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Scalable quantum computing in decoherence-free subspaces with Cooper-pair box qubits

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ABSTRACT

We theoretically propose a feasible scheme to perform quantum computing in decoherence-free subspaces (DFSs) with Cooper-pair box (CPB) qubits arrayed in a circuit QED architecture. Based on the cavity-bus assisted interaction, the selective and controllable interqubit couplings occur only by adjusting the individual gate pulses, by which we obtain the scalable DFS-encoded universal quantum gates to resist certain collective noises. Further analysis shows the protocol may implement the scalable fault-tolerant quantum computing with current experimental means.

1. Introduction

Due to potential advantages such as convenient control, flexible design and accessible scalability, superconducting Josephson qubits have been identified as promising candidates for quantum information processing [1–3]. As elementary superconducting qubits, Cooper pair box (CPB) systems are insensitive to the dephasing effects caused by the first-order charge noise, and then Vion et al. experimentally obtained a high quality factor of quantum coherence Q_φ [4]. Recently, circuit quantum electrodynamics (QED) emerged in superconducting nanocircuits further inspired the rapid development of quantum information science [5–9]. This is mainly because transmission line resonator in the circuit QED can effectively generate quantized cavity field, which gives rise to the stronger couplings between the quantized field and the artificial atoms in contrast with the conventional cavity QED [10–12].

In the field of superconducting quantum computing, two crucial issues have attracted considerable attention. One is how to get the fault-tolerant quantum computing. As is well known, solid-state circuits easily interact with the environmental noises, which impair greatly the capability of the qubits to process quantum information. Many valuable strategies to fight against decoherence effects have been put forward, such as geometric quantum computation [13,14], optimal control approach [15,16], topologically protected qubits [17,18]. Particularly, decoherence-free subspace (DFS) encoding as quantum error avoiding way is an interesting treatment [19–25]. Another issue is how to scale up to many qubits. To achieve this goal, some theoretical proposals and experimental attempts have been investigated, in which cavity-bus assisted couplings open the novel opportunities to address the spatially remote qubits [26–32]. However, towards physically implementing a practical quantum computer, one should effectively combine the above two aspects to realize the fault-tolerant quantum operations on multiqubit systems.

Motivated by performing the scalable fault-tolerant quantum computing, in this Letter we propose a theoretical scheme to realize the quantum computing in DFSs with CPB qubits. Many CPB systems acting as effective three-level atoms (TLAs) are arrayed in a circuit QED that plays the role of quantum data bus. Under the irradiation from gate pulses, the controllable couplings between any pair of selected qubits dispersively coupled to cavity-bus can be achieved via Raman transitions. Based on the circuit QED-assisted interqubit couplings, we implement the universal gate operations on the DFS-encoded logic qubits to eliminate some kinds of col-

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lective noises, as well as obtain the scalable quantum gates in the DFSs. The scheme provides the possibility to obtain the scalable fault-tolerant quantum computing with CPB qubits.

This Letter is organized as follows. In Section 2, we present CPB circuit as an effective TLA. In Section 3, circuit QED-assisted coupling between any pair of CPB qubits is studied. The DFS-encoded logic gates are given in Section 4. In Section 5, we analyze the scalability of quantum computing in the DFSs. Finally, discussions and conclusions are drawn in Section 6.

2. CPB system as an effective three-level atom

The CPB circuit under consideration, as shown in Fig. 1, consists of a superconducting box with extra Cooper-pairs n. The box is coupled by two symmetric Josephson junctions each with capacitance C_J and coupling energy E_J to a segment of a superconducting ring. The characteristic parameters satisfy [3,4] $\Delta \gg E_c \sim E_J \gg k_B T$, where the energy gap Δ is large enough to suppress the quasiparticle tunneling, E_c is the charging energy with the same order of E_J, and $k_B T$ represents the lower thermal excitation. To achieve the adjustable Josephson coupling, an external magnetic flux Φ_x threads the ring with a small inductance L. Through the gate capacitance C_g, a bias voltage $V = V_d + \tilde{V}_a$ is applied to the box, where the dc component V_d changes the static system levels by inducing offset charges, and ac one $\tilde{V}_a$ as a classical microwave pulse aims at coupling the system levels.

Without the irradiation from classical microwave field, the static system Hamiltonian is given by $H_0 = E_c(n - n_d)^2 - \bar{E}_J \cos\theta$, where the first term is the electrostatic energy, and the second one is the Josephson coupling. Here the charging energy scale is $E_c = 2e^2/C_t$, with $C_t = (C_g + 2C_J)$ being the total capacitance of the box, and $n_d = C_g V_d/2e$ represents the gate charges induced by dc voltage. The effective Josephson energy is $\bar{E}_J = 2E_J \cos(\varphi/2)$, where $\varphi = 2\pi\Phi_t/\Phi_0$ characterizes the total phase difference, $\Phi_t = \Phi_x + LI_s$ is the total flux, with LI_s being the induced flux by supercurrent I_s, and $\Phi_0 = h/2e$ stands for the flux quantum. The average phase difference θ of the two junctions is conjugate to the extra number n, this yields the relation of $[\theta, n] = i$. Therefore, within the Cooper-pair state representation $\{|n\rangle, |n+1\rangle\}$, the static Hamiltonian can be written as [33,34]

$$H_0 = \sum_n \left[E_c(n - n_d)^2 |n\rangle\langle n| - \frac{\bar{E}_J}{2}\big(|n\rangle\langle n+1| + \text{h.c.}\big)\right]. \tag{1}$$

