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Impact of quantum–classical correspondence on entanglement enhancement by single-mode squeezing

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ABSTRACT

Quantum entanglement between two field modes can be achieved through the collective squeezing of the two respective modes. If single-mode squeezing is performed prior to such a two-mode squeezing, an enhancement of entanglement production can happen. Interestingly, the occurrence of this enhancement can be implicitly linked to the local classical dynamical behavior via the paradigm of quantum–classical correspondence. In particular, the entanglement generated through quantum chaos is found to be hardly enhanced by prior squeezing, since it is bounded by the saturation value of the maximally entangled Schmidt state with fixed energy. These results illustrate that entanglement enhancement via initial squeezing can serve as a useful indicator of quantum chaotic behaviour.

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1. Introduction

In the last decades, a number of quantum information protocols which utilizes continuous-variable (CV) entanglement have been developed [2,6,44]. The performance of these protocols is often constrained by the achievable degree of the entanglement that is being produced. In particular, a stable control of entanglement generation is necessary before quantum cryptography with a finite number of samples can be secured against the most general coherent attacks [15]. It is noteworthy that while various schemes of generating controllable CV entanglement have been proposed, a major scheme of interest is that of two-mode squeezing. In fact, it has been shown via diverse quantum systems that the generation of entanglement can be enhanced by performing single-mode squeezing prior to two-mode squeezing. In the Jaynes–Cummings model for example, it has been demonstrated that a stronger entanglement between a two-level atom and an electromagnetic field mode can be achieved by using a squeezed state rather than a coherent state as the initial photon state [16]. Note that in this case the enhancement is observed only when the initial state of the field mode is sufficiently squeezed. Similar threshold has also been observed in systems of coupled harmonic oscillators [13]. Beyond the threshold, the maximum attainable entanglement is found to grow steadily with an increase in the initial squeezing parameter. In another interesting investigation, the enhancement in entanglement via unequal single-mode squeezing performed separately on the two field modes was studied [40]. Notably, entanglement was found to persist even in a decohering environment with high temperature when the normal modes are squeezed [18].

Experimental schemes for generating CV entanglement was first proposed and realized in non-degenerate parametric amplifiers [38,47]. Later, non-degenerate three-level cascade laser was suggested as an alternative optical system for the experimental generation of entangled quantum state. For this setup, an enhancement of intra-cavity quadrature squeezing was observed by coupling the cavity mode to a squeezed vacuum reservoir. The effect of the squeezed vacuum was studied and the result is a large enhancement of the intra-cavity squeezing and entanglement in the two-mode light [1]. Further investigations on this topic were focused on the search for effective ways to increase the initial single-mode squeezing with easily implementable schemes that can generate a high degree of squeezing and entanglement. On the other hand, our interest is to examine into new schemes which could exploit the effectiveness of initial single-mode squeezing on entanglement enhancement beyond the control of the amplitude and orientation of prior squeezing or the existence of a critical squeezing parameter. A particular novel idea is to employ the fundamental physics of quantum-to-classical correspondence to guide the process of entanglement enhancement with initial squeezing through the perspective of classical dynamics. The potential effectiveness of this new approach would be surprising and counter-intuitive since both squeezing and entanglement are purely quantum phenomena.

Indeed, the correspondence between the physics of quantum systems and its classical counterparts has been well-established for

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decades [5,17,33,42]. Notable examples include the manifestation of chaos in the energy-level distribution [42] of atomic systems, as well as the wave patterns that are exhibited in quantum chaotic systems which are known as 'scars' [5]. In recent years, there has been an increasing interest in correlating the entanglement production of a quantum system with the corresponding dynamical behaviour in the classical domain. For example, the dynamical production of entanglement was studied on the N-atoms Jaynes–Cummings model [17] for initial coherent states whose centers lie in different regions of the corresponding classical phase space. The entanglement production was found to be a good indicator of the regular-to-chaotic transition that happens in the classical domain. Similar studies were undertaken for kicked tops [3,4,8,14,31,34], the 4D standard maps [28], nonlinear oscillators [10,46], the Dicke model [41], Rydberg molecule [30], triatomic molecules [24,45], integrable dimers [23] and interacting spins [35]. In this paper, we shall show the effectiveness of entanglement enhancement through initial squeezing, and more importantly, demonstrate its dependence on the local dynamical behavior of the corresponding classical phase space. Interestingly, for initial coherent states whose centers lie in the regular regimes of the classical phase space, the maximum attainable entanglement can be enhanced significantly by performing prior single-mode squeezing. In addition, the amount of entanglement enhancement is found to increase monotonically with the degree of prior squeezing. Conversely, for initial coherent states whose centers lie in chaotic regions of the classical phase space, prior single-mode squeezing is observed to have negligible effects in enhancing the quantum entanglement.

2. Model

In this study, we consider bipartite system composed of two coupled anharmonic oscillators [9,11,37]. Specifically, we focus on the following Hamiltonian:

$$H = \frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{1}{2}q_1^2 + \frac{1}{2}q_2^2 + \lambda q_1^2 q_2^2. \tag{1}$$

In the equation, p_1 and p_2 denote the kinetic momenta, while q_1 and q_2 denote the oscillators' positions, with λ being the coupling parameter. The classical dynamics of this model has been shown to range from regular, to mixed regular, and chaotic [37]. Upon quantization, the corresponding dynamical production of entanglement with initial separable coherent states was found to relate closely to the classical trajectories [46]. Specifically, the maximum value of the entanglement production is found to correspond systematically to the classical invariant tori and is the largest when the initial state lies at the edge of the regular islands or in the chaotic sea.

