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Stability of the charged radiating cylinder

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ABSTRACT

We discuss the dynamical instability of cylindrically symmetric isotropic geometry under the effect of electromagnetic field. The interior geometry of the dynamical collapse is matched with an exterior geometry through Darmois junction conditions. The perturbation scheme is used to describe the collapse equation and categorize the Newtonian and post-Newtonian regions in radiating as well as non-radiating eras. It is concluded that energy density, pressure, radiation density and electromagnetic field control the stability of the cylinder leading to more unstable configuration.

1. Introduction

General relativity and relativistic astrophysics have provided useful insights while discussing the instability of the massive stars in recent decades. Strong gravitational effects play vital role in the dynamics of compact objects. The problem of dynamical instability is closely associated with formation and evolution of self-gravitating objects. If a stationary black hole is stable under perturbations, such a solution describes a possible final state of dynamical evolution of a gravitating system.

During the collapse, a large amount of energy radiates which gradually increases during the evolution of self-gravitating objects. This radiated energy can be described by two approximations: diffusion and free streaming approximations. The physical parameters like dissipation can affect the stability of self-gravitating stars. The instability range increases by the dissipating quantities at Newtonian (N) regions and makes the system stable at relativistic corrections.

It is true that extensions from spherical to other kind of symmetries provide important information about self-gravitating fluids. In particular, cylindrical systems in general relativity puzzle relativists since the time Levi-Civita found its vacuum solution. Different physical aspects of fluids play a key role in the dynamical instability and evolution of self-gravitating systems. The motivation for including charge in the stability analysis of compact objects is well justified in the light of some theoretical evidences based on new mechanisms allowing the presence of huge electric charge in self-gravitating systems, e.g., the possibility of huge electric field in strange stars.

The inclusion of electromagnetic field provides an interesting outcome to discuss the stability of self-gravitating objects. Many attempts have been made to describe the interaction between electromagnetic and gravitational fields. Bekenstein [1] was the first who extended the work from neutral to charged case. Since then a large amount of work has been done in the scenario of electromagnetic field [2]. The dynamics of non-adiabatic collapsing process leads to the emission of gravitational radiations in the presence of electromagnetic field [3]. The effects of electromagnetic field for the dissipative plane collapse with anisotropic fluid [4] and the dissipative spherical collapse with perfect fluid [5] are also studied.

Regge and Wheeler [6] furnished the foundation to discuss the stability regions by providing evidence for the stability of the Schwarzschild black hole. Chandrasekhar [7] was the pioneer who discussed the dynamical instability of spherical star with perfect fluid. Instability range depends upon the critical value $\frac{4}{3}$ for isotropic spheres as described by Chadrasekhar. For cylindrical systems, it is based on the critical value 1. However, with the inclusion of electromagnetic field, the instability range depends upon the physical parameters like energy density, pressure etc. Moreover, electromagnetic field increases the instability range and makes the system more unstable. Recently, we have investigated the instability range for a restricted class of non-static axial symmetry with anisotropic matter configuration [8]. It is found that the adiabatic index depends upon the energy density and different stresses of the fluid distribution and contains static terms of the axial geometry.

It is interesting to mention here that different ranges of instability may lead to different patterns of evolution of stars. Different

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authors [9] extended Chandrasekhar work for dissipative fluids. Chan et al. [10] found that anisotropy and viscosity affect the instability range at N and post-Newtonian (pN) regions. Since then a lot of work has been done to investigate the dynamical stability of spherical stars with isotropic pressure in the interior [11]. Bisnovatyi-Kogan and Tsupko [12] discussed different criterion for the stability of stars. Roupas [13] studied the dynamical stability of spherical system with perfect fluid.

The adiabatic index Γ is known to be the key factor to discuss the dynamical instability whose value is smaller than $\frac{4}{3}$ in N limit for perfect fluid. Sorkin et al. [14] explored the instability of spherical star for the particular case of radiation. Chan et al. [15] studied the dynamical instability of spherical star model with perfect fluid and found that this leads to collapse for all values of $\Gamma < \frac{4}{3}$. Dev and Gleiser [16] investigated the stability under radial perturbation of fluid sphere. Boehmer and Harko [17] examined the instability of the spherical system with perfect fluid in the presence of cosmological constant.

Herrera et al. [18] studied the stability of dissipative spherically symmetric viscous star. They also explored the instability range of expansion-free spherical geometry using perturbation scheme and found the independence of Γ in this scenario [19]. Sharif and Azam [20] have investigated the stability of cylindrical geometry in Newtonian and post-Newtonian approximations under different scenarios. Recently, they [21] explored the instability range of the cylinder for radiative and non-radiative perturbations. In a recent paper [22], we have examined the role of electromagnetic field on the stability of the expansion-free cylinder. Sharif and Yousaf [23, 24] discussed the instability ranges of collapsing models with the help of perturbation scheme in $f(R)$ gravity.

In this paper, we investigate instability of the radiating collapsing cylinder under the effect of electromagnetic field. The plan of the paper is as follows. Section 2 is devoted to study the matter distribution. In Section 3, we formulate the Einstein–Maxwell field equations, the corresponding conservation laws as well as junction conditions and deduce the instability range through perturbation scheme. The radiative and non-radiative cases under the effect of electromagnetic field are also explored. In the last section, we conclude our results.

