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Soliton–antisoliton interaction in a parametrically driven easy-plane magnetic wire

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ABSTRACT

In the present work we study the soliton–antisoliton interaction in an anisotropic easy-plane magnetic wire forced by a transverse uniform and oscillatory magnetic field. This system is described in the continuous framework by the Landau–Lifshitz–Gilbert equation. We find numerically that the spatio-temporal magnetization field exhibits both annihilative and repulsive soliton–antisoliton interactions. We also describe this system with the aim of the associated Parametrically Driven and Damped Nonlinear Schrödinger amplitude equation and give an approximate analytical solution that roughly describes the repulsive interaction.

1. Introduction

Localized states are observed in different systems, e.g., chiral bubbles in liquid crystals, current filaments in gas discharges, spots in chemical reactions, localized states in fluid surface waves, oscillons in granular media, isolated states in thermal convection, and solitary waves [1,2]. The simplest dynamic localized structure that appears in a restricted spatial region and that connects asymptotically time-independent states in the rest of the space is called *soliton* [1]. In magnetism, the state of the art for conservative and for dissipative systems can be found in Refs. [3–5]. Here, we study ferromagnetic dissipative systems that can have spatially localized, stable, dynamic excitations [4]. Here we are interested in dissipative solitons. These perturbations were found experimentally in magnetic systems [6,7]. Recently, magnetic solitonic modes in nano-oscillators were observed [8–10], and dissipative magnetic droplet solitons were theoretically predicted in Ref. [11] and then experimentally found and studied [12]. Moreover, dark solitons and their interactions were also studied [13–16]. Lately, the effects of disturbances of the kick (tilt) type on two-dimensional dissipative solitons were also studied [17,18]. In addition to ordinary single soliton solutions, there exist other localized states [19–24]. In particular, there are complex time-dependent localized states, called *breathers* [25–29], which are solitons such that their amplitude and width oscillate non-monotonically. In this work we deal with ordinary solitons.

We call *antisoliton* the soliton with the opposite polarity of a given soliton. Parametrically excited soliton–antisoliton systems (or double-solitons of opposite polarity) were observed and numerically studied in oscillating water channels of finite length showing a repulsive behavior and synchronous oscillations [30]. We study the soliton–antisoliton precession states of an anisotropic easy-plane ferromagnetic wire subject to a combined, constant and oscillatory, applied magnetic field. In order to achieve this goal, we perform numerical simulations to characterize the interaction and to obtain the region of existence of these localized solutions in the space of parameters. Additionally, we take advantage of the associated amplitude equation to compare numerical results and to obtain an approximate analytical description to the interaction law of these systems.

2. Theoretical model

In order to study the dynamics of the macroscopic magnetization, $\mathbf{m}$, in the continuous framework, we use the standard approach given by the Landau–Lifshitz–Gilbert (LLG) equation [31]

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{H}_{eff} + \lambda \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}. \tag{1}$$

We consider the magnetization of a long ferromagnetic anisotropic wire of length L oriented along the $\hat{\mathbf{z}}$ axis such that the normalized magnetization field is given by $\mathbf{m} = \mathbf{m}(z,t)$, where z and

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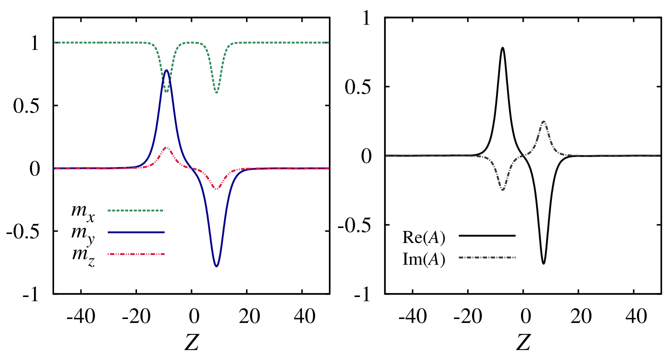

Fig. 1. Instantaneous profiles of magnetization $\mathbf{m}(z,t)$ (left) and complex amplitude $A(Z,t)$ (right) at $t = 77.488$ for the soliton–antisoliton system with parameters $\nu = -0.198$, $h_0 = 0.394$, $d(0) = 1.59$, $f = 1.00$, and $k_0 = 0$.

