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Efficient generation of maximally entangled states via four-wave mixing in a semiconductor quantum-dot nanostructure

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ABSTRACT

We investigate the generation of the maximally entangled state of two weak-light pulses (the probe and generated pulses) via four-wave mixing (FWM) in a semiconductor quantum dot (SQD) with a biexciton–exciton cascade configuration. The results show that this maximally entangled state can propagate with an ultraslow group velocity under suitable parameter conditions. For application, our proposed scheme is probably achievable with the present technology by applying the standard GaAs/InGaAs self-assemble quantum dots (QDs). Furthermore, our calculations provide a guideline for the realization of the maximally entangled state in the SQD solid-state system, which can be much more practical than that in an atomic system because of its flexible design and the wide tunable parameters.

1. Introduction

Quantum entanglement, as one of the most important resources in quantum information processing, has generated a great deal of interest in the past few years. The entangled state which is the basis of the quantum information protocols, such as quantum cryptography [1–3], quantum teleportation [4–6], and quantum computing [7–9], can be generated in various quantum systems [10–17], including trapped ions [14], cavity quantum electrodynamics [15], semiconductor quantum dots (SQDs) [16], and so on. In the previous schemes, the preparation of entangled states is often realized using spontaneous parametric down-conversion (SPDC) [18–20], which has been envisaged as one of the efficient ways to produce entanglement, but its conversion efficiency is very low. Recently, it is proved theoretically and experimentally that four-wave mixing (FWM) is another effective way for the generation of entanglement [21–25]. For example, an ultra-slowly propagating maximally entangled state of two light beams via FWM in a double-Λ system [21], the generation of polarization-entangled photon pairs using spontaneous FWM in a fiber loop [22], the entanglement of two-mode fields generated from FWM with the help of auxiliary atomic transition [23], the preparation of multiparty entangled states using pairwise perfectly efficient single-probe photon FWM [24], and quantum entanglement of Fock states with FWM [25] have been reported.

As mentioned above, SQDs, regarded as "artificial atoms", appear to be good candidates to implement a solid state source of entangled state [26–28]. In recent years, the rapid evolution of quantum dot (QD) studies has paved the way for applications in quantum information processing. Single QDs are particularly appealing since they are fixed in place, scalable, and have long coherence times [26]. So far, many scholars have done a lot of research in this area. The generation of entangled photon pairs in a QD has been reported [26,27]. Chen et al. have already demonstrated the entanglement of excitons in a single QD [28]. The generated quantum logic gate using the biexciton–exciton correlation in a single GaAs/AlGaAs QD has been demonstrated [29]. An optically controlled device utilizing excitons in a semiconductor nanostructure [30,31] is a promising candidate for solid-state quantum entanglement. From this point of view, we propose a scheme with QD as a source of entanglement, and the single QD is described as a four-level double-cascade configuration.

In this Letter, we use the general theory of FWM in the biexciton–exciton cascade QD. FWM is a third-order optical nonlinear process, in which two planar counter-propagating pump waves interact with a probe wave in a nonlinear medium and produce the fourth wave, i.e., the so-called mixing wave. Here, we use a single QD as the nonlinear coherent solid medium. Based on the coupled Maxwell–Liouville equations, analytical and explicit expressions are well developed to describe the generation of entan-

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gled state, as well as its ultraslow propagating group velocity. To the best of our knowledge, no related theoretical or experimental work has been carried out to study the efficient generation of the entangled state of two weak-light pulses with an ultraslow group velocity via FWM in such a SQD system with biexciton–exciton cascade configuration, which motivates the current work.

The remainder of this Letter is organized into three parts as follows. In Section 2, we establish the theoretical model under study, and derive the equations of motion for the electron density operator and the quantized electromagnetic fields. In Section 3, the analytical expressions of the two quantized pulsed fields including group velocities, phase shifts and absorption coefficients are solved numerically and analyzed by taking some realistic experimental parameters. Finally, our main conclusions are summarized in Section 4.

