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Large amplitude electron plasma oscillations

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ABSTRACT

We consider a cold plasma in order to find new large-amplitude wave solutions in the long-wavelength limit. Accordingly we derive two generic coupled equations which describe the energy exchange between the electrostatic and electromagnetic waves. A new kind of quasi-periodic behavior is found. Our derivations may be considered as a prerequisite to extended studies of stimulated Raman scattering for cases where the wave amplitudes are so large that standard perturbation techniques are not applicable.

1. Introduction

In order to understand plasma dynamics it may be advisable to start with an analysis of the most simple plasma, namely a cold unmagnetized electron plasma with a fixed ion background. Linearizing the equations one then easily finds the well-known longitudinal mode with frequency $\omega = \omega_{p0}$ where $\omega_{p0} = (n_0 e^2/\varepsilon_0 m)^{1/2}$, n_0 is the equilibrium number density, e is the elementary charge, ε_0 is the vacuum permittivity and m is the electron mass. Moreover, the transverse mode has the frequency $\omega = (\omega_{p0}^2 + k^2 c)^{1/2}$, where k is the wavenumber and c is the speed of light in vacuum. Adopting then the results of nonlinear theory, it is well-known that these modes interact with each other, leading to, for example, stimulated Raman Scattering (e.g. Refs. [1–6]).

If the amplitudes are increased too much, one cannot use standard nonlinear theory. Fortunately, however, it is possible to solve a few interaction processes *exactly* (e.g. [7]). Thus, for longitudinal oscillations one can find the solution

$$n(t) = \frac{n_0(1+\Delta)}{1+\Delta-\Delta\cos(\omega_{p0}t)} \tag{1}$$

where Δ is a parameter describing the initial electron density perturbation, or $\Delta = [n(0) - n_0]/n_0$. This solution was then extended to satisfy the boundary conditions of a plasma slab. However, in the presence of both longitudinal and transverse disturbances, the solution turns out to be much more complex than that of Ref. [7]. It is the purpose of the present paper to shed some more light on this situation.

2. Basic equations

Let us start from the basic equations for a cold nonrelativistic electron plasma with one-dimensional spatial variations (the z-direction). Thus we have

$$\frac{\partial n}{\partial t} + \frac{\partial (n v_z)}{\partial z} = 0 \tag{2}$$

$$\frac{\partial \mathbf{v}}{\partial t} + v_z \frac{\partial \mathbf{v}}{\partial z} = -\frac{e}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \tag{3}$$

$$\hat{\mathbf{z}} \times \frac{\partial \mathbf{E}}{\partial z} = -\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

and

$$\hat{\mathbf{z}} \times \frac{\partial \mathbf{B}}{\partial z} = -e\mu_0 n\mathbf{v} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{5}$$

In order to find *exact* solutions we then make the ansatz $n = n(t)$, $\mathbf{B} = B_y(t)\hat{\mathbf{y}}$, $\mathbf{v} = (u_x(t)\hat{\mathbf{x}} + u_z(t)\hat{\mathbf{z}})z$ and $\mathbf{E} = (\epsilon_x(t)\hat{\mathbf{x}} + \epsilon_z(t)\hat{\mathbf{z}})z$. Substituting this ansatz into Eqs. (2)–(5) we obtain

$$\frac{\partial n}{\partial t} + n u_z = 0 \tag{6}$$

$$\frac{\partial u_z}{\partial t} + u_z^2 = -\frac{e}{m}(\epsilon_z + u_x B_y) \tag{7}$$

$$\frac{\partial u_x}{\partial t} + u_z u_x = -\frac{e}{m}(\epsilon_x - u_z B_y) \tag{8}$$

$$\epsilon_x = -\frac{\partial B_y}{\partial t} \tag{9}$$

$$0 = -\mu_0 e n u_x + \frac{1}{c^2}\frac{\partial \epsilon_x}{\partial t} \tag{10}$$

