effect [4–8]. It was discovered experimentally and studied on SLAC and CERN accelerators [7–9].

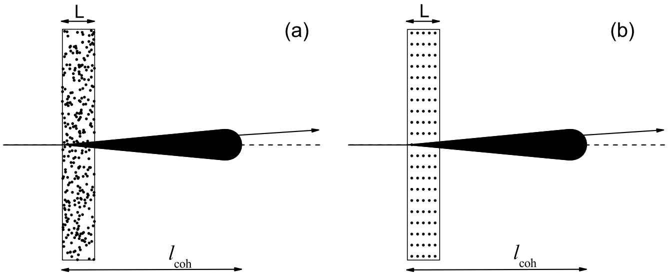

Fig. 1. Radiation by ultrarelativistic electron at its interaction with thin layer of amorphous (a) and crystalline matter (b).

An analogous effect is possible at radiation by high energy electrons in thin crystal (see Fig. 1b) [2,10]. In this case, nevertheless, not the bremsstrahlung will be suppressed, but the coherent radiation by ultrarelativistic electrons. Than the conditions of appearance of such effect in crystal, as compared to those in amorphous medium, may be fulfilled at lower electron energies and in larger

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http://dx.doi.org/10.1016/j.physleta.2014.08.034

frequency range of radiated photons. This is connected with the fact that crystal atoms form a regular structure, and at passing of electrons through it the correlations between consequent collisions of electron with the lattice atoms are substantial. Thanks to these correlations, a scattering of the particle in crystal occurs in a more intensive way than in amorphous medium. At this, the mean value of the scattering angles of the particles in crystal, $\overline{\vartheta_e}(L)$, becomes comparable to the characteristic radiation angle by relativistic electron $\vartheta_\gamma \sim m/\varepsilon$ at much smaller target thicknesses than in amorphous medium (in crystal $\overline{\vartheta_e}(L) \sim L$, whereas in amorphous medium $\overline{\vartheta_e^2}(L) \sim L$). An attention to the possibility of such effect was paid yet in works [11,12] at study of conditions of applicability of the theory of coherent radiation by relativistic electrons in crystals [13–15].

The results obtained in [10–12], however, were only qualitative, so far as they were related to the interaction of electron with a single isolated atomic string. An experimental study of this effect was previously impossible in view of absence of ultrathin crystals indispensable for observing this effect.

In the last years new possibilities opened in this domain, connected with the development of the technology of creation of ultrathin silicon crystals with thicknesses beginning from 100 nm which can be used in the experiments on high energy accelerators, and, in particular, at the energies accessible on CERN accelerator [16]. In this connection a necessity appeared to develop a quantitative theory of the effect of coherent radiation suppression by high energy electrons in ultrathin crystal and to execute a detailed analysis of conditions for appearance of this effect. The present paper is dedicated to the solution of these problems.

The work is based on the use of the theorem of factorization of radiation cross-section into a product of the radiation probability and the elastic scattering cross-section of a particle in external field and upon the description of the scattering process on the basis of the eikonal approximation of quantum electrodynamics. This lets do an analysis of the radiation process with taking into account the recoil effect at radiation. An important property of the developed approach is the fact that it lets describe quantitatively the effects of suppression of both coherent radiation in crystal and bremsstrahlung in amorphous media on the basis of the same method. At this, in the considered approach a concept of continued potential reveals itself automatically. Obtained results let estimate the strength of the effect at different crystal thicknesses and formalize optimal conditions for discovery of the effect that is important for the setting of corresponding experiments.

2. Differential cross-section of radiation in eikonal approximation

In ultrathin crystal the conditions may be satisfied, at which the radiation formation length $l_{\text{coh}} = 2\varepsilon\varepsilon'/m^2\omega$ is larger than the target thickness L (see Fig. 1), where ε and $\varepsilon' = \varepsilon - \omega$ are initial and final electron energies and ω – the energy of radiated photon (we use the system of units in which the light speed and the Planck constant are equal to one). In this case the condition of applicability of the factorization theorem [2,17] is satisfied, according to which the cross-section of radiation is equal to the probability of radiation $dW(q_\perp)/d\omega$ multiplied by the elastic scattering cross-section $d\sigma_e(q_\perp)$:

$$d\sigma(q_\perp) = dW(q_\perp) \cdot d\sigma_e(q_\perp), \tag{1}$$

where $q_\perp$ is the transversal component of momentum transmitted to external field,

$$\frac{dW(q_\perp)}{d\omega} = \frac{2e^2}{\pi\omega}\frac{\varepsilon'}{\varepsilon}\left[\frac{2\xi^2(1+\omega^2/2\varepsilon\varepsilon')+1}{\xi\sqrt{\xi^2+1}}\ln\left(\xi+\sqrt{\xi^2+1}\right)-1\right]$$

and $\xi = q_\perp/2m$.