t stand for the space coordinate and time, respectively. The effective torque field is given by $\mathbf{H}_{eff} = \nabla^2\mathbf{m} - \beta(\mathbf{m}\cdot\hat{\mathbf{z}})\hat{\mathbf{z}} + \mathbf{h}_{ext}$, where the Laplacian term accounts for the coupling of the magnetization with the first neighbors, $\beta > 0$ (easy-plane) measures the anisotropy along the $\hat{\mathbf{z}}$ axis, and $\mathbf{h}_{ext}$ is the external magnetic field, which comprises both, a constant and an oscillatory part, $\mathbf{h}_{ext} = (h_c + h_0\cos(\Omega t))\hat{\mathbf{x}}$, where $\{h_c, h_0, \Omega\}$ are constants. Here λ denotes the dimensionless phenomenological Gilbert damping coefficient which is a material property. Throughout this manuscript and as given in Ref. [29], we use dimensionless quantities having scaled the magnetization and magnetic fields by the saturation magnetization M_s; the time t by $T_s = 1/\gamma_0 M_s$, where $\gamma_0 = 2.2\times10^5$ $\mathrm{A^{-1}ms^{-1}}$ is the electron gyromagnetic ratio [31]; and the space coordinates $\mathbf{r}$ by the exchange length $\ell_{ex} = \sqrt{2A/\mu_0 M_s^2}$, where A is the exchange stiffness constant. In the long wire approximation, the dimensionless anisotropy parameter becomes $\beta = -(1/2 + 2K_u/\mu_0 M_s^2)$, where K_u is the uni-axial anisotropy constant and the $1/2$ term corresponds to the dipole field contribution [32]. Taking, e.g., material values for $CsNiF_3$ [33–35]: $M_s = 2.2\times10^5$ A/m, $K_u = -1.2\times10^6$ J/m^3, $A = 0.8$ pJ/m, we have $\ell_{ex} = 5$ nm, $T_s = 20$ ps, and $\beta = 39$.

A simple homogeneous state of model (1) is $\mathbf{m} = \hat{\mathbf{x}}$, which represents a uniform magnetization parallel to the magnetic forcing. Small perturbations of this homogeneous state are characterized by damped dispersive waves, with frequencies close to the natural frequency $\Omega_0 = \sqrt{h_c(h_c+\beta)}$. When the wire is forced at about twice this frequency, $\Omega \equiv 2(\Omega_0 + \nu)$, ν being the detuning parameter, this uniform state becomes unstable by means of an oscillatory instability. This bifurcation gives rise to a uniform attractive periodic solution, which corresponds to a parametric resonance [23]. More precisely, the bifurcation occurs at $h_{0,c}^2 = (4\Omega_0)^2[\nu^2 + (\lambda q/2)^2]/\beta^2$ with $q = \beta + 2h_c$; this relationship defines the first Arnold tongue. Close to this parametric resonance the simplest description to our magnetic system can be given by an amplitude equation of the envelope of the z-component of $\mathbf{m}$: $m_z \propto \mathrm{Re}(A\exp(i(\Omega_0+\nu)t)) + \ldots$, where the complex amplitude $A(Z,t)$ satisfies

$$\partial_t A = -(i\nu + \mu)A - iA|A|^2 + \gamma\bar{A} - i\partial_Z^2 A, \qquad (2)$$

where $\mu = \lambda q/2$, $\gamma = \beta h_0/4\Omega_0$ and $Z = \sqrt{2\Omega_0/q}z$. The last equation is known as the *parametrically driven and damped nonlinear Schrödinger equation* (PDDNLS) [4,5]. In general, amplitude equations give a qualitatively correct description, although often quantitative agreement is not obtained [5]. Below the first Arnold tongue, and for negative detuning values, Eqs. (1) and (2) admit single soliton and antisoliton solutions [4,5] which consist of single up and down bump localized structures, respectively. We study numerically the interaction between soliton–antisoliton

