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Adaptive approach to global synchronization of directed networks with fast switching topologies

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ABSTRACT

Global synchronization of directed networks with switching topologies is investigated. It is found that if there exists at least one directed spanning tree in the network with the fixed time-average topology and the time-average topology is achieved sufficiently fast, the network will reach global synchronization for appreciate coupling strength. Furthermore, this appreciate coupling strength may be obtained by local adaptive approach. A sufficient condition about the global synchronization is given. Numerical simulations verify the effectiveness of the adaptive strategy.

1. Introduction

Recently, synchronization of complex dynamical networks has been a hot research topic [1–12]. By means of the method of the master stability function the local synchronization of complex networks is investigated when the initial states of nodes are near to the synchronization manifold [1–7]. However, when the initial states of nodes are randomly distributed, the global synchronization of complex networks is proposed [8–12]. If there is a spanning directed tree in the network, Chua found that a directed network may achieve global synchronization for sufficiently large coupling strength between nodes [8].

The network topologies of the above researches are fixed. However, the network topologies of many real world networks are time varying [13–20]. Recently, the local synchronization of complex networks with time-varying coupling strengths is proposed in [15]. Although there exists an instantaneously disconnected topology, Skufca et al. found that a time-varying network could propagate sufficient information to make the complex networks achieve local synchronization [17]. Based on the Skufca's results, Stiwell et al. found that if the network with the fixed time-average of the topology may achieve local synchronization and the time-average is achieved sufficiently fast, the network with switching topologies achieves local synchronization [8]. Furthermore, at the same conditions, if there is a spanning directed tree in the network, we found that the directed network with fast switching topologies may reach global synchronization for sufficiently large coupling strength between nodes [13]. In these researches of network with fast switching topologies, there is a common condition: the network may reach synchronization for sufficiently large coupling strength between nodes. However, how to obtain these coupling strengths is difficult, especially when the network has big size and switching topologies are complex.

Recently, adaptive method is proposed to apply in the synchronization of complex networks [19–23]. At first, global adaptive method, which uses the global dynamical information of the network, was applied to the synchronization of complex networks [8,9]. However, when the nodes of the complex network increase rapidly, it is difficult to obtain this global information. Therefore, these adaptive techniques are unrealistic to be realized. Then the local adaptive strategies, based only on local information, are proposed to make the network with fixed network topology achieve synchronization in [13,14]. Naturally, there is a problem to ask: does the network with fast switching topologies achieve global synchronization with local adaptive method?

In this Letter, a local adaptive method, which only uses the corresponding local information, is used to investigate global synchronization of directed networks with fast switching topologies. If there exists a directed spanning tree in the fixed time-average of network topology and the time-average is achieved sufficiently

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fast, it is found that the network with switching topologies may achieve global synchronization for appreciate coupling strength between nodes.

The rest of the Letter is organized as follows. In Section 2, a theorem is proposed to make the directed network achieve synchronization with fast switching topologies. Simulations are investigated in Section 3. In the last section, conclusions are presented.

2. Adaptive synchronization of directed networks with switching topologies

Assume a network made up of N linearly coupled identical oscillators, with each oscillator being an n-dimensional dynamical system. The state equations of the network are

$$\dot{x}(t) = \big(f\big(x_1(t), t\big), \ldots, f\big(x_N(t), t\big)\big)^T + \big(A(t) \otimes D(t)\big)x(t) \tag{1}$$

where $x(t) = (x_1(t), \ldots, x_N(t))^T$, $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in R^n$ is the state variable of node i. f describes the dynamics of each isolated node, κ is coupling strength between nodes, $D(t) = (d_{ij}(t))_{n\times n}$ is an inner coupling matrix. $A(t) = (a_{ij}(t))_{N\times N}$ is the coupling matrix which is defined as following: If there is a connection between node i and node j, then $a_{ij}(t) > 0$ $(i \neq j)$; otherwise, $a_{ij}(t) = 0$ $(i \neq j)$. Moreover, $a_{ii}(t) = -\sum_{j=1,\, j\neq i}^{N} a_{ij}(t)$ $(i = 1, 2, \ldots, N)$. The coupling strength $a_{ij}(t) = (a_{ij}^1(t), \ldots, a_{ij}^n(t))^T$ between nodes changes according to the edge-strategy [22]

$$\dot{a}_{ij}(t) = \beta\big\|x_j(t) - x_i(t)\big\| \tag{2}$$

where β is the adaptive gain. If the coupling strength of one edge is fixed, which means that the adaptive gain is $\beta = 0$. Obviously, when the network has fixed network topology, the adaptive gain is $\beta = 0$ for all edges of network. It is known that if there exists at least one spanning directed tree in the network, a sufficient condition of a directed network achieving global synchronization for sufficiently large coupling strength between nodes is given in [8].

Local synchronization and global synchronization of networks with fast switching topologies were proposed in [18] and [13], respectively. If the complex network with fixed time-average topology may achieve synchronization and the time-average topology is achieved sufficiently fast, the network with switching topologies will reach synchronization for sufficiently large coupling strength [18,13]. However, it is difficult to acquire the exact value of the sufficiently large coupling strength. According to the local adaptive strategy in (2), a sufficient condition of global synchronization in directed network with switching topologies is given.

