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Overlap amplitude and localization properties in aperiodic diluted and non-diluted direct electric transmission lines

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ABSTRACT

In this work we study the relationship existing between the localization properties of the diluted and non-diluted direct electrical transmission lines with the overlap amplitude $C_{ij}^{\omega} = 2\left|I_i^{\omega} I_j^{\omega}\right|$, where I_j^{ω} is the amplitude of the electric current function at jth cell of the transmission line for the state with frequency ω. We distribute two values of inductances L_A and L_B, according to the generalized aperiodic Thue–Morse m-tupling sequence. We find that the behavior of $C_{i,j}^{\omega}$ is directly related to the localization properties of the aperiodic sequences measured by the ξ normalized participation number, the R_q Rényi entropies and the μ_q moments. In addition, we generalize the scaling relationship for the overlap amplitude $C_{i,j}^{\omega}$, i.e., $\left\langle \left(C_{i,j}^{\omega}\right)^{2q} \right\rangle = \left(\frac{2}{N}\right)^{2q}$.

1. Introduction

The study of localization properties of an aperiodic system is a subject of great interest because these systems present an intermediate behavior between periodic and quasiperiodic systems. The aperiodic Thue–Morse model and its generalizations [1–15] has received great attention in the literature. In addition, the localization behavior of non-periodic electrical transmission lines (TL) has been recently studied [16–24]. In Ref. [24], the generalized Thue–Morse sequence (the m-tupling sequence) has been used to distribute two values of inductances L_A and L_B of the TL in aperiodic way. This aperiodic sequence can be defined using the inflation rule $L_A \to L_A L_B^{m-1}$ and $L_B \to L_B L_A^{m-1}$, where m is an integer $m \geq 2$. The case $m = 2$ corresponds to the usual Thue–Morse sequence. In that reference it was demonstrated that the localization behavior of the usual Thue–Morse sequence ($m = 2$), does not belong to the m-tupling family with $m \geq 3$, because when m changes from $m = 2$ to $m = 3$, the number of extended states drastically diminishes. By increasing the m values, the m-tupling family begins to regain its extended states, in such a way that for $m \gg 3$, the localization behavior of the m-tupling resembles the $m = 2$ case (the usual Thue–Morse case).

In the context of quantum correlation, for the case of two-state systems (qubit), the C_{ij}^{α} concurrence has been proposed as a new way to measure the entanglement [25–27]. Entanglement is a kind of quantum correlation without a classical counterpart, which appears from the direct or indirect interaction between two or more quantum systems. If the systems are correlated, then a measurement process performed on one affects the other, even if individual systems are spatially separated. For the general one-particle state, the entanglement between a pair of qubits, qubit i and j, called pairwise entanglement, can be quantified by the concurrence C_{ij}^{α}, and the entanglement amongst the qubits is then the entanglement between the sites themselves [27,28]. In addition, in the single-particle space the concurrence is named pairwise concurrence, and states that have a large minimum pairwise concurrence can be said to share entanglement better [25]. In general, the one-particle state is the superposition $|\phi(E)\rangle = \sum_{j=1}^{N} \phi_j(E)\,|j\rangle$, where $\phi_j(E)$ is the amplitude of the quantum wave function at jth site for the state with energy E. In this case, the C_{ij}^{E} pairwise concurrence in this state is given by $C_{ij}^{E} = 2\left|\phi_i(E)\,\phi_j(E)\right|$. As an application to the electronic case, the relation between localization properties and entanglement has been recently studied in one-particle states [28–35].

Motivated by this quantum formalism we can define the "overlap amplitude" $C_{ij}^{\omega} = 2\left|I_i^{\omega} I_j^{\omega}\right|$ to describe the overlap between states i and j in the study of classical electric transmission lines. Here I_j^{ω} is the amplitude of the electric current function at jth cell of the transmission line for the state with frequency ω. We study the relationship between localization properties, measured by the

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participation number $P(\omega)$, the normalized participation number $\xi(\omega) = \frac{1}{N} P(\omega)$, the $\mu_q(\omega)$ moments, the $R_q(\omega)$ Rényi entropies, and the power to $2q$ of the overlap amplitude $\left\langle \left(C_{ij}^{\omega}\right)^{2q} \right\rangle$. To do a better comparison between the localization properties and the behavior of the overlap amplitude C_{ij}^{ω}, we study the same model and we use the same numerical values of the parameters of the m-tupling family used in Ref. [24].

This paper is organized as follows. Section 2 describes the model and methods. Section 3 shows the most important numerical results and in Section 4 we give the conclusions of our work.

2. Model and method

2.1. Direct electrical transmission lines

The dynamic equation for the direct diagonal transmission lines formed by horizontal inductances L_j and vertical constant capacitances $C_j = C_0$, $\forall j$ is given by

$$\left(2 - \omega^2 C_0 L_j\right) I_j - I_{j-1} - I_{j+1} = 0 \tag{1}$$

where ω is the frequency. For each ω frequency belonging to the spectrum, we can define the overlap amplitude C_{ij}^{ω} between two sites i and j of the transmission line in the following form

