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On natural frequencies of non-uniform beams modulated by finite periodic cells

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A B S T R A C T

It is well known that an infinite periodic beam can support flexural wave band gaps. However, in real applications, the number of the periodic cells is always limited. If a uniform beam is replaced by a non-uniform beam with finite periodicity, the vibration changes are vital by mysterious. This paper employs the transfer matrix method (TMM) to study the natural frequencies of the non-uniform beams with modulation by finite periodic cells. The effects of the amounts, cross section ratios, and arrangement forms of the periodic cells on the natural frequencies are explored. The relationship between the natural frequencies of the non-uniform beams with finite periodicity and the band gap boundaries of the corresponding infinite periodic beam is also investigated. Numerical results and conclusions obtained here are favorable for designing beams with good vibration control ability.

1. Introduction

A lot of perfect or imperfect periodic structures can be found in nature and engineering fields. When elastic waves propagate in them, the phenomenon of band gap can happen, which means acoustic or elastic waves are barred to spread in certain frequency ranges. Many researchers have studied this phenomenon and tried to apply it to vibration and noise control. For example, Ruzzene's group investigated the characteristics of wave propagation in periodic lattice structures by finite element method and homogenization method [1,2]. Inspired by the hierarchy of biological material, Xu et al. [3] simulated wave propagation in the hierarchical hexagonal lattice structures. They found that the introduction of secondary-level structure is conducive to the formation of band gaps. The flexural wave propagations in rods, beams and plates were also studied. For example, Xiao et al. [4,5] proposed using beams or reinforced resonant structures as resonant units to explore longitudinal and flexural waves propagating in the periodic rods and plates, explaining the conversion mechanism of the Bragg band gaps and the locally resonant band gaps. Yu et al. [6] recently investigated the vibration band gaps of the periodic pipeline system by considering the fluid–structure interactions.

The above studies are focused on the calculation methods, formation mechanisms and enlargement of band gaps. And one should notice that the theories and methods used in these studies are based on the concept of phononic crystal [7–10] which is referred to the structure with infinite periodic cells. Although the finite periodic models are used to illustrate the existence of band gaps in experiments, it is still based on the theory of wave propagation. For elastic rods, beams, and slab structures which are commonly used in engineering, there are restrictions on sizes and boundary conditions. Wave propagation in these objects should be converted into a vibration problem of a non-uniform beam modulated by finite periodic cells [11–14]. In fact, the non-uniform beam model widely exists in the projects like towering chimneys, buildings, and drive bearings with variable cross sections. Researchers have proposed several methods such as finite element method, differential transformation method [15], AMD method [16], and the transfer matrix method (TMM) [17,18] to solve the natural frequencies of these non-uniform beam structures.

Earlier, Mead [19] considered the link between locations of band gaps of an infinite periodic structure and the natural frequencies of its finite slender segments. Recently, Hvatov and Sorokin [20] employed a hierarchy of four mathematical models to compare the natural frequencies of finite periodic beams with locations of band gaps for their infinite counterparts. The existing studies ignored designing the non-uniform beam in a subjective initiative way, especially modulating the beam with finite periodic cells. To the authors' knowledge, the influence of modulation by finite periodic cells on the natural frequencies of the non-uniform beams

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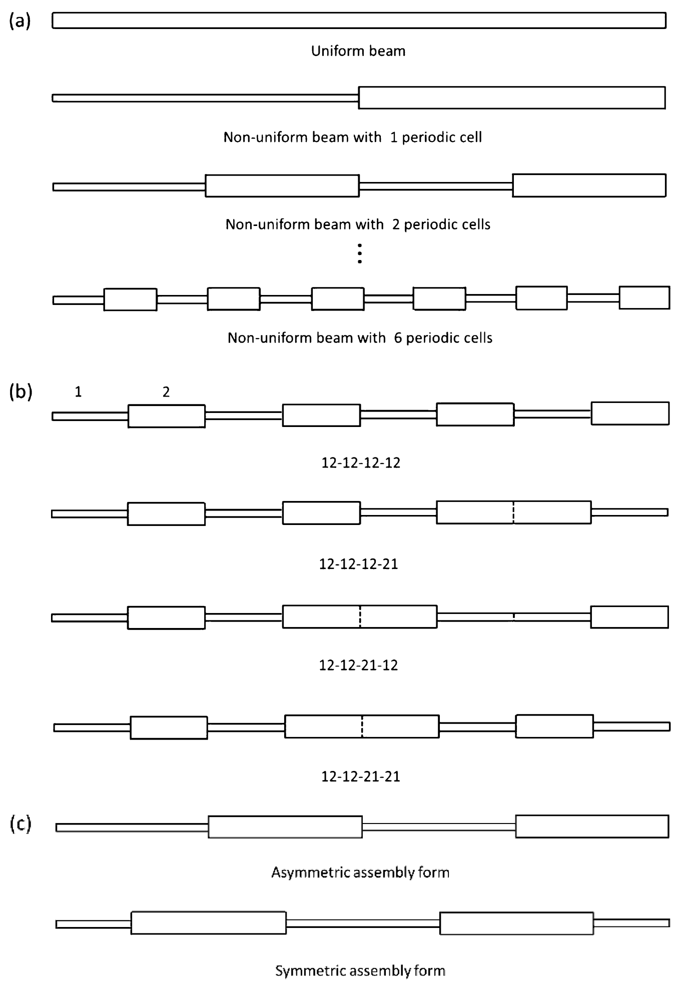

Fig. 1. The schematic of non-uniform beams with modulation by finite periodic cells. (a) The uniform beam and the non-uniform beams modulated by 1 to 6 periodic cells, respectively. All the beams have the same length and mass. (b) Four different assembly forms of the non-uniform beams with four periodic cells. (c) Asymmetric and symmetric assembly forms for the non-uniform beams with two asymmetric and symmetric unit cells, respectively.

has yet to be studied. To elucidate this point, the natural frequencies of the non-uniform beams with the same mass as the uniform beam but modulated by finite periodic cells are investigated. The influence of the amounts, cross section ratios, and assembly forms of the finite periodic cells on the natural frequencies of the modulated non-uniform beam are considered numerically. Compared to the work of Hvatov and Sorokin [20], the relationship of the natural frequencies of the finite non-uniform beams and the band gap boundaries of the corresponding periodic beam is also reviewed and commented in a direct and parametric way.