2. Model and equations of the SQD system

The scheme under consideration is a lifetime broadened four-level double-cascade SQD system, as depicted in Fig. 1. In our system, the SQD is naturally formed in a narrow (4.2 nm) GaAs layer grown between two 25-nm $Al_{0.3}Ga_{0.7}As$ barriers and is probed through submicron apertures at 6 K [32–34]. The GaAs substrate was removed to allow transmission. The energy levels $|0\rangle, |1\rangle$ and $|2\rangle, |3\rangle$ are the crystal ground state, intermediate states, and excited state of the four-level cascade configuration, respectively. The value of the biexciton binding energy ΔE is several meV [35]. For more details on this SQD system, we refer the reader to Refs. [32–35] and references therein. Effects of the electron–electron interactions are expected to be rather weak in the present analysis, as a result, many body effects arising from electron–electron interactions are not included in our study. This method has described quantitatively the results of several experimental papers [36–39] and has been used in several theoretical papers [40–43]. In fact, the effects of electron–electron interactions in the dynamics of intersubband transitions in SQWs have been studied in several recent publications, see e.g. [44–47]. These works have shown that this dynamics can be significantly altered but for much larger electron doping than those of interest here. The sample interacts with two continuous-wave (cw) laser pump fields c, d and a single-photon pulsed probe field p, and a radiation field e is generated. From the model of energy levels, we can see that these fields p, c, and d can induce FWM among energy levels $|0\rangle \xrightarrow{\hat{E}_p} |1\rangle \xrightarrow{\Omega_c} |3\rangle \xrightarrow{\Omega_d} |2\rangle \xrightarrow{\hat{E}_e} |0\rangle$, thus the quantized radiation field e can be generated efficiently. Due to the effect of strong coupling fields, the electromagnetically induced transparency (EIT) effect can be produced among energy levels $|0\rangle \to |1\rangle \to |3\rangle \to |2\rangle$. Thereby, the absorption of the probe field becomes very weak because of the EIT. Furthermore, the group velocity of the probe field decreases sharply due to the existence of EIT, and keeps consistent with the group velocity of the generated field.

Under the rotating-wave approximation (RWA) and the electric-dipole approximation (EDA), according to the spirit of Refs. [48–50] via choosing the proper free Hamiltonian, turning to the interaction picture, with the assumption of $\hbar = 1$, the resulting interaction Hamiltonian describing the SQD system can be given by

$$\begin{aligned}\hat{H} = {} & \Delta_p|1\rangle\langle 1| + (\Delta_p + \Delta_c - \Delta_d)|2\rangle\langle 2| + (\Delta_p + \Delta_c)|3\rangle\langle 3| \\ & - \left(\hat{\Omega}_p e^{i\mathbf{k}_p\cdot\mathbf{r}}|1\rangle\langle 0| + \hat{\Omega}_e e^{i\mathbf{k}_e\cdot\mathbf{r}}|2\rangle\langle 0| + \text{H.c.}\right) \\ & - \left(\Omega_c e^{i\mathbf{k}_c\cdot\mathbf{r}}|3\rangle\langle 1| + \Omega_d e^{i\mathbf{k}_d\cdot\mathbf{r}}|3\rangle\langle 2| + \text{H.c.}\right), \end{aligned} \tag{1}$$

where we designate by H.c. the hermitian conjugate. $\Delta_p = \omega_1 - \omega_p$, $\Delta_c = \omega_3 - \omega_1 - \omega_c$, and $\Delta_d = \omega_3 - \omega_2 - \omega_d$ are the frequency detunings, with $\hbar\omega_j$ being the energy of the subband level $|j\rangle$ $(j = 1, 2, 3)$. $\omega_{c(d)}$ and $2\Omega_{c(d)}$ are the carrier frequency and Rabi frequency of the classical pump laser field $c(d)$, and $2\hat{\Omega}_{p(e)} = \mu_{10(20)}\hat{E}^{(+)}_{p(e)}/\hbar$ is the Rabi frequency operator for the quantized probe (radiation) field with frequency $\omega_p(\omega_e)$, here $\mu_{10} = \vec{\mu}_{10}\cdot\vec{e}_L$ $(\mu_{20} = \vec{\mu}_{20}\cdot\vec{e}_L)$ ($\vec{e}_L$ is the unit polarization vector of the corresponding laser field) denotes the interband dipole moment for the transition between subbands $|1\rangle$ and $|0\rangle$ ($|2\rangle$ and $|0\rangle$). $\mathbf{k}_j$ $(j = p, c, d$, and $e)$ is the wave vector of the corresponding laser field.

