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Experimental analysis of shape deformation of evaporating droplet using Legendre polynomials

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ABSTRACT

Experiments involving heating of liquid droplets which are acoustically levitated, reveal specific modes of oscillations. For a given radiation flux, certain fluid droplets undergo distortion leading to catastrophic bag type breakup. The voltage of the acoustic levitator has been kept constant to operate at a nominal acoustic pressure intensity, throughout the experiments. Thus the droplet shape instabilities are primarily a consequence of droplet heating through vapor pressure, surface tension and viscosity. A novel approach is used by employing Legendre polynomials for the mode shape approximation to describe the thermally induced instabilities. The two dominant Legendre modes essentially reflect (a) the droplet size reduction due to evaporation, and (b) the deformation around the equilibrium shape. Dissipation and inter-coupling of modal energy lead to stable droplet shape while accumulation of the same ultimately results in droplet breakup.

1. Introduction

Understanding droplet dynamics and heat transfer is critical in industrial and scientific applications such as spray applications [1,2], pharmaceuticals [3,4], materials processing with microstructures, crystal growth [5,6], fuel injection/atomization, and micro gravity. Therefore, droplet vaporization has been extensively studied. While some have incorporated non equilibrium effects [7], or adopted molecular dynamics approach [8] others have considered quasi-steady vaporization, involving spray combustion [9,10] and, solute precipitation using plasma jet [11]. The present work deals with quasi-steady droplet vaporization through acoustic levitation which is a convenient way to study droplet vaporization, fluid recirculation, shape change, atomization and agglomeration in a non-contact containerless environment. Suspension of droplet in an acoustic levitator works on the principle of generating a standing wave that can be amplitude-tuned to achieve greater control over the stability of a droplet without any surface contact. However, the pressure field is highly non-homogeneous which can lead to surface oscillations due to acoustic streaming, and bulk deformation due to the sound pressure level (SPL). The increase in sound intensity has been shown to flatten the droplet to membrane-like structures [12,13]. Surface ripples have been observed in some droplets leading to atomization, while in others, the onset of buckling instability results in bulk shattering. The instability mechanisms are complex functions of the pressure field, initial droplet shape, instantaneous surface tension and viscosity in addition to temperature gradients (for heated droplets). Extensive analyses on the influence of acoustic forces on droplets and subsequent evaporation enhanced by acoustic convection have been previously reported [14]. Theoretical prediction of the droplet shape deformation and the levitated droplet position in an acoustic field based on the boundary integral technique have revealed the effect of droplet aspect ratio on the acoustic pressure wave [15]. Further extension leads to the calculation of heat and mass transfer from a levitated droplet and the effect of acoustic streaming on these fluxes [16]. The effects of varying acoustic intensity on vaporization rate and droplet surface temperature have revealed dependence of these properties on droplet sizes [17]. The dependence of d^2-law on the droplet wet bulb and surface temperatures was also reported.

Alternatively it has been found that oscillations in levitated droplets can also be induced without modulating SPL. Secondary atomization can occur through non-intrusive laser heating at the equatorial plane. This may also ultimately lead to catastrophic droplet breakup. The shape deformation and instability criteria of a single acoustically levitated droplet induced by laser heating were first investigated in our previous publications [18,19]. In our earlier study [18], we reported three distinct stages of droplet deformation prior to breakup: continuous volume regression due to evaporation with negligible shape deformation; initial bulk deformation through shedding of satellite droplets; and secondary atomization with eventual bag-type breakup [19]. We proposed a universal criterion for the uncontrolled bulk

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deformation of heated, levitated droplet in terms of Weber number = (non-homogeneous pressure force/surface tension force). It was determined that below a critical Weber number throughout the entire droplet lifecycle, there was no catastrophic breakup. The nature of the fluid, input sound pressure level, instantaneous shape/size and external heating rate (laser power) determine the variation of surface tension and viscosity in addition to the pressure non-homogeneity around the droplet. This leads to a local variation of Weber number, which for fluids like kerosene and FC43 crosses the threshold value at some point of the droplet lifespan. Although stability criteria were established, the exact mechanism of evolution of the droplet shape was not quantified in [19]. Our earlier work focused on the physical explanations of the process, and in this paper, we extend the analysis using a semi-quantitative approach for droplet shape decomposition.

In the current work, a general framework has been introduced for predicting stability of heated droplets against acoustic forcing. The experiments were carried out in the absence of intensity modulation of the acoustic wave. Even under a preset value of the r.m.s. pressure the droplet shape was disturbed primarily by radiation heating. The deformation of the heated levitated droplet is found to be initiated by specific modes of bulk oscillations. Evolution of these modes will determine whether the bulk deformation will decay to a stable droplet shape or grow uncontrollably and breakup. For identifying these modes, a quantitative approach employing Legendre polynomials (for approximating the droplet shape) has been adopted. Legendre polynomials have not been previously used to describe thermally induced instabilities in a levitated droplet. So this classical approach has been used for the first time to analyze the droplet deformation induced by thermal means in an acoustic field. Earlier use of Legendre polynomials in droplet dynamics have been restricted to large amplitude free [20], and forced oscillations (set up by pressure or frequency modulation in acoustic levitator). First suggested by Rayleigh for describing axisymmetric oscillations of droplet, these independent spherical harmonics, when imposed over an equilibrium shape give a near complete description of the transient droplet shape [21]. Lamb's treatment of this method has yielded the expression of mode frequency for the inviscid approximation [22]. Subsequent modifications incorporated the effect of viscosity of the droplet [23,24]. The higher order modes damp out quite rapidly due to lower decay time, and thus for experimental analysis, the first few harmonics are usually sufficient in providing a good estimate of the droplet shape [20].