In terms of Eq. (1), we have chosen the appropriate parameters and numerically calculated the first four levels versus gate charge n_d (see Fig. 2). At bias point $n_d = 0.5$, we denote the three lower levels by $|s_j\rangle$, which are the superpositions of many Cooper-pair states, $|s_j\rangle = \sum_n c_{jn}|n\rangle$, with c_{jn} being superposition coefficients, $j = 1, 2$ and 3. The result demonstrates that the third level $|s_3\rangle$ is well separated from the fourth one, and then the CPB system can be considered as an artificial three-level atom (TLA) effectively. The two lowest levels $|s_1\rangle$ and $|s_2\rangle$ compose qubit eigenbasis physically, and $|s_3\rangle$ is the ancillary state, respectively.

Note that the reasons for choosing dc bias at the magic point ($n_d = 0.5$) are as follows. Since the qubit eigenbasis at magic point is decoupled from the first-order charge noise, we should keep the advantage to combat the dephasing effects as much as possible. On the other hand, the selection rules determined by the parity symmetry of energy levels [8,35] do not impede the desired transitions induced by the external fields. As will be discussed below, the quantized cavity field causes the coupling $|s_1\rangle \leftrightarrow |s_3\rangle$, and the transition $|s_2\rangle \leftrightarrow |s_3\rangle$ can be realized via classical microwave pulse.

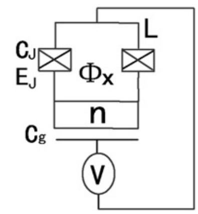

Fig. 1. Schematic diagram of the considered CPB circuit.

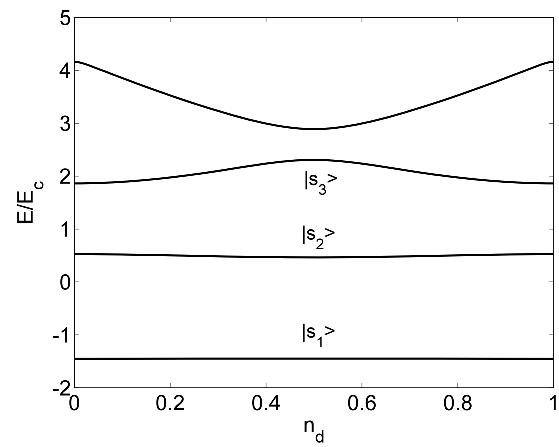

Fig. 2. The first four levels of the CPB system as a function of static gate charges n_d for the fixed $\bar{E}_J = 2.5E_c$, energies are given in units of E_c ($= 14.0$ GHz). At bias point $n_d = 0.5$, the CPB system containing the three lower levels $|s_1\rangle$, $|s_2\rangle$ and $|s_3\rangle$ forms a TLA.

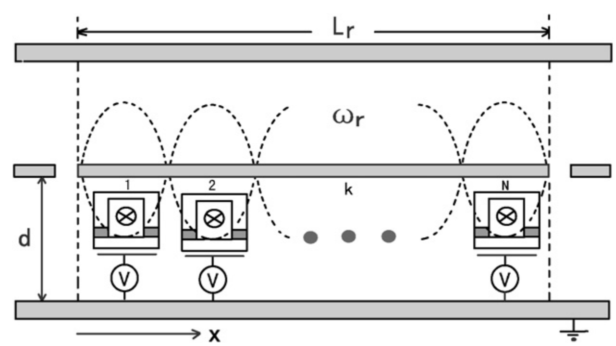

Fig. 3. Schematic representation of many CPB circuits arrayed in a circuit QED setup.

3. Circuit QED-assisted interqubit couplings

As schematically shown in Fig. 3, many CPB circuits are arrayed in a high-Q circuit QED architecture, in which the strip-line resonator generates single-mode cavity field ω_r [25,29,30,36]. The distance between the line connected with circuits and the center line is d, and L_r denotes the geometric length of the one-dimensional transmission line resonator (along the x direction). As a quantum data bus, circuit QED interacts with each TLA simultaneously. For a standing-wave cavity field, appropriate boundary condition of the coplanar line resonator makes the electric field zero at the antinodes. Hence the maximal magnetic coupling strengths between the CPB systems and the cavity mode can be achieved when systems are situated at the antinodes of the magnetic field. Since the linear dimension of each CPB is much smaller than the wavelength [37], we can consider CPB circuits that are located at the antinodes approximately.

The Hamiltonian of the single-mode cavity field is described by $H_r = \hbar\omega_r(a^\dagger a + 1/2)$, where $a^\dagger(a)$ is photon creation (annihilation) operator. The kth circuit is coupled inductively to the cavity field,