2. Matter distribution and field equations

In this section, we discuss the fluid configuration and construct field equations under the effect of electromagnetic field. For this purpose, we take the non-static cylindrical geometry in the interior region as [25]

$$ds_-^2 = -W^2(t,r)\,dt^2 + X^2(t,r)\,dr^2 + Y^2(t,r)\,d\theta^2 + dz^2, \tag{1}$$

where the constraints $-\infty < t < \infty$, $0 \leqslant r < \infty$, $0 \leqslant \theta \leqslant 2\pi$, $-\infty < z < \infty$ have been considered on the coordinates of the cylinder. We consider perfect fluid with radiation in the interior of the collapsing cylinder described by the energy-momentum tensor of the form

$$T^-_{\alpha\beta} = (\mu + p)v_\alpha v_\beta + p g_{\alpha\beta} + \varepsilon l_\alpha l_\beta, \tag{2}$$

where μ, p and ε are the energy density, pressure and radiation density, respectively. Also, v_α and l_α are the unit four-velocity and the null four-vector satisfying

$$l^\alpha l_\alpha = 0, \qquad v^\alpha v_\alpha = -1.$$

These quantities in comoving coordinates can be written as

$$l^\alpha = W^{-1}\delta^\alpha_0 + X^{-1}\delta^\alpha_1, \qquad v^\alpha = W^{-1}\delta^\alpha_0.$$

The energy–momentum tensor for electromagnetic field is

$$E_{\alpha\beta} = -\frac{1}{4\pi}\left(\frac{1}{4}F^{\gamma\delta}F_{\gamma\delta}g_{\alpha\beta} - F^\gamma_\alpha F_{\beta\gamma}\right), \tag{3}$$

where $F_{\alpha\beta} = \phi_{\beta,\alpha} - \phi_{\alpha,\beta}$ is an anti-symmetric strength field tensor and ϕ_α corresponds to four-potential.

The Maxwell field equations are

$$F_{[\alpha\beta;\gamma]} = 0, \qquad F^{\alpha\beta}_{\ \ ;\beta} = \mu_0 J^\alpha,$$

with J^α and $\mu_0 = 4\pi$ as the four-current and magnetic permeability, respectively. The magnetic field will be zero in comoving coordinate system, as the charge per unit length of the system is assumed to be at rest. Accordingly, we take the four-potential and four-current as

$$\phi_\alpha = \phi\delta^0_\alpha, \qquad J^\alpha = \zeta v^\alpha,$$

here ϕ and ζ are functions of t as well as r representing the scalar potential and charge density, respectively. The only non-vanishing component of the strength tensor is

$$F_{10} = -F_{01} = \phi',$$

where prime indicates differentiation with respect to r. Using these values, the Maxwell field equations become

$$\left(\frac{\partial\phi}{\partial r}\right)\left(\frac{W'}{W} - \frac{Y'}{Y} + \frac{X'}{X}\right) - \left(\frac{\partial^2\phi}{\partial r^2}\right) = -4\pi\zeta W X^2, \tag{4}$$

$$\left(\frac{\partial\phi}{\partial r}\right)\left(\frac{\dot W}{W} - \frac{\dot Y}{Y} + \frac{\dot X}{X}\right) - \left(\frac{\partial^2\phi}{\partial t\partial r}\right) = 0, \tag{5}$$

here dot corresponds to differentiation with respect to t. Integration of Eq. (4) with respect to r gives

$$\frac{\partial\phi}{\partial r} = \frac{qWX}{Y},$$

$q(r) = 4\pi\int_0^r \zeta XY\,dr$ is the total amount of charge per unit length of the cylinder. Also, ϕ' identically satisfies Eq. (5). The non-vanishing components of $E_{\alpha\beta}$ turn out to be

$$E_{00} = -\frac{\pi}{2}E^2W^2, \qquad E_{11} = \frac{\pi}{2}E^2X^2,$$
$$E_{22} = -\frac{\pi}{2}E^2Y^2, \qquad E_{33} = -\frac{\pi}{2}E^2,$$

where $E = \frac{q}{2\pi Y}$.

The Einstein–Maxwell field equations corresponding to Eq. (1) yield

$$\kappa\left(\mu + \varepsilon - \frac{\pi}{2}E^2\right)W^2 = \frac{\dot X\dot Y}{XY} + \left(\frac{W}{X}\right)^2\left(\frac{X'Y'}{XY} - \frac{Y''}{Y}\right), \tag{6}$$

$$\kappa WX\varepsilon = \frac{\dot Y'}{Y} - \frac{\dot X Y'}{XY} - \frac{\dot Y W'}{YW}, \tag{7}$$

$$\kappa\left(p + \varepsilon + \frac{\pi}{2}E^2\right)X^2 = \frac{W'Y'}{WY} + \left(\frac{X}{W}\right)^2\left(-\frac{\ddot Y}{Y} + \frac{\dot W\dot Y}{WY}\right), \tag{8}$$

$$\kappa\left(p - \frac{\pi}{2}E^2\right)Y^2 = \left(\frac{Y^2}{WX}\right)\left(\frac{\dot W\dot X}{W^2} - \frac{\ddot X}{W} - \frac{W'X'}{X^2} + \frac{W''}{X}\right), \tag{9}$$

$$\kappa\left(p - \frac{\pi}{2}E^2\right) = -\frac{\ddot X}{W^2X} + \frac{W''}{WX^2} - \frac{\ddot Y}{W^2Y} - \frac{W'X'}{WX^3} + \frac{\dot W\dot Y}{W^3Y} + \frac{\dot W\dot X}{W^3X} - \frac{X'Y'}{X^3Y} - \frac{\dot X\dot Y}{W^2XY} + \frac{W'Y'}{WX^2Y} + \frac{Y''}{X^2Y}. \tag{10}$$