In the current study, volume change due to evaporation/atomization is an additional parameter present alongside deformation on the equilibrium shape. So the equilibrium spherical radius becomes a transient variable. The two major effects of thermal perturbations have been identified as modes of droplet oscillations. A theoretical framework has been provided to incorporate the bulk effects that include (i) evaporative size decay and (ii) the deformation of the spherical droplet, and the coupled interplay between the two modes to account for the stability of levitated droplets undergoing shape oscillations due to radiative heating. The short wavelength surface ripples if present have not been modeled or considered in the analysis.

2. Modeling description and experimental validation

The model has been verified using results from our previous work [19], maintaining the same experimental parameters (ambient temperature ∼ 25% and relative humidity ∼ 66%). Droplets of initial diameter 500 μm were suspended using a micro syringe near the pressure node in a 100 KHz Tec5 acoustic levitator. Fluids used were ethanol, diesel, kerosene. The droplets were heated with a constant laser flux of 1.25 MW/m^2 from a 30 W output

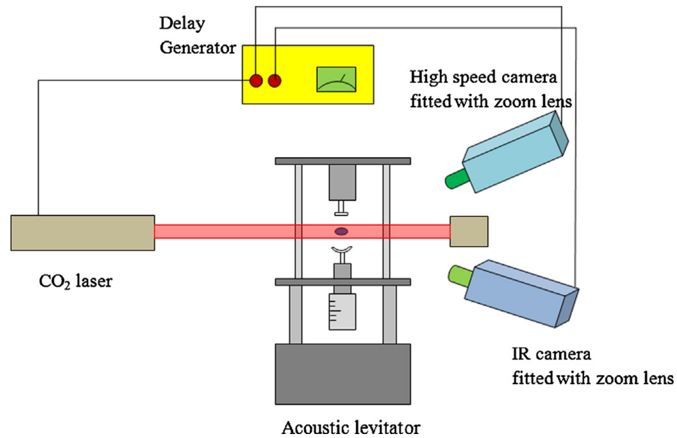

Fig. 1. Schematic of the experimental setup.

CW CO_2 laser with beam diameter of 4 mm. The laser flux was kept below than what is required to initiate any boiling phenomena, and indeed no formation of any bubble was observed that would suggest the occurrence of boiling in any of the experimental runs described here. A high speed Phantom V12 camera was used to capture the droplet dynamics. Droplet surface temperature was recorded with an infra-red camera operated at 100 fps. The frame rate for high speed imaging was fixed at 1000 fps for ethanol and 3000 fps for kerosene and diesel. Spatial resolution was enhanced in every pixel to 2 μm using microscopic lenses for both the high speed and the IR camera. A delay generator was used to synchronize the laser with the cameras. Further details are discussed in [19]. A schematic of the experimental setup is shown in Fig. 1.

The droplet images have been analyzed as follows – the grayscale images were first converted to binary images. Sobel edge detection function in MATLAB has been used to isolate the droplet edge. Some of the additional domains detected were removed by suitably choosing a threshold value of domain pixels. Legendre polynomial approximation has been applied to describe the edge details which are discussed in the following section. Our analysis concerns ethanol, kerosene and diesel droplets with more emphasis on the first two since they exhibit two extreme behaviors as will be seen subsequently. The other fluids such as camelina, FC-43, etc. have been included only to show the variation of their thermo-physical properties. FC-43 has substantial reduction in surface tension similar to Kerosene and like the latter undergoes breakup [19]. Ethanol shows stable shape as it vaporizes and remains spherical. Camelina, bio-diesel and diesel exhibit intermediate response.

2.1. Mathematical description for droplet shape analysis

Rayleigh's mathematical model is used to describe axisymmetric droplet oscillations –

$$R(\theta,t) = R_0 + \sum_{l=1}^{\alpha} a_l(t) P_l\big(\cos(\theta)\big) \tag{1}$$

where R_0 = radius of spherical droplet having equal volume as that of the deformed shape, θ = angle from the equatorial plane of the droplet, $R(\theta,t)$ = the instantaneous droplet radius (Fig. 2), $P_l(\cos\theta)$ = Legendre polynomials, $a_l(t)$ = Legendre mode amplitudes. Now since $R_0 = R_0(t)$, it can be modeled as an independent harmonic with a time varying amplitude. Recasting it as $R_0 = a_0(t)P_0(\cos\theta)$ and including harmonics $l = 0$–5, the modified equation becomes