Adopting the one-electron density-matrix approach, we begin to describe this SQD system. Making good use of the standard formalism [51] and the phase-matching condition, i.e., $\mathbf{k}_p + \mathbf{k}_c = \mathbf{k}_d + \mathbf{k}_e$, the coupled Maxwell–Liouville equations for the electron density matrix operator $\hat{\rho}$ and two quantized electromagnetic fields $\hat{\Omega}_{p,e}$ can be easily obtained as follows

$$\begin{aligned}\frac{\partial\hat{\rho}_{10}}{\partial t} = {} & (-\gamma_1 - i\Delta_p)\hat{\rho}_{10} + i\hat{\Omega}_p\hat{\rho}_{00} + i\Omega_c^*\hat{\rho}_{30} \\ & - i\hat{\Omega}_e\hat{\rho}_{12} - i\hat{\Omega}_p\hat{\rho}_{11},\end{aligned} \tag{2a}$$

$$\begin{aligned}\frac{\partial\hat{\rho}_{20}}{\partial t} = {} & \left[-\gamma_2 - i(\Delta_p + \Delta_c - \Delta_d)\right]\hat{\rho}_{20} + i\hat{\Omega}_e\hat{\rho}_{00} \\ & + i\Omega_d^*\hat{\rho}_{30} - i\hat{\Omega}_e\hat{\rho}_{22} - i\hat{\Omega}_p\hat{\rho}_{21},\end{aligned} \tag{2b}$$

$$\begin{aligned}\frac{\partial\hat{\rho}_{30}}{\partial t} = {} & \left[-\gamma_3 - i(\Delta_p + \Delta_c)\right]\hat{\rho}_{30} + i\Omega_c\hat{\rho}_{10} \\ & + i\Omega_d\hat{\rho}_{20} - i\hat{\Omega}_e\hat{\rho}_{32} - i\hat{\Omega}_p\hat{\rho}_{31},\end{aligned} \tag{2c}$$

$$\frac{\partial\hat{\Omega}_p}{\partial z} + \frac{1}{c}\frac{\partial\hat{\Omega}_p}{\partial t} = i\kappa_{02}\hat{\rho}_{20}, \tag{2d}$$

$$\frac{\partial\hat{\Omega}_e}{\partial z} + \frac{1}{c}\frac{\partial\hat{\Omega}_e}{\partial t} = i\kappa_{01}\hat{\rho}_{10}, \tag{2e}$$

where γ_j denotes the total decay rate of subband $|j\rangle$ $(j = 1, 2, 3)$, which are added phenomenologically in the above density matrix equations (2a)–(2c). In SQDs, the overall decay rate is given by $\gamma_j = \gamma_{jl} + \gamma_{jd}$. The former γ_{jl} denotes the lifetime broadening linewidth, which is due primarily to longitudinal optical photon emission at low temperature. The latter γ_{jd} is the dephasing broadening linewidth, which may originate from electron–electron scattering, electron–phonon scattering, as well as inhomogeneous broadening due to scattering on interface roughness. Generally, γ_{jd} is the dominant mechanism in a semiconductor solid-state system in contrast to the atomic systems. And $\kappa_{01(02)} = 2N\omega_{p(e)}|\mu_{10(20)}|^2/(\hbar c)$ with N denoting the average density of electrons in the QD layer are the propagation constants of two quantized fields.

Under the condition of two small-signal quantized fields, we can adopt the nondepleted ground-state approximation $\hat{\rho}_{00} \simeq 1$ in solving the coupled Maxwell–Liouville equations (2a)–(2e) for the electrons and the quantized electromagnetic fields. Note that the last two terms in the right-hand side (RHS) of Eqs. (2a)–(2c) are higher-order terms of small quantities $\hat{\Omega}_p$ and $\hat{\Omega}_e$. For a small signal treatment, we can neglect these higher-order terms [17]. By taking the time Fourier transform of Eqs. (2a)–(2e), we can directly obtain

$$(\omega + \Delta_1 + i\gamma_1)\hat{\alpha}_{10} + \hat{\Lambda}_p + \Omega_c^*\hat{\alpha}_{30} = 0, \tag{3a}$$

$$(\omega + \Delta_2 + i\gamma_2)\hat{\alpha}_{20} + \hat{\Lambda}_e + \Omega_d^*\hat{\alpha}_{30} = 0, \tag{3b}$$

$$(\omega + \Delta_3 + i\gamma_3)\hat{\alpha}_{30} + \Omega_d\hat{\alpha}_{20} + \Omega_c\hat{\alpha}_{10} = 0, \tag{3c}$$

$$\frac{\partial\hat{\Lambda}_p}{\partial z} - i\frac{\omega}{c}\hat{\Lambda}_p = i\kappa_{01}\hat{\alpha}_{10}, \tag{3d}$$

$$\frac{\partial\hat{\Lambda}_e}{\partial z} - i\frac{\omega}{c}\hat{\Lambda}_e = i\kappa_{02}\hat{\alpha}_{20}, \tag{3e}$$