In a thin target at high energies the electron motion is close to be rectilinear. In this case the differential cross-section of scattering can be considered in the eikonal approximation of quantum electrodynamics. In this approximation the scattering cross-section has the following form [2]:

$$\frac{d^2\sigma_e(q_\perp)}{d^2q_\perp} = \left|a(q_\perp)\right|^2, \tag{2}$$

where $a(q_\perp)$ is the scattering amplitude,

$$a(q_\perp) = -\frac{i}{2\pi}\int d^2\rho\, e^{i\vec{q}_\perp\vec{\rho}}\left(e^{i\chi_0(\vec{\rho})}-1\right),$$

and

$$\chi_0(\vec{\rho}) = -\frac{1}{v}\int_{-\infty}^{\infty} dz\, U(\vec{\rho}, z).$$

Here z and $\vec{\rho} = (x, y)$ are the coordinates which are consequently parallel and orthogonal to the incident electron momentum, v is the particle speed and $U(\vec{\rho}, z)$ is the potential energy of interaction of the particle with an external field. In the considered problem

$$U(\vec{r}) = \sum_{n=1}^{N_C} u(\vec{r} - \vec{r}_n), \tag{3}$$

where $u(\vec{r} - \vec{r}_n)$ is the potential energy of interaction of a particle with a single medium atom situated in the point $\vec{r}_n$ and N_c is the number of crystal atoms.

At $q_\perp \neq 0$ the differential cross-section of elastic scattering (2) with taking into account of (3) may be expressed in the form

$$4\pi^2\frac{d^2\sigma_e}{d^2q_\perp} = \int d^2\rho\, d^2\rho'\, e^{i\vec{q}_\perp(\vec{\rho}-\vec{\rho}')}e^{i[\chi_c(\vec{\rho})-\chi_c(\vec{\rho}')]}, \tag{4}$$

where

$$\chi_c(\vec{\rho}) = \sum_{n=1}^{N_C}\chi(\vec{\rho} - \vec{\rho}_n),$$

$$\chi(\vec{\rho} - \vec{\rho}_n) = -\frac{1}{v}\int_{-\infty}^{\infty} dz\, u(\vec{\rho} - \vec{\rho}_n, z).$$

The positions of atoms in crystal $\vec{r}_n = \vec{r}_n^{\,0} + \vec{u}_n$ have some spread $\vec{u}_n$ relatively their equilibrium positions $\vec{r}_n^{\,0}$, which is connected with the heat oscillations of atoms in the lattice, therefore, the cross-section (4) must be averaged over the values $\vec{u}_n$. In supposing for simplicity that the spread of positions of the atoms is the same for each lattice atom we come to the following expression for the average value of the elastic scattering cross-section (4):

$$4\pi^2\left\langle\frac{d^2\sigma_e}{d^2q_\perp}\right\rangle = \int d^2\rho\, d^2\rho'\, e^{i\vec{q}_\perp(\vec{\rho}-\vec{\rho}')}F_{N_c}(\vec{\rho}, \vec{\rho}'), \tag{5}$$

where

$$F_{N_c}(\vec{\rho}, \vec{\rho}') = \prod_{n=1}^{N_c}\int d^2\rho_n\, f(\vec{\rho}_n)e^{i[\chi(\vec{\rho}-\vec{\rho}_n)-\chi(\vec{\rho}'-\vec{\rho}_n)]}$$

and $f(\vec{\rho}_n)$ is the distribution function of positions of nth atom in the lattice by coordinates $\vec{\rho}_n$.

The crystal has a periodic structure. At passing of a particle through ultrathin crystal with cubic elementary diamond-like cell along one of its main crystal axes its collision with the crystal may be considered as a collision with a set of crystal atomic strings