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Efficient generation of N-photon binomial states and their use in quantum gates in cavity QED

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ABSTRACT

A high-fidelity scheme to generate N-photon generalized binomial states (NGBSs) in a single-mode high-Q cavity is proposed. A method to construct superpositions of exact orthogonal NGBSs is also provided. It is then shown that these states, for any value of N, may be used for a realization of a controlled-NOT gate, based on the dispersive interaction between the cavity field and a control two-level atom. The possible implementation of the schemes is finally discussed.

1. Introduction

In the framework of cavity quantum electrodynamics (CQED) [1–3] several schemes have been proposed to generate nonclassical states of the cavity electromagnetic field [4–6]. Using standard atom–cavity interactions, Fock states $|N\rangle$ with a maximum number of photons $N = 1$ [7,8], $N = 2$ [9,10] and recently up to $N = 4$ [11] were for example experimentally produced and probed. Superpositions of coherent states with different phases were also obtained [11,12].

Quantum superpositions of orthogonal quasiclassical states play a crucial role to investigate the quantum–classical border. To this purpose, superpositions of coherent and Fock (number) states have been considered. However, coherent states, although naturally suited to give a classical limit, are never exactly orthogonal, while Fock states with a different photon number are orthogonal but present nonclassical features even at high N. Among other classes of electromagnetic field states, the so-called N-photon generalized binomial states (NGBSs), has been introduced [13,14] and successively used for measuring the canonical phase of quantum electromagnetic field states in theoretical proposals [15,16]. These NGBSs are in a sense intermediate between coherent states and number states, sharing with the first ones the property of naturally giving a classical limit and with the second ones the possibility of building superpositions of exactly orthogonal states [17]. Therefore, NGBSs are the natural candidates to construct quasiclassical superpositions of orthogonal states [18].

It has been also pointed out [19] that the NGBSs are the exact electromagnetic correspondent of the coherent atomic states of N two-level atoms [20]: correspondence suggesting that NGBSs could be naturally produced, in the context of CQED, by interactions between two-level atoms and a quantum electromagnetic field. Recently it has been shown that, within this context, 2GBSs can be efficiently generated [21] and their orthogonal quantum superpositions obtained [18]. Entangled 2GBSs can also be obtained, in two spatially separated cavities, by standard resonant atom–cavity interactions [22,23]. These quantum superpositions of NGBSs could also play a role in the context of quantum information [24]. The recent experimental CQED capabilities, that permit the control of a sequence of many atoms (~ 100), appear to make possible the production of NGBSs with N rather larger than 1 [11,12]. Thus, finding efficient experimental schemes to generate NGBSs with $N > 2$ is an up-to-date issue.

In this work, after recalling the definition of a NGBS and some of its properties in Section 2, we propose a scheme to realize NGBSs with $N > 2$ in CQED developed in Section 3. Successively, we show that NGBSs may be exploited for a realization of a controlled-NOT (CNOT) gate as described in Section 4, where we also provide a way to construct superpositions of two orthogonal NGBSs. In Section 5 we analyze the experimental feasibility of the proposed schemes and in Section 6 we conclude.

2. N-photon generalized binomial state (NGBS)

The normalized N-photon generalized binomial state, denoted by $|N, p, \phi\rangle$, is defined as [13]:

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$$|N,p,\phi\rangle=\sum_{n=0}^{N}b_n^{(N)}\left[p^n(1-p)^{N-n}\right]^{1/2}e^{in\phi}|n\rangle, \tag{1}$$

where $0\leqslant p\leqslant 1$ is the probability of single photon occurrence, ϕ the mean phase [14] and $b_n^{(N)}$ are the binomial coefficients

$$b_n^{(N)}=\binom{N}{n}^{1/2}=\left[\frac{N!}{(N-n)!\,n!}\right]^{1/2}. \tag{2}$$

Thus, a NGBS is a finite superposition of number states weighted by a binomial distribution with a fixed relative phase between two consecutive number states.

In the case $\phi=0$, the properties of these states [13,14,25,26] and the effects of their interaction with atoms [27,28] have been studied. It is important to observe that the NGBS of Eq. (1) becomes the vacuum state $|0\rangle$ when $p=0$ and the number state $|N\rangle$ when $p=1$. For $N\to\infty$ and $p\to 0$, so that $Np=\text{const}=|\alpha|^2$, the NGBS coincides with the coherent state $||\alpha|e^{i\phi}\rangle$. In this sense we say that NGBSs interpolate between the number and the coherent states.

It is important to point out that an orthogonality property exists for NGBSs. To each state $|N,p,\phi\rangle$ it corresponds the state $|N-1,p,\pi+\phi\rangle$ such that $\langle N,p,\phi|N,1-p,\pi+\phi\rangle=0$ [17].

3. *N*GBS generation scheme

A conditional scheme for the generation of NGBSs in a cavity, based on the resonant interaction of N consecutive monokinetic two-level atoms with the single-mode cavity, initially in its vacuum state, has been proposed [29]. This scheme requires that all the N atoms are detected in their ground state after the interaction with the cavity, and the probability of success turns to be of the order of $1/2^N$, thus making the scheme difficult to experimentally implement when $N>2$.

A nonconditional scheme to generate a 2GBS in a cavity, that avoids this drawback, has been proposed [21]. This scheme is a variation of the previous ones and takes advantage of the possibility to utilize suitable different interaction times for the two atoms. However, its generalization to N larger than two is not obvious. Because of the intrinsic interest and of the recent experimental possibility to reach the region of N larger than two (even of some order of magnitude [11,12]), we have examined the possibility of appropriately extending the previous $N=2$ results to arbitrary sufficiently large N. We describe an efficient generation scheme for the NGBS of Eq. (1) with $N>2$.

