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Discrete parametric oscillation and nondiffracting beams in a Glauber–Fock oscillator

Z. Oztas, C. Yuce *

Physics Department, Anadolu University, Eskisehir, Turkey

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ABSTRACT

We consider a Glauber–Fock oscillator and show that diffraction can be managed. We show how to design arrays of waveguides where light beams experience zero diffraction. We find an exact analytical family of nondiffracting localized solution. We predict discrete parametric oscillation in the Glauber–Fock oscillator.

1. Introduction

Diffraction is a fundamental process in physics. It is well known that a wave is diffracted if it passes through an opening or if there is an obstacle in its path. Diffraction leads to the broadening of an initial intensity profile in space. Propagation-invariant waves have attracted a considerable interest over the years. The most familiar example of nondiffracting wave is that of a plane wave propagating in free space. Other nontrivial nondiffracting waves for various physical systems have been intensively investigated by many authors. Durnin introduced diffractionless (nonspreading) solution of the free-space scalar wave equation almost three decades ago [1]. His solution is in the form of a Bessel function in the transverse direction and the corresponding intensity profile remains invariant during propagation unlike other beams that spread during propagation. In the framework of quantum mechanics, only the plane wave had been known as a nondiffracting wave in 1-D before Berry and Balazs. They theoretically showed another nonspreading solution was available for the Schrodinger equation describing a free particle [2]. Their solution is unique in the sense that it self-accelerates although no external potential exists. The accelerating behavior is not consistent with the Ehrenfest theorem, which describes the motion of the center of mass of the wave packet. The reason why the Ehrenfest theorem doesn't work is because of the nonintegrability of the Airy function.

The physics of photon propagation in discrete lattices is very rich and has been extensively studied by many authors. Nondiffracting waves in media such as waveguide or nonlinear materials are interesting to study. Diffraction in such an array of waveguides is governed by hopping light from site to site through optical tunneling. Discrete diffraction is different from continuous diffraction. As an example, if light is initially excited at only one waveguide of a 1-D periodic array of waveguide, then light spreads into two main lobes with several secondary peaks between them. The idea to control discrete diffraction has attracted a special attention. The case of optical field propagation in a linearly coupled, infinite array of one dimensional waveguides was considered in [3] and anomalous diffraction (negative discrete diffraction) and diffraction-free cases were theoretically discussed and experimentally realized. Discrete diffraction was shown to be controlled in size and sign by the input conditions and diffractionless beams and focusing of normally diverging beams were discussed in homogeneous waveguide arrays [4]. The self-collimation effect where the spatial width of a light beam does not change over hundreds of free-space diffraction lengths was realized in a macroscopic photonic lattice [5]. Periodic photonic structures where the strength of diffraction can be made normal, anomalous or zero in a very broad frequency range were introduced [6].

One special system where discrete diffraction can be studied is the semi-infinite and asymmetric Glauber Fock lattice that has recently been introduced into optics community [7,8]. The system is composed of an array of evanescently coupled waveguides with a square-root distribution of the coupling between adjacent guides [7]. The first experimental realization with a direct observation of the classical analogue of Fock state displacements was presented in [8]. The Glauber–Fock photonic lattice is interesting in the sense that every excited waveguide represents a Fock state and the system admits an exact analytical solution. In [9], the Ermakov–

* Corresponding author.
E-mail address: cyuce@anadolu.edu.tr (C. Yuce).

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Lewis invariant is constructed for the Glauber–Fock oscillator with propagation distance dependent tunneling amplitude and refractive index gradients. Some interesting effects can be observed in a Glauber–Fock system in the presence of the refractive index gradient, which was shown to be accomplished by varying the waveguide writing velocity of the femtosecond laser [10]. For example both periodic collapses and revivals in a discrete Glauber–Fock oscillator were observed in the intensity evolution. It was shown that periodically changing tunneling amplitude leads to Bloch-like oscillations and dynamic delocalization depending on the oscillation frequency and strength of refractive index gradient [11]. It is interesting to observe the Bloch-like oscillations in spite of the fact that the evanescently coupled waveguide array has nonuniform coupling and semi-infinite. Dynamic localization and quantum self-imaging are other interesting effects that occur in periodical lattice. These two effects were theoretically discussed and shown to be possible in a Glauber–Fock oscillator [12]. Glauber–Fock oscillator was shown to be engineered by the method of shortcuts to adiabaticity [13]. Recently, geometric phase for a Glauber–Fock oscillator lattice was measured [14]. Quantum Rabi model based on light transport in two decoupled semi infinite binary tight binding photonic lattice with a square-root distribution of the coupling between neighboring sites was experimentally realized in [15]. The standard Glauber–Fock oscillator is semi-infinite and all the experiments mentioned above were realized in an effectively semi infinite system (the dimension in the transverse direction is long enough). The effect of truncation in a finite Glauber–Fock oscillator was discussed in [16]. The Glauber–Fock oscillator was generalized to include the nonlinear interaction and the corresponding system was theoretically explored in [17]. In this paper, we consider Glauber–Fock oscillator with propagation distance dependent tunneling amplitude and refractive index gradient. The purpose of this paper to find a way to obtain difractionless propagating initial excitations. We will find a solution that is capable of maintaining its spatial form during propagation. In this way, engineered diffraction can be realized. Secondly, we study parametric oscillation in the system.

