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Effect of small scale motions on dynamo actions generated by the Beltrami-like flows

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ABSTRACT

The geodynamo and solar dynamo are driven by the turbulent flows which involve motions of various scales. Of particular interest is what role is played by the small scale motions in these dynamos. In this paper, the integral equation approach is employed to investigate the effect of the small scale motions on dynamo actions driven by multiscale Beltrami-like flows in a cylindrical vessel. The result shows that some small scale motions can trigger a transition of a dynamo from a steady to an unsteady state. Our results also show that when the poloidal components of the small and large scale flows share the same direction in the equatorial plane, the small scale flows have more positive or less detrimental effect on the onsets of the dynamo actions in comparison with the case that the poloidal components have different directions. These findings shed light on the effect of the small scale turbulence on dynamo actions.

1. Introduction

It is believed that the origin of the magnetic fields of most celestial bodies is the dynamo action which converts the kinetic energy of the electrical conducting fluids to the electromagnetic energy [1,2]. Usually, these dynamos are driven by the electrically conducting turbulent flows [3]. It is well known that the turbulent flow involves motions of various scales. An important question is what effect is of the small scale turbulent flows on the dynamo actions.

Recently, Ponty and Plunian [4] investigated the dynamo action driven by a helical forcing corresponding to the Roberts flow, their numerical results showed that the turbulence has weak effect on the mean-flow dynamo onset, and beyond the onset of the large scale dynamo, the further increase of the magnetic Reynolds number can give rise to a small scale dynamo. Peyrot et al. [5] have studied the dynamo driven by a helical flow made of mean flow plus a fluctuating one, and found that the dynamo threshold depends on the frequency and the strength of the fluctuating flow. In [6], an asymptotic method was developed for giving the growth rates and frequencies of the oscillating Ponomarenko dynamo in the highly conducting limit of large magnetic Reynolds number. Leprovost et al. [7] found that a modified Bullard dynamo can be modeled by a nonlinear oscillator subject to a multiplicative noise. Based on this model, they have investigated the bifurcation phenomena of the modified Bullard dynamo. Priede et al. [8] showed that a Bullard-type disc dynamo can be explained by a parametric resonance mechanism when the disc rotation rate is subject to harmonic oscillations. Laval et al. [9] found that for the dynamo action driven by a time-dependent Taylor–Green force the addition of the small scale noise to the mean velocity field can significantly increase the critical magnetic Reynolds number. Pétrélis and Fauve [10] found that the fluctuations of the phase of a cellular flow always impede the dynamo process. Bayliss et al. [11] performed a numerical simulation of a mechanically forced turbulent flow, and found that when the turbulence becomes strong, the dynamo action driven by the large scale motion is suppressed. Gissinger [12] numerically investigated the Taylor–Couette dynamo, and showed that the small scale turbulent flow tends to increase the critical magnetic Reynolds number.

It is well known that the numerical simulation becomes a powerful tool for investigating the hydromagnetic dynamo actions. Especially for the geodynamo the numerical simulations have revealed many features of the geomagnetic fields, such as the axial dipole structure and geomagnetic reversals [13–16]. However, direct numerical simulations are not able to reach the realistic parameter regime with the power of today's supercomputers [16]. Therefore, laboratory dynamo experiment is another option of exploring the geodynamo, although it is also not possible to fulfill all of the dimensionless numbers in laboratory.

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In the past decade, three liquid sodium dynamo experiments in Riga, Karlsruhe, and Cadarache [17–19] have successfully demonstrated dynamo actions. However, in the Riga and Karlsruhe dynamo experiments, the laminar flows play the dominant role and suffer many constraints, which are different from the natural dynamos, such as the geodynamo. Although in the VKS experiment carried out in Cadarache (France) the fully turbulent flow is used to drive the dynamo action, the generated magnetic field is a pure toroidal magnetic field [20], and the dynamo is successful only when one of the impellers driving the flow is made of the soft-iron [21]. Furthermore, the liquid metal is usually used as the working fluid in the current dynamo experiments. Unfortunately, it is still lack of the reliable measurement technique for measuring the velocity field of the liquid metal flow.

In order to avoid touching the thorny turbulent flow which is still one of the unsolved problems in classical physics, Tilgner [22] used a 2D periodic flow comprising of periodic arrays of helical eddies with various scales to approximately model the turbulent flow, and found that the small scale eddies have a detrimental effect on the dynamo action driven by the large scale eddies, except in some appropriate geometry, the small scale eddies can be more efficient for driving a dynamo action than the large scale eddies. In the present work, the integral equation approach is employed to investigate the dynamo actions driven by some multiscale Beltrami-like flows in a cylindrical vessel. The main focus is on the effect of small scale motions on the dynamos.

2. Integral equation approach

Consider the dynamo action to be driven by the flow of an electrically conducting fluid with the electrical conductivity σ and magnetic permeability μ. The fluid occupies the region D. Outside D is the insulating region denoted as D' which extends to infinity. The magnetic field $\mathbf{B}$ satisfies the following induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\mu\sigma}\triangle \mathbf{B} \tag{1}$$

where $\mathbf{u}$ is the velocity field. t the time, ∇ the gradient operator, $\triangle$ the Laplace operator. Furthermore, the magnetic field satisfies the following conditions:

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\mathbf{B} = O(r^{-3}),\ \text{as } r = |\mathbf{r}| \to \infty \tag{3}$$

Equations (1)–(3) describe the induction process in terms of the partial differential equations.