$$C_{ij}^{\omega} = 2\left|I_i^{\omega} I_j^{\omega}\right| \tag{2}$$

This expression defines the overlap amplitude C_{ij}^{ω} between the electric current of two cells of the transmission line, where I_j^{ω} is the amplitude of the electric current function at jth cell of the transmission line for the state with frequency ω and the probability distribution $\left\{\left|I_j^{\omega}\right|^2\right\}$ satisfies the normalization condition $\sum_{j=1}^{N}\left|I_j^{\omega}\right|^2 = 1$. Using the dynamic equation (1), the ω frequencies belonging to the spectrum fulfill the condition $\left|2 - \omega^2 C_0 L_j\right| \leq 2$. For the homogeneous distribution of I_j^{ω}, namely for $I_j^{\omega} = \frac{1}{\sqrt{N}}, \forall j$, we obtain $C_{ij}^{\omega} = \frac{2}{N}$. This case correspond to the fully extended function. On the contrary, for fully localized electric current function, such that $I_j^{\omega} = 1$ and $I_{i\neq j}^{\omega} = 0, \forall i$, the overlap amplitude becomes zero, i.e., $C_{ij}^{\omega} = 0$. As a consequence, for extended and localized $I(\omega)$ electric current function it holds that $C_{ij}^{\omega} \leq \frac{2}{N}$. In this way we can see a close relationship between localization properties and the overlap amplitude. Given that the overlap amplitude C_{ij}^{ω} depends on each pair of sites, for each specific ω frequency, we can define the average overlap amplitude $C_{\omega} = \left\langle C_{ij}^{\omega}\right\rangle$ [28]

$$C_{\omega} = \left\langle C_{ij}^{\omega}\right\rangle = \left\langle 2\left|I_i^{\omega} I_j^{\omega}\right|\right\rangle = \frac{1}{d}\sum_{i<j} C_{ij}^{\omega} \tag{3}$$

where $d = N(N-1)/2$. In addition, we can define the overlap amplitude $C = \langle C_{\omega}\rangle$ averaged over all frequencies belonging to the spectrum in the following way [33]

$$C = \langle C_{\omega}\rangle = \frac{1}{N_{\omega}}\sum_{\omega} C_{\omega} \tag{4}$$

where N_{ω} is the number of frequencies belonging to the spectrum.

2.2. The relationship between the μ_q moments, the R_q Rényi entropies and the C_{ω} overlap amplitude

In what follows, we want to find the general relationship existing between the overlap amplitude $C_{\omega} = \left\langle C_{ij}^{\omega}\right\rangle$, the μ_q moments and the R_q Rényi entropies, as a function of the arbitrary index q. In the first place, let us calculate the power to $2q$ of the overlap amplitude C_{ij}^{ω}, namely

$$\left(C_{ij}^{\omega}\right)^{2q} = 2^{2q}\left|I_i^{\omega}\right|^{2q}\left|I_j^{\omega}\right|^{2q} \tag{5}$$

The average of this quantity, $\left\langle \left(C_{ij}^{\omega}\right)^{2q}\right\rangle$, can be obtained using the definition (3), i.e.,

$$\left\langle \left(C_{ij}^{\omega}\right)^{2q}\right\rangle = \frac{1}{d}\sum_{i<j}\left(C_{ij}^{\omega}\right)^{2q} \tag{6}$$

where $d = N(N-1)/2$. Using (5) we obtain

$$\left\langle \left(C_{ij}^{\omega}\right)^{2q}\right\rangle = \frac{2^{2q-1}}{d}\left(2\sum_{i<j}\left|I_i^{\omega}\right|^{2q}\left|I_j^{\omega}\right|^{2q}\right) \tag{7}$$

Now, let us define the μ_q moments of the electric current function $I(\omega)$ in the usual way, i.e.,

$$\mu_q(\omega) = \sum_{j=1}^{N}\left|I_j^{\omega}\right|^{2q} \tag{8}$$

The squared μ_q moments are given by

$$\left(\mu_q\right)^2 = \left(\sum_{j=1}^{N}\left|I_j^{\omega}\right|^{2q}\right)^2 \tag{9}$$

which can written as

$$\left(\mu_q\right)^2 = \left(2\sum_{i<j}\left|I_i^{\omega}\right|^{2q}\left|I_j^{\omega}\right|^{2q}\right) + \sum_{j=1}^{N}\left|I_j^{\omega}\right|^{4q} \tag{10}$$

The last term of the right side corresponds to the μ_{2q} moments, i.e., $\mu_{2q} = \sum_{j=1}^{N}\left|I_j^{\omega}\right|^{4q}$, then

$$\left(\mu_q\right)^2 = \left(2\sum_{i<j}\left|I_i^{\omega}\right|^{2q}\left|I_j^{\omega}\right|^{2q}\right) + \mu_{2q} \tag{11}$$

Finally, the sum can be expressed as a function of the moments μ_q and μ_{2q},

$$\left(2\sum_{i<j}^{N}\left|I_i\right|^{2q}\left|I_j\right|^{2q}\right) = \left(\mu_q\right)^2 - \mu_{2q} \tag{12}$$

Inserting this result in relation (7), we obtain $\left\langle \left(C_{ij}^{\omega}\right)^{2q}\right\rangle$ as a function of the moments μ_q and μ_{2q}, namely,

$$\left\langle \left(C_{ij}^{\omega}\right)^{2q}\right\rangle = \frac{2^{2q-1}}{d}\left\{\left(\mu_q\right)^2 - \mu_{2q}\right\} \tag{13}$$

This general result is a new indication of the relationship existing between the localization properties contained in the μ_q moments and the overlap amplitude $\left\langle \left(C_{ij}^{\omega}\right)^{2q}\right\rangle$. This general result is still valid for the quantum case of concurrence.

For the special case $q = 1/2$ we obtain [28,30]

$$C_{\omega} = \frac{1}{d}\left\{\left(\sum_{j=1}^{N}\left|I_j^{\omega}\right|\right)^2 - 1\right\} \tag{14}$$