2. Model

We consider a semi-infinite Glauber–Fock oscillator array consisting of evanescently coupled waveguides. The tunneling amplitude through which particles are transferred from site to site increases with the square root of the site number n. We suppose that tunneling amplitude and linearly increasing refractive index gradient are z-dependent, where z is the normalized propagation distance. The equation satisfied by the complex field amplitude at the n-th waveguide is given by [9]

$$i\partial_z c_n + F\, n\, c_n + J(\sqrt{n+1}c_{n+1} + \sqrt{n}c_{n-1}) = 0 \tag{1}$$

where $n = 0, 1, 2, ...,$ $J = J(z)$ is the z-dependent first order tunneling amplitude, $F = F(z)$ is the z-dependent refractive index gradient and c_n is the field amplitude at the n-th waveguide. Note that $c_n(z) = 0$ for $n < 0$. Therefore our system is semi-infinite and asymmetric.

To find the solution, we follow the method introduced in [9]. Let us write the state vector as $|\psi\rangle = \sum_{n=0}^{\infty} c_n(z)|n\rangle$, where the Fock state $|n\rangle$ corresponds to situation when only the waveguide with number n is excited [8]. Substituting this solution into the equation (1) yields the Schrodinger equation $H\psi = i\frac{\partial\psi}{\partial z}$ with $\hbar = 1$. The corresponding Hamiltonian reads

$$H = -\left(F(z)\hat{n} + J(z)\left(\hat{a} + \hat{a}^\dagger\right)\right) \tag{2}$$

where the bosonic creation and annihilation operators satisfy $\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ and $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$, respectively and the number operator satisfies $\hat{n}|n\rangle = n|n\rangle$. We can transform this Hamiltonian using $\hat{a} = \frac{q+ip}{\sqrt{2}}$ and $\hat{a}^\dagger = \frac{q-ip}{\sqrt{2}}$, where q and p normalized position and momentum operators, respectively. Then the Hamiltonian can be rewritten in the following form

$$H = -\left(\frac{p^2}{2m} + \frac{m}{2}\omega^2 q^2 + \sqrt{2}Jq - \frac{F}{2}\right) \tag{3}$$

where the z-dependent mass and frequency are defined by $m = 1/F(z)$ and $\omega^2 = F(z)^2$.

The exact analytical solution of this Hamiltonian is available if we change $z \to Z = -z$. This is the Hamiltonian of a quantum harmonic oscillator with Z-dependent mass, frequency, and external driving force. Let us now obtain an exact analytical solution. We first transform the coordinate according to $q' = \frac{q-q_c}{L}$, where the Z-dependent function $q_c(Z)$ describes translation and $L(Z)$ is a Z-dependent dimensionless scale factor to be determined later. More precisely, the center of the wave packet moves according to $q_c(Z)$ and the width of the wave packet changes according to $L(Z)$. Under this coordinate transformation, the Z-derivative operator transforms as $\partial_Z \to \partial_Z - L^{-1}(\dot{L}q' + \dot{q_c})\partial_{q'}$, where dot denotes derivation with respect to Z. In the accelerating frame, we will seek the solution of the form

$$\psi_n(q', Z) = \exp(i\Lambda)\,\frac{\phi_n(q', Z)}{\sqrt{L}} \tag{4}$$

where the position dependent phase reads $\Lambda(q', Z) = m(\alpha q' + \frac{\beta}{2}q'^2 + S)$, α, β and S are Z-dependent functions to be determined. Substitute this ansatz into the corresponding Schrodinger equation and demand that the resulting equation includes harmonic and linear potential terms. Therefore we choose $\alpha = L\dot{q_c}$, $\beta = L\dot{L}$ and $\dot{S} + \frac{\dot{m}}{m}S = \frac{1}{2}\dot{q_c}^2 - \frac{\omega^2}{2}q_c^2 - \frac{\sqrt{2}Jq_c}{m} + \frac{F}{2m}$. The resulting equation reads

$$-\frac{1}{2mL^2}\frac{\partial^2\phi}{\partial q'^2} + (\frac{m}{2}\Omega^2 q'^2 + Uq')\phi = i\frac{\partial\phi}{\partial Z} \tag{5}$$

where $\Omega^2 = L(\ddot{L} + \frac{\dot{m}}{m}\dot{L} + \omega^2 L)$ and $U = mL(\ddot{q_c} + \frac{\dot{m}}{m}\dot{q_c} + \omega^2 q_c + \sqrt{2}\frac{J}{m})$. We can now determine L and q_c.

$$\ddot{L} + \frac{\dot{m}}{m}\dot{L} + \omega^2 L = \frac{1}{m^2L^3} \tag{6}$$

$$\ddot{q_c} + \frac{\dot{m}}{m}\dot{q_c} + \omega^2 q_c + \sqrt{2}\frac{J}{m} = 0 \tag{7}$$

The former one (known as the Ermakov equation) is easy to solve for the initial condition $\dot{L}(Z=0) = 0$. It is given by $L(Z) = 1$. Therefore, the quadratic term in $\Lambda(q', Z)$ disappears. The solution of the latter equation will be discussed below.

With the choices (6), (7), the linear potential is eliminated from the equation (5) and the resulting equation for $\phi_n(q')$ can be solved analytically. It is given by

$$\phi_n(q', Z) = N_n \exp\left(i\int \frac{E_n}{mL^2}dZ - \frac{q'^2}{2}\right) H_n(q') \tag{8}$$

where $E_n = (n + \frac{1}{2})$ and H_n are the Hermite polynomials and N_n is the normalization constant. Transforming backwards yields the exact solution. We have analytically found the exact solution. Our solution and the one in [9] are equivalent but our solution is advantageous since it is written in terms of the width and the center of mass of the wave packet. As we shall see below, the equation (7) enables us to see the dynamics of the system clearly.

Let us now write the exact solution

$$\psi_n = N_n \exp\left(im\dot{q_c}q + i\epsilon_n - \frac{(q-q_c)^2}{2}\right) H_n(q - q_c) \tag{9}$$