In recent years, we have developed an integral equation approach to simulating dynamo actions [20,23–28]. This approach has been examined by the Riga, Karlsruhe, and VKS experiments [20,23,24,29]. In this approach, the governing equations of the induction process in terms of the partial differential equations are firstly changed to the following integral equations [24,25]:

$$\mathbf{B} = \frac{\mu\sigma}{4\pi}\left[\int_D \frac{(\mathbf{u}\times\mathbf{B})\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dv - \frac{\partial}{\partial t}\int_D \frac{\mathbf{A}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dv - \int_S \phi\mathbf{n}\times\frac{\mathbf{r}-\mathbf{s}'}{|\mathbf{r}-\mathbf{s}'|}ds\right] \tag{4}$$

$$\frac{1}{2}\phi = \frac{1}{4\pi}\left[\int_D \frac{(\mathbf{u}\times\mathbf{B})\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dv - \frac{\partial}{\partial t}\int_D \frac{\mathbf{A}\cdot(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dv - \int_S \phi\mathbf{n}\cdot\frac{\mathbf{r}-\mathbf{s}'}{|\mathbf{r}-\mathbf{s}'|^3}ds\right] \tag{5}$$

$$\mathbf{A} = \frac{1}{4\pi}\left[\int_D \frac{\mathbf{B}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dv + \int_S \mathbf{n}\times\frac{\mathbf{B}}{|\mathbf{r}-\mathbf{s}|}ds\right] \tag{6}$$

where $\mathbf{A}$ and ϕ are the magnetic vector potential and electrical potential, respectively.

For the velocity field, we consider the Beltrami-like flows in a finite cylinder with radius R and height $H = 2h$. In the following, the spatial variables are normalized by the radius R, and we set $h/R = 1.0$. In dynamo theory, there is a particular interest in such flows for they are helicity maximizing. Therefore, they are likely to have small critical Reynolds number. We use the notation $s_m^{\pm}t_n$ to characterize flows with m poloidal vortices and n toroidal vortices. The sign $\pm$ indicates that the poloidal flow in the equatorial plane is directed inward (+) or outward (-), respectively. These flows are expressed in the cylindrical coordinate system $[\rho, \varphi, z]$ as follows:

$$u_\rho = c_1 J_1(\gamma\rho)\cos(m\pi(z+h)/(2h)), \tag{7}$$

$$u_\varphi = 2J_1(\gamma\rho)\sin(n\pi(z+h)/(2h)), \tag{8}$$

$$u_z = -c_1 c_2\,\gamma/\pi\ J_0(\gamma\rho)\sin(m\pi(z+h)/(2h)), \tag{9}$$

where $\gamma = 3.8317$ is the first root of the Bessel function J_1, $c_1 = 1$ for all $s_m^+t_n$ flows, $c_1 = -1$ for all $s_m^-t_n$ flows, $c_2 = 2h/m$, $z \in [-h, h]$. In the following, we shall take the $s_1^{\pm}t_1$ or $s_2^{\pm}t_2$ as the large scale flow and the $s_m^{\pm}t_m$ with larger m as the small scale flows. Thus the multiscale Beltrami-like flow to be considered is written as

$$\begin{aligned} u_\rho &= c_1\ J_1(\gamma\rho)\cos(k\pi(z+h)/(2h)) \\ &\quad + f\,d_1\ J_1(\gamma\rho)\cos(m\pi(z+h)/(2h)), \end{aligned} \tag{10}$$

$$\begin{aligned} u_\varphi &= 2\ J_1(\gamma\rho)\sin(k\pi(z+h)/(2h)) \\ &\quad + 2\ f\ J_1(\gamma\rho)\sin(n\pi(z+h)/(2h)), \end{aligned} \tag{11}$$

$$\begin{aligned} u_z &= -c_1\,c_2\,\gamma/\pi\ J_0(\gamma\rho)\sin(k\pi(z+h)/(2h)) \\ &\quad - f d_1\,d_2\,\gamma/\pi\ J_0(\gamma\rho)\sin(m\pi(z+h)/(2h)), \end{aligned} \tag{12}$$

where $k = 1$ or $k = 2$, $m > 1$ for $k = 1$ or $m > 2$ for $k = 2$, f is the magnitude of the small scale motion, $d_1 = 1$ for all $s_m^+t_n$ flows, $d_1 = -1$ for all $s_m^-t_n$ flows, $d_2 = 2h/m$, $z \in [-h, h]$. It is easy to show that the above velocity field is divergence free. Note that the length scale of the small scale flow is inversely proportional to m appearing in the above express of the velocity field. Therefore, we call the Beltrami-like flow with the larger m as the small scale flow. In some sense, this multiscale Beltrami-like flow reflects the feature of turbulent flow which also comprises the large scale and small scale flows. Based on the velocity field under consideration, the magnetic Reynolds number is defined as

$$R_m = \mu\sigma R\sqrt{2E},\ E = \int_D 0.5(v_\rho^2 + v_\varphi^2 + v_z^2)dV \tag{13}$$

Since the multiscale Beltrami-like flows under consideration are steady, the magnetic field can be expressed as $\mathbf{B}(\mathbf{r})exp(\lambda t)$ where the real part of λ is the growth rate of the magnetic field, its imaginary part is the frequency. Therefore, Eqs. (4)–(6) are rewritten as follows:

$$\mathbf{B} = \frac{\mu\sigma}{4\pi}\left[\int_D \frac{(\mathbf{u}\times\mathbf{B})\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dv - \lambda\int_D \frac{\mathbf{A}\times(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}dv - \int_S \phi\mathbf{n}\times\frac{\mathbf{r}-\mathbf{s}'}{|\mathbf{r}-\mathbf{s}'|}ds\right] \tag{14}$$