The paper is organized as follows. Section 2 presents the beam models and the TMMs which are used to calculate the natural frequencies of the non-uniform beams with finite periodic cells and the band structures of the corresponding infinite periodic beams. In Section 3, the numerical results are provided. The effects of the amounts, cross section ratios and arrangements of the periodic cells on the frequencies are discussed. At last, Section 4 comes the conclusions.

2. Models and methods

To study the influence of the modulation by finite periodic cells on the natural frequencies of the uniform beam, the finite periodic non-uniform beams are designed to keep the same amount of mass with the original one. The modulation by finite periodic cells is realized by the stepped cross sections changed alternately. The unit cells can be used to compose the finite periodic non-uniform beams in asymmetric and symmetric forms. Fig. 1(a) is the schematic of non-uniform beams assembled by several finite periodic unit cells. The length and thickness of the original uniform beam are L and h, respectively. For the non-uniform beam with N periodicity, to keep the same length and same mass as the original one, the length of the unit cell Λ is $L/(2N)$ and the thicknesses of the sub-cell 1 and sub-cell 2 in the unit cell are εh and $(2-\varepsilon)h$, respectively. Fig. 1(b) provides several examples of different assembly forms for the non-uniform beams with four-cell periodicity. And different assembly forms can give asymmetric and symmetric non-uniform beams illustrated by using two-cell periodicity as shown in Fig. 1(c).

The flexural wave equation in a uniform beam considering Bernoulli–Euler theory is

$$\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 y(x,t)}{\partial t^2} = 0 \tag{1}$$

where E and ρ are the Young's modulus and density of the beam, respectively, A is cross-sectional area, and I is the inertia moment of the cross section. Assuming the separable solution is $y(x,t) = \phi(x)e^{i\omega t}$ with ω the angular frequency and substituting this solution into the wave equation, one can have the governing equation in the frequency domain

$$\frac{\partial^4 \phi(x)}{\partial x^4} - \lambda^4 \frac{\partial^2 \phi(x)}{\partial t^2} = 0 \tag{2}$$

where λ^4 is equal to $(\rho A\omega^2)/(EI)$. The general solution for the fourth-order ordinary differential equation is

$$\phi(x) = A\cos(\lambda x) + B\sin(\lambda x) + C\cosh(\lambda x) + D\sinh(\lambda x) \tag{3}$$

where A, B, C, and D are constants. Then, one can get the slope, moment, and shear force as follows, respectively.

$$\begin{aligned}
\theta(x) &= \phi'(x) \\
&= \lambda\big(-A\sin(\lambda x) + B\cos(\lambda x) + C\sinh(\lambda x) + D\cosh(\lambda x)\big) \\
M(x) &= EI\phi''(x) \\
&= EI\lambda^2\big(-A\cos(\lambda x) - B\sin(\lambda x) + C\cosh(\lambda x) + D\sinh(\lambda x)\big) \\
F(x) &= EI\phi'''(x) \\
&= EI\lambda^3\big(A\sin(\lambda x) - B\cos(\lambda x) + C\sinh(\lambda x) + D\cosh(\lambda x)\big)
\end{aligned} \tag{4}$$

Physical quantities ϕ, θ, M, and F on the boundaries of each sub-cell can be organized as a state vector: $\mathbf{v} = \{\phi, \theta, M, F\}^{\mathrm{T}}$. The state vector on the left boundary $\mathbf{v}_{\mathrm{L}}$ and that on the right boundary $\mathbf{v}_R$ of the sub-cell can be connected by a transfer matrix $\mathbf{T}$, according to the general solutions of Eqs. (3) and (4). The transfer matrix is a function of the frequency and sub-cell's thickness. For sub-cell 1, one has $\mathbf{v}_{\mathrm{R}}^{(1)} = \mathbf{T}^{(1)}\mathbf{v}_{\mathrm{L}}^{(1)}$, and sub-cell 2, $\mathbf{v}_{\mathrm{R}}^{(2)} = \mathbf{T}^{(2)}\mathbf{v}_{\mathrm{L}}^{(2)}$. Note that the joint between sub-cell 1 and sub-cell 2 are perfectly bonded, one has the continuity condition $\mathbf{v}_{\mathrm{R}}^{(1)} = \mathbf{v}_{\mathrm{L}}^{(2)}$. So the relationship of the state vector of the left boundary in sub-cell 1 and that of the right boundary in sub-cell 2 can be written as

$$\mathbf{v}_{\mathrm{R}}^{(2)} = \mathbf{T}^{(2)}\mathbf{v}_{\mathrm{L}}^{(2)} = \mathbf{T}^{(2)}\mathbf{v}_{\mathrm{R}}^{(1)} = \mathbf{T}^{(2)}\mathbf{T}^{(1)}\mathbf{v}_{\mathrm{L}}^{(1)} = \mathbf{T}\mathbf{v}_{\mathrm{L}}^{(1)} \tag{5}$$

The transfer matrix $\mathbf{T}^{(1)}$ for the sub-cell 1 is given in Appendix A.