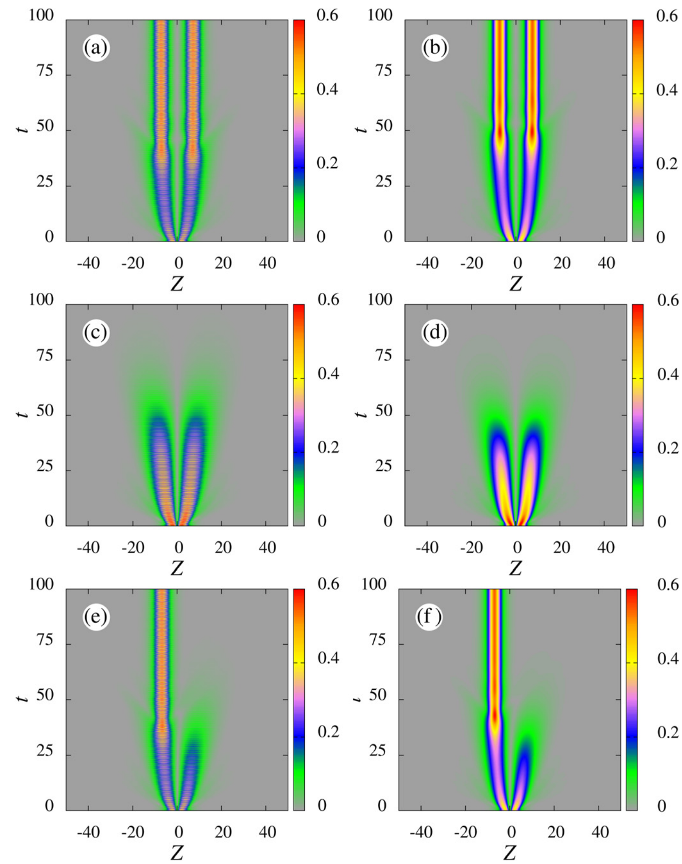

Fig. 2. (Color online.) Spatio-temporal maps of the transverse magnetization, $(m_y^2 + m_z^2)^{1/2}$ (frames (a), (c) and (e)), and complex amplitude modulus, $|A|$ (frames (b), (d) and (f)), for the soliton–antisoliton systems with parameters $\nu = -0.198$ and $h_0 = 0.394$. (a) and (b): Repulsive interaction for $d(0) = 1.59$, $f = 1.00$, and $k_0 = 0$. (c) and (d): Complete annihilation for $d(0) = 1.58$, $f = 1.00$, and $k_0 = 0$. (e) and (f): Partial annihilation for $d(0) = 1.59$, $f = 0.99$, and $k_0 = 0$.

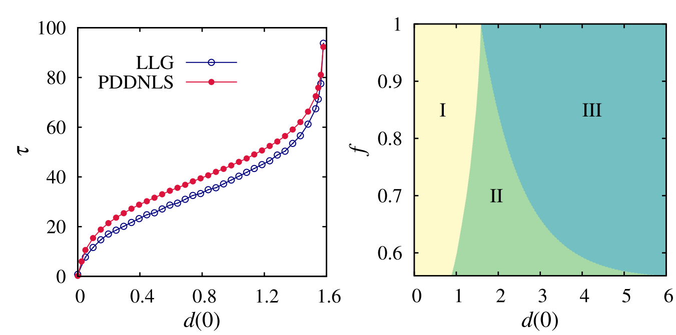

Fig. 3. Characteristic time for annihilation with $f = 1$ (left), and regions of complete annihilation (I), partial annihilation (II), and soliton–antisoliton repulsive interaction (III) (right) for the soliton–antisoliton system with parameters $\nu = -0.198$, $h_0 = 0.394$, and $k_0 = 0$. These regions are the same for LLG and PDDNLS models.

pairs. However we also use Eq. (2) to construct an approximate model in order to give, when possible, a quantitative description of the interaction. Let us describe in some detail this model: Writing the amplitude in the polar form, $A = R\exp(i\theta)$, Eq. (2) becomes the system

$$\frac{\partial_t R}{R} = -\mu + 2\partial_Z\theta\frac{\partial_Z R}{R} + \partial_Z^2\theta + \gamma\cos(2\theta), \qquad (3a)$$

$$\partial_t\theta = -\nu - R^2 - \frac{\partial_Z^2 R}{R} + (\partial_Z\theta)^2 - \gamma\sin(2\theta). \qquad (3b)$$