The initial state is chosen to be a tensor product of the coherent state, $|\psi(0)\rangle = |\alpha_1\rangle \otimes |\alpha_2\rangle$, whose center lies precisely on a classical phase point (q_1, p_1, q_2, p_2) with $\alpha_k = (q_k + ip_k)/\sqrt{2}$, where $k = 1, 2$. In other words, the classical phase point (q_1, p_1, q_2, p_2) gives the center of the initial coherent state. Prior to the dynamical generation of entanglement through the Hamiltonian given by Eq. (1), single-mode squeezing is performed individually on each subsystem initial state by the following squeezing operator:

$$\hat{S}(\zeta_k) = e^{\frac{1}{2}\zeta_k{}^* \hat{a}_k^{\dagger 2} - \frac{1}{2}\zeta_k \hat{a}_k^2}. \tag{2}$$

The result is a product state of single-mode squeezed coherent state: $|\psi(0)\rangle = |\alpha_1, \zeta_1\rangle \otimes |\alpha_2, \zeta_2\rangle$ where

$$|\alpha_k, \zeta_k\rangle = \hat{S}(\zeta_k)|\alpha_k\rangle, \tag{3}$$

with $k = 1, 2$. Note that $\hat{a}_k = (\hat{q}_k + i\hat{p}_k)/\sqrt{2}$, with $\hat{q}_k$ and $\hat{p}_k$ being the position and momentum operators respectively. Also, $\zeta_k = |\zeta_k| \exp(i2\theta_k)$ denotes the squeezing parameter for mode k which quantifies the degree of single-mode squeezing.

The time evolution of the quantum state $|\psi(t)\rangle$ is given by

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle, \tag{4}$$

where the time evolution operator $\hat{U}(t)$ is given by $\hat{U}(t) = e^{-i\hat{H}t/\hbar}$, with $\hat{H}$ being the quantized Hamiltonian:

$$\hat{H} = \left(\hat{a}_1^\dagger \hat{a}_1 + \frac{1}{2}\right) + \left(\hat{a}_2^\dagger \hat{a}_2 + \frac{1}{2}\right) + \frac{\lambda}{4}\left(\hat{a}_1^\dagger + \hat{a}_1\right)^2 \left(\hat{a}_2^\dagger + \hat{a}_2\right)^2 \tag{5}$$

of Eq. (1). The time evolved density matrix is then determined as follow:

$$\rho(t) = \hat{U}(t)\rho(0)\hat{U}(t)^\dagger, \tag{6}$$

where $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$. By taking the partial trace of $\rho(t)$ over the l-th subsystem, the reduced density matrix $\rho_l(t)$ is obtained. The von Neumann entropy of entanglement $S_{vn}(t)$ is then evaluated via

$$S_{vn}(t) = -\mathrm{Tr}\left[\rho_l(t) \log_2 \rho_l(t)\right] = -\sum_{i=1}^{N} \lambda_i \log_2 \lambda_i, \tag{7}$$

where $l = 1$ or 2 and N is the basis size employed in the numerical simulation.

3. Effect of squeezing on entanglement enhancement

To probe the dependence on local classical dynamics, we consider specifically the effect of prior single-mode squeezing on entanglement enhancement for the situation when the classical phase space exhibits both a mixture of regular and chaotic behaviour. For this, the energy $E = 150.75$ and the coupling constant $\lambda = 0.0075$ is selected. The Poincaré surface is shown in Fig. 1 where we observe groups of regular islands within a sea of chaos. Specifically, the dynamics displayed by the trajectories of this system is very different depending on the initial condition. While a regular orbit is restricted within a small region of the regular tori, the temporal position of a chaotic trajectory spreads out unpredictably within the chaotic sea. In Fig. 2, we show entanglement dynamics of four initial coherent states with centers lie at different positions of this classical mixed phase space. For each coherent state, a single-mode squeezing is performed on both the oscillator field modes (with $\zeta_1 = \zeta_2 = \zeta$) before the dynamical generation of quantum entanglement. Interestingly, the enhancement of entanglement production is not uniform for the four chosen initial coherent states although each of them is subjected to the same amplitude of single-mode squeezing prior to two-mode squeezing which generates entanglement. In particular, entanglement production is found to be higher for initial states with centers lie in the chaotic region compared to the regular region. When equal amount of prior single-mode squeezing are performed on the initial states, enhancement of entanglement is found to be larger for quantization of the regular orbit versus that of the chaotic orbit (see Fig. 2). In addition, we found that entanglement can be effectively enhanced when a higher degree of single-mode squeezing is injected prior to the dynamical generation of entanglement as shown in Fig. 3. However, prior squeezing has minimal enhancement effect on entanglement generation when the center of the initial state lies in the chaotic regime of the classical counterpart. Note that similar results are obtained for numerical computation performed for prior squeezing with different angles ($\theta = \pi/4, \pi/2$ and $3\pi/4$).

4. Dependence of entanglement enhancement on local classical dynamical behaviour

While the correspondence between the quantum system and its classical counterpart has been witnessed in various contexts,

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