Theorem. *Assume there exists a constant T for which the matrix-valued function $A(t)$, there exists*

$$\bar{A} = \frac{1}{T}\int_t^{t+T} A(\tau)\,d\tau \tag{3}$$

for all t and the system

$$\dot{x}(t) = \big(f\big(x_1(t), t\big), \ldots, f\big(x_N(t), t\big)\big)^T + \big(\bar{A} \otimes D(t)\big)x(t) \tag{4}$$

where the coupling strength between nodes changes according to the local adaptive approach (2) and the following conditions satisfy

(i) *$\bar{A}$ is a zero row sums matrix with nonnegative off-diagonal elements.*
(ii) *$f(x(t), t) + D(t)x(t)$ is V-uniformly decreasing for some symmetric positive definite V, that is*

$$(x-y)^T V\big(f(x,t) + D(t)x - f(y,t) - D(t)y\big) \leqslant -\mu\|x-y\|^2 \tag{5}$$

for some $\mu > 0$ and all $x, y \in R^n$ and all t.

(iii) *$VD(t) \leqslant 0$ and is symmetric for all t.*
(iv) *The underlying weighted directed graph exists at least one spanning directed tree.*

The system (3) may achieve global synchronization. Furthermore, assume that the oscillator has bounded slope such that

$$\big|\big(f(v,t) - f\big(v',t\big)\big)/\big(v - v'\big)\big| \leqslant l \tag{6}$$

where $l > 0$ for all $v, v' \in R^n$ and $v \neq v'$.

Then there exists $\varepsilon^* > 0$ such that for all fixed $\varepsilon \in (0, \varepsilon^*)$, the system

$$\dot{x}(t) = \big(f\big(x_1(t), t\big), \ldots, f\big(x_N(t), t\big)\big)^T + \big(A(t/\varepsilon) \otimes D(t)\big)x(t) \tag{7}$$

may achieve global synchronization for sufficiently large coupling strength between nodes.

Proof. Since the system (4) satisfies the conditions (i)–(iv) in the theorem, there exists a symmetric irreducible zero row sums matrix $U_{N\times N}$ with nonpositive off-diagonal elements. Construct a Lyapunov function

$$g(t) = \frac{1}{2}x(t)^T(U \otimes V)x(t) + \frac{1}{2\beta}\sum_{\varepsilon}\big(c_{ij} - a_{ij}(t)\big)^T\big(c_{ij} - a_{ij}(t)\big)$$

Then, the derivation of $g(t)$ on the time t is

$$\begin{aligned}
\dot{g}(t) &= x(t)^T(U \otimes V)\dot{x}(t) - \frac{1}{\beta}\sum_{\varepsilon}\big(c_{ij} - a_{ij}(t)\big)^T\dot{a}_{ij}(t)\\
&= x(t)^T(U \otimes V)\begin{pmatrix} f(x_1(t),t) + D(t)x_1(t)\\ \vdots \\ f(x_N(t),t) + D(t)x_N(t)\end{pmatrix}\\
&\quad + x(t)^T(U \otimes V)\big(\bar{C} \otimes D(t) - I \otimes D(t)\big)x(t)\\
&\quad - \frac{1}{\beta}\sum_{\varepsilon}(c_{ij} - a_{ij})^T\dot{a}_{ij}(t)\\
&\leqslant \sum_{i<j} -U_{ij}\big(x_i(t) - x_j(t)\big)^T V\big(f\big(x_i(t), t\big)\\
&\quad + D(t)x_i(t) - f\big(x_j(t), t\big) - D(t)x_j(t)\big)\\
&\quad - \sum_{\varepsilon}\big(c_{ij} - a_{ij}(t)\big)^T\|x_j - x_i\|
\end{aligned}$$

Furthermore, for any edge (i, j) in the network, there exists the value c_{ij} bigger than the corresponding edge strength a_{ij}, it is obtained that

$$\dot{g}(t) \leqslant \sum_{i<j} -U_{ij}\big(-\mu\big\|x_i(t) - x_j(t)\big\|^2\big)^T$$

Note that $-U_{ij} \geqslant 0$ for $i < j$. For each $-U_{ij} \geqslant 0$ and $\delta > 0$, and sufficiently large t such that if $\|x_i(t) - x_j(t)\| \geqslant \delta$, then $\dot{g} \leqslant -(\mu/2)\|x_i(t) - x_j(t)\|^2$. This implies that for large enough t, $\|x_i(t) - x_j(t)\| \leqslant \delta$. Therefore, $\lim_{t\to\infty}\|x_i(t) - x_j(t)\| = 0$.

For system (4), suppose $\bar{e}(t) = (\bar{e}_1(t), \ldots, \bar{e}_N(t))^T$, $\bar{e}_i(t) = x_i(t) - s(t)$, where $s(t)$ is the synchronization state $s(t) = x_i(t)$, $i = 1, \ldots, N$, which satisfies $\dot{s}(t) = f(s(t))$. Then construct the following Lyapunov function as follows

$$\begin{aligned}
g\big(\bar{e}(t)\big) &= \frac{1}{2}\bar{e}(t)^T(U \otimes V)\bar{e}(t) + g(t)\\
&= \frac{1}{2}x(t)^T(U \otimes V)x(t)\\
&\quad + \frac{1}{2\beta}\sum_{\varepsilon}\big(c_{ij} - a_{ij}(t)\big)^T\big(c_{ij} - a_{ij}(t)\big)
\end{aligned} \tag{8}$$