3.1. Atom–cavity interaction model

The scheme we propose here is once again based on the resonant interaction of N consecutive two-level atoms with a single-mode cavity field. The resonant interaction of each two-level atom with a single-mode high-Q cavity is described by the usual Jaynes–Cummings Hamiltonian [30]

$$H_{JC}=\hbar\omega\sigma_z/2+\hbar\omega a^\dagger a+i\hbar g\left(\sigma_+a-\sigma_-a^\dagger\right) \tag{3}$$

where ω is the cavity field mode, g the atom–field coupling constant, a and $a^\dagger$ the field annihilation and creation operators and $\sigma_z=|\uparrow\rangle\langle\uparrow|-|\downarrow\rangle\langle\downarrow|$, $\sigma_+=|\uparrow\rangle\langle\downarrow|$, $\sigma_-=|\downarrow\rangle\langle\uparrow|$ the pseudo-spin atomic operators, $|\uparrow\rangle$ and $|\downarrow\rangle$ being the excited and ground state respectively of the two-level atom. As well known the Hamiltonian H_{JC} generates the transitions [31]

$$\begin{aligned}|{\uparrow}n\rangle&\to\cos\left(g\sqrt{n+1}t\right)|{\uparrow}n\rangle-\sin\left(g\sqrt{n+1}t\right)|{\downarrow}n+1\rangle,\\|{\downarrow}n\rangle&\to\cos\left(g\sqrt{n}t\right)|{\downarrow}n\rangle+\sin\left(g\sqrt{n}t\right)|{\uparrow}n-1\rangle,\end{aligned} \tag{4}$$

where $|\uparrow\rangle|n\rangle\equiv|{\uparrow}n\rangle$, $|\uparrow\rangle|n\rangle\equiv|{\downarrow}n\rangle$, $a^\dagger a|n\rangle=n|n\rangle$ and t is the atom–cavity interaction time.

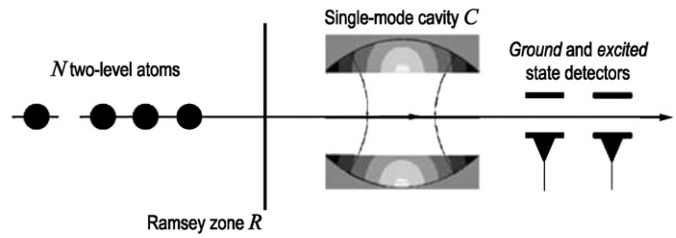

Fig. 1. Sketch of the experimental setup to generate a NGBS in a high-Q cavity. The Ramsey zone R prepares each atom in a desired superposition of its two level.

3.2. Description of the scheme

A sketch of the scheme we propose is presented in Fig. 1. It describes N consecutive two-level atoms which are injected, one at a time, in a Ramsey zone [32], where they are appropriately prepared in a superposition of their two levels. Successively they interact with the cavity, initially in the vacuum state, and after leaving the resonator they are detected by field ionization detectors. Adjusting the atomic separation and the distances between the interaction zones it is possible to neglect the free evolution times of both atoms and cavity field [33]. The probability to generate the desired NGBS coincides with the probability to detect all the atoms in their respective ground state. The key point, as in Ref. [21], is to find suitable atom–cavity interaction times, different in general for each atom, such that the generation probabilities are high enough not to require a measurement. This point makes the difference with respect to previous conditional schemes [4,29]. Different interaction times can be obtained by selecting different velocities for each atom or by applying an appropriate Stark shift inside the cavity when a monokinetic atomic beam is used [33]. The status of current CQED technology allows to know the position of each atom along its trajectory, and thus inside the cavity, with a ± 1 mm precision [1]. Being the atomic beam off-resonant with the cavity mode, the duration of the interaction can be adjusted by a Stark shift pulse which tunes the atom in resonance with the cavity mode for a desired interaction time [2]. The current CQED capabilities also permit to produce monoenergetic atomic beams with a Poisson distribution [3] where the probability of occurrence of two atoms per sample is $\sim 6\times 10^{-2}$ that of single atom occurrence. The one-atom occurrence probability is also small (~ 0.2 atom per sample). However, by data acquisition softwares and detections of the atom beam before the entrance in the Ramsey zone, it is possible to reject most of pulses giving no atoms, so that currently single atom events are selected with high probability (90%) [1].

In our scheme the k-th two-level atom ($k=1,\dots,N$) is prepared by the Ramsey zone in the following superposition:

$$|\chi_k\rangle=\sqrt{p}|\uparrow\rangle+e^{i\varphi_k}\sqrt{1-p}|\downarrow\rangle\quad(k=1,\dots,N) \tag{5}$$

where $0\leqslant p\leqslant 1$ is equal for all the atoms whereas the relative phase φ_k will be *a posteriori* related to the mean phase ϕ of the NGBS we wish to generate. The values of p and φ_k can be fixed by adjusting the Ramsey zone settings, i.e. the classical field amplitude and the atom–field interaction time [1].

Now let us assume that, before the injection of the k-th atom in the cavity, the $(k-1)$-th atom has left the cavity in a state that for convenience we write in the form

$$|\psi_{k-1,p,\phi}\rangle=\frac{1}{\mathcal{N}_{k-1}}\sum_{n=0}^{k-1}c_n^{(k-1)}\sqrt{p^n(1-p)^{k-1-n}}e^{in\phi}|n\rangle, \tag{6}$$

where $\mathcal{N}_{k-1}$ is a normalization constant. For $k=1$ this state reduces to the vacuum state $|0\rangle$. Before the k-th atom enters the cavity, the total atom–cavity state is therefore factorized as