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Edge effects in electrostatic calibrations for the measurement of the Casimir force

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ABSTRACT

We have performed numerical simulations to evaluate the effect on the capacitance of finite size boundaries realistically present in the parallel plane, sphere–plane, and cylinder–plane geometries. The potential impact of edge effects in assessing the accuracy of the parameters obtained in the electrostatic calibrations of Casimir force experiments is then discussed.

1. Introduction

The Casimir force [1] has been demonstrated in a variety of experimental setups and geometries, yet there is an ongoing reanalysis of the level of accuracy with which it has been determined, which is crucial for assessing reliable limits to the existence of Yukawa forces of gravitational origin predicted by various models [2–4]. In performing Casimir force measurements a crucial role is played by the related electrostatic force calibrations, and mastering all possible systematic effects in the latter is mandatory to assess the precision of the former. The presence of anomalous exponents in the power-law dependence upon distance and the dependence on distance of the minimizing potential [5] have been identified as a possible source of systematic effects, to be carefully scrutinized in each experimental setup [6]. The distance-dependence of the minimizing potential has been recently modeled theoretically, after a first effort reported in [7], in terms of non-equipotential conducting surfaces due to random patterns of patch charges [8]. Here we report results on a further potential source of systematic error by studying, by means of numerical simulations using finite element analysis, the influence of the unavoidable presence of edges on electrostatic calibrations in various geometries (for the influence of edge effects on Casimir forces in peculiar geometries see [9,10]). We limit the attention to the three geometries that, apart from the crossed-plane investigated in [11], have been extensively discussed for measuring Casimir force so far, i.e. the parallel plates [12,13], the sphere–plane [14–17], and the recently proposed cylinder–plane [18] (see Fig. 1). We discuss the deviations from ideality in all these geometries through the essential knowledge of the capacitance dependence on distance, and its general implications for determining the parameters used in the measurement of the Casimir force. This is performed under the simplifying assumptions that the surfaces are equipotential, i.e. by omitting any superimposed effect due to electrostatic patches, and by neglecting the tensorial nature of the capacitance among all the conducting surfaces realistically involved in concrete experimental setups.

2. Parallel plate geometry

We start our analysis with the parallel plates configuration since this is the simplest geometry even in terms of possible deviations from the ideal, infinite plane case. Moreover, besides having an exact expression in the ideal case of infinite plates, analytical approximate expressions are also available for the capacitance including edge effects [19], allowing to obtain reliable numerical benchmarks. We consider two identical parallel square plates of length L, and boundary conditions set in such a way that the two plates are at a constant electric potential difference. The total electrostatic potential energy is then computed by numerically solving the Laplace equation using a dedicated finite element analysis software (COMSOL). We evaluate the total electrostatic energy W_{el} by summing the electrostatic energy density over a selected volume surrounding the two equipotential surfaces. The capacitance C can then be calculated from the relationship $C = 2W_{\mathrm{el}}V^2$. The parameters of the mesh (mesh size, rate of growth etc.) are carefully chosen and extensively tested to ensure the accuracy of the numerical results. When the distance between the two plates is much smaller than L, the capacitance obtained from the numerical simulation agrees within 0.01% with the well-known analytical formula $C_{\mathrm{pp}} = \epsilon_0 A/d$, where A is the surface area of the plates, d their separation, and ϵ_0 the vacuum electric permittivity. However, when the distance becomes larger, the value of the capacitance begins to deviate from the analytical formula. The left plot in Fig. 2 shows the capacitance of two parallel square plates *vs*. distance, together with the expected capacitance for two plates following the

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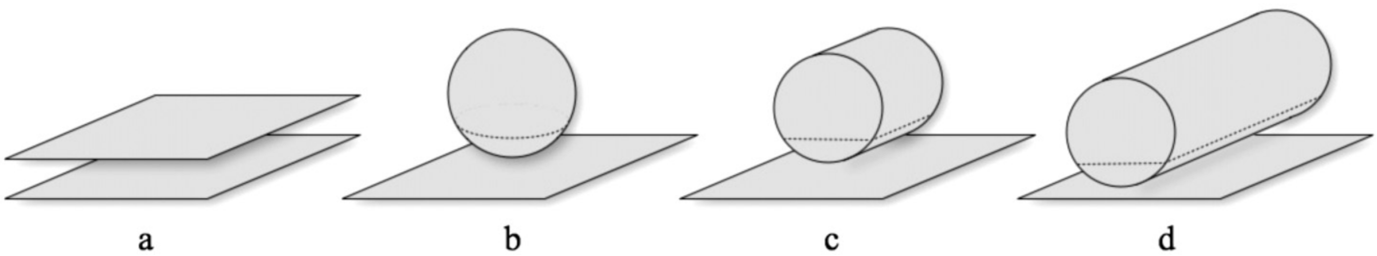

Fig. 1. Selected geometries for the numerical study of finite size effects. Parallel plates of finite size (a); a sphere in front of a finite size plane (b); a cylinder in front of a plane, with width smaller (c) and larger (d) than the size of the plane. In the latter three configurations, a truncated sphere and short and long truncated cylinders, as shown by the dashed lines, have been also studied.

infinite surface formula, appearing as a dashed line. The data at large distance deviate significantly from the dashed line, and the fact that they are all above implies that the power-law exponent is softer, i.e. in between 0 and −1. This deviation is ascribed to the finite size of the parallel plates. In fact, if only the field lines in the volume delimited by the two plates are used for the sum of the electrostatic energy, which corresponds to ignoring the contribution of the outer region, the capacitance obtained from the numerical simulation would still agree with the analytical formula even at the largest explored distances. In order to better quantify this deviation we have fitted the capacitance curve by progressively removing points at the largest distance. As shown in the right plot in Fig. 2, the optimal exponent becomes smaller than unity when data at large distance are progressively included in the fit.