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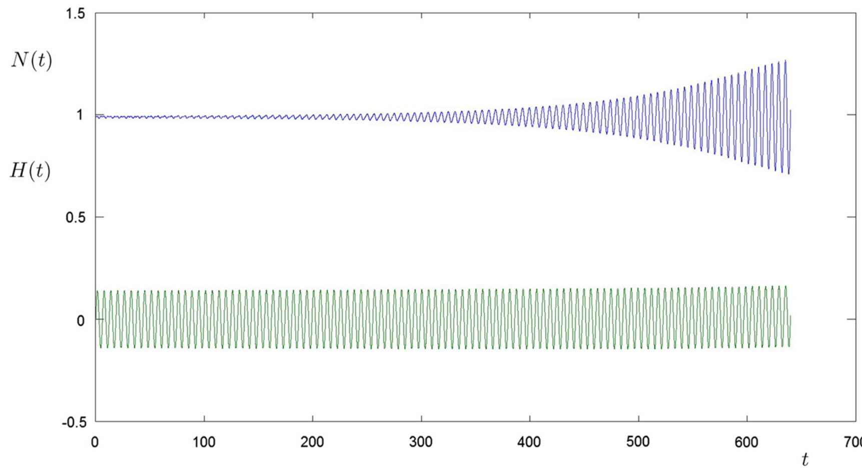

Fig. 1. The evolution of N (upper curve) and H (lower curve) for $a = 0$ and initial amplitudes $H_0 = 0.2$ and $N_0 = 0.98$.

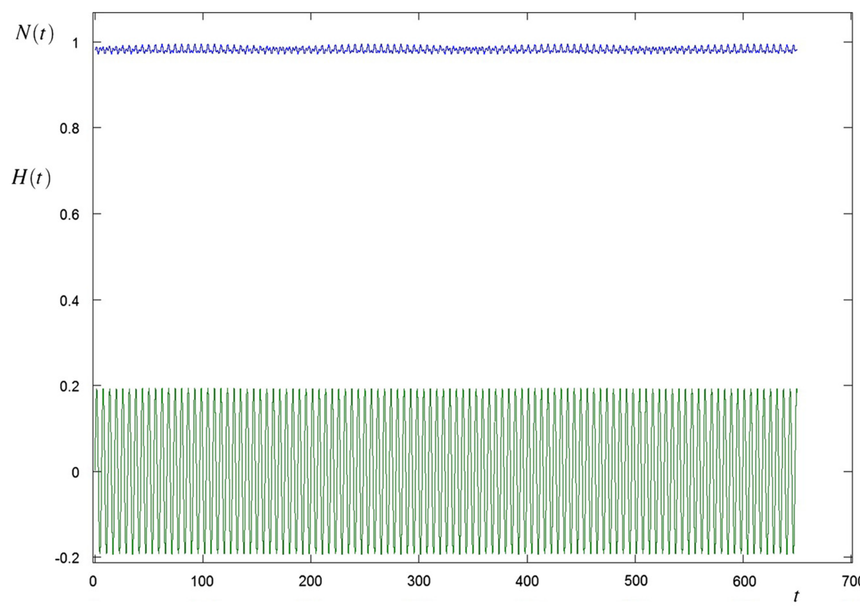

Fig. 2. The evolution of N (upper curve) and H (lower curve) for $a = 0.25$ and initial amplitudes $H_0 = 0.2$ and $N_0 = 0.98$.

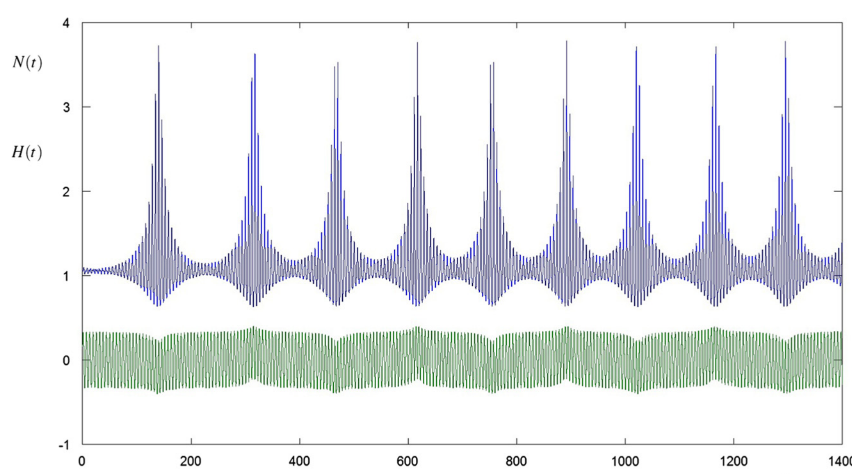

Fig. 3. The evolution of N (upper curve) and H (lower curve) for $a = 0$ and initial amplitudes $H_0 = 0.33$ and $N_0 = 1.05$.