3. Sphere–plane geometry

In the case of the sphere–plane geometry, the exact expression for the capacitance between a sphere and an infinite plane is written in terms of a series [20]:

$$C_{sp} = 4\pi\epsilon_0 R \sinh(\alpha) \sum_{n=1}^{+\infty} \frac{1}{\sinh(n\alpha)} \tag{1}$$

with $\cosh(\alpha) = 1 + d/R$, R is the sphere radius, and d the separation distance. In the limit of small separations, $d/R \ll 1$, an approximate expression for the capacitance can be obtained [20]:

$$C_{sp} \approx 2\pi\epsilon_0 R \left(\ln\frac{R}{d} + \ln 2 + \frac{23}{20} + \frac{\theta}{63} \right) \tag{2}$$

where $0 \leqslant \theta \leqslant 1$. By neglecting the distance-independent terms one obtains an expression often appearing in relationship to the so-called Proximity Force Approximation (PFA) [21,22] in electrostatics. The formulas above both assume an infinite plane and a whole sphere. In real experiments involving microresonators the size of the plane is not necessarily large enough to be considered infinite and thus edge effects could be present. In some measurements [17] the sphere is located close to one end of the squared plane, to increase the torque exerted on the underlying microresonator. Also, in other measurements a lens, schematized as a truncated sphere was used in lieu of a whole sphere [14,23]. Fig. 3 shows the numerical results for configurations taking into account these deviations from the idealized case, as well as the curves expected from the exact (1) and the approximate (2) expressions. For the whole sphere, it is worth remarking that neither the exact nor the approximate expression can give an accurate value for the capacitance between a sphere and a finite plane, even at small distance. We have checked that the idealized case of a sphere and an infinite plane is approached by considering square plates of progressively larger size. The capacitance in the realistic case still preserves a logarithmic dependence at small distance, with the slopes very close to each other. For the truncated sphere, the capacitance is significantly smaller than both the exact and the approximate expressions, which is expected considering that there is less conducting surface available in this case. Moreover,

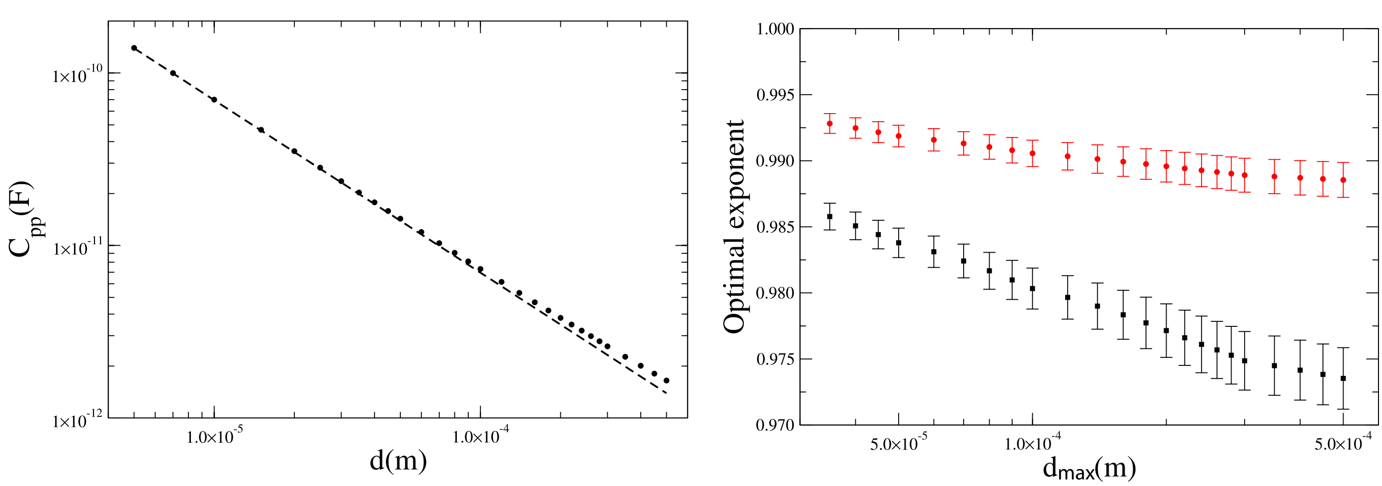

Fig. 2. (Left) Capacitance *vs.* distance for two parallel square plates. Dots indicate the results of the numerical simulation, while the dashed line is the capacitance expected using the formula $C_{pp} = \epsilon_0 A/d$. The length of the square plates L was chosen to be 8.86 mm. (Right) Optimal exponent ϵ_c coming from the best fit of a set of numerical data with $C_{pp} = \epsilon_0 A/(d - d_0)^{\epsilon_c}$ *vs.* the distance of the farthest data point used in the fitting. Red dots are obtained by fixing $d_0 = 0$, corresponding to an *a priori* knowledge of the absolute distance, while the black squares are obtained if d_0 is considered as a fitting parameter, as usually done when analyzing real experimental data with no *a priori* independent knowledge of the absolute distance between the two surfaces. Considering the size of the square plates used in the experiment reported in [13] of 1.1 mm, all separation distances in that case are obtained by scaling down the horizontal axis by a factor $\simeq 65$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)