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Field effects on the vortex states in spin–orbit coupled Bose–Einstein condensates

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ABSTRACT

Multi-quantum vortices can be created in the ground state of rotating Bose–Einstein condensates with spin–orbit couplings. We investigate the effects of external fields, either a longitudinal field or a transverse field, on the vortex states. We reveal that both fields can effectively reduce the number of vortices. In the latter case we further find that the condensate density packets are pushed away in the horizontal direction and the vortices finally disappear to form a plane wave phase.

1. Introduction

Quantum vortices can be created in Bose–Einstein condensates (BECs) by stirring trapped condensates with lasers, both in one-component [1] and two-component systems [2,3]. If the rotating velocity is fast, a large amount of vortices is created to form a lattice: from a triangular lattice to a square lattice depending on the relative value of the intra-species (g) and inter-species (g_{12}) interaction strength [4]. It is theoretically shown that for immiscible condensates ($g_{12} > g$) it is possible to form vortex sheets [5]. Other ground states in the rotating BECs such as giant Skyrmion [6,7] and meron-pair [8] are explored.

In recent years, vortices in BECs created by the spin–orbit (SO) coupling are extensively investigated [9–16] since experimental realization of the SO coupling in neutral ^{87}Rb atoms [17]. Although the SO coupling in this experiment is an equal mixing of the Rashba and the Dresselhaus couplings, in [17] the authors proposed feasible schemes with tunable combinations of the Rashba and the Dresselhaus SO couplings. Unlike the SO coupling in the electron systems which origins from the relativity effect, the SO coupling in the neutral atoms is induced by coupling the pseudo-spin of two dressed states with the momentum through Raman lights. For the bosonic ^{87}Rb atom, the total spin separates into $F = 1$ and $F = 2$ by hyperfine coupling of the electron spin $S = \frac{1}{2}$ and the nuclear spin $I = \frac{3}{2}$. In the experiment designed by Ref. [17], the researchers generate the coupling between the two hyperfine states $|F = 1, m_F = -1\rangle$ and $|F = 1, m_F = 0\rangle$, with the hyperfine state $|F = 1, m_F = +1\rangle$ is neglected because of a large energy difference with the preceding two hyperfine states. The coupling induces the SO coupling of a pseudospin-$\frac{1}{2}$ system which is described by an effective Hamiltonian (1).

The ground state exhibits rich phase structures according to the relative value of g_{12} and g in the SO coupling BECs. Fundamentally, the spin-$\frac{1}{2}$ BEC is divided into a plane-wave phase ($g_{12} < g$) and a stripe phase ($g_{12} > g$) [18,19]. A hexagonally-symmetric lattice phase is suggested for a proper SO coupling strength [9]. A Skyrmion lattice phase is proposed at strong SO coupling [10, 20], in analogy to that in the rotating spinor condensate [21,22]. For rotating BECs with strong SO coupling, researchers found that the density profile exhibits a ring-shape domain structure which is insensitive to the relative value of g_{12} and g [11,14,23].

In this paper we investigate the effect of external fields on the vortex states of the two dimensional (2D) rotating SO coupled condensates. The condensates are trapped in an isotropic harmonic potential $V(\vec{r}) = m\omega^2 r^2/2$, $r^2 = x^2 + y^2$. Two types of external fields, the longitudinal field (the field perpendicular to the plane) and the transverse field (the in-plane field), are incorporated into the Hamiltonian as $H = H_0 + H_{\text{int}}$, where

$$H_0 = \int \Psi^\dagger[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r}) - \Omega L_z + \frac{\gamma}{m}(p_x\hat{\sigma}_y + p_y\hat{\sigma}_x) + \alpha\hat{\sigma}_z + \beta\hat{\sigma}_x]\Psi d\vec{r} \quad (1)$$

is the single particle Hamiltonian. $\Psi = (\psi_1, \psi_2)$ denotes the condensate wave function. Ω is the rotating frequency and $L_z = -i\hbar(xp_y - yp_x)$ is the z-component of the angular momentum operator. We adopt the SO coupling of the form $\gamma(p_x\hat{\sigma}_y + p_y\hat{\sigma}_x)$,

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where p_i is the momentum, $\hat{\sigma}_i$ is the Pauli matrix and γ is the SO coupling strength. $\alpha\hat{\sigma}_z$ is the longitudinal field term describing the Zeeman field along z axis and $\beta\hat{\sigma}_x$ is the transverse field depicting the Zeeman field along x-axis. The interaction part of the Hamiltonian is

$$H_{\text{int}} = \int (U_1|\psi_1|^4 + U_2|\psi_2|^4 + 2U_{12}|\psi_1|^2|\psi_2|^2)d\vec{r}, \tag{2}$$

where U_1, U_2 are the intra-species interactions and U_{12} the inter-species interaction, respectively. $U_i = 4\pi\hbar^2 a_i/m$ with a_i denoting the s-wave scattering length.

For simplicity, we consider that $a_1 = a_2$ in the present work. By introducing the harmonic oscillator scale length $l = \sqrt{\hbar/m\omega}$, the Hamiltonian takes the dimensionless form,

$$\begin{aligned} H_0 = \int \Psi^\dagger[-\frac{1}{2}\nabla^2 + \frac{1}{2}(x^2+y^2) + ib(x\partial_y - y\partial_x) \\ - i\kappa(\partial_x\hat{\sigma}_y + \partial_y\hat{\sigma}_x) + \mu\hat{\sigma}_z + \nu\hat{\sigma}_x]\Psi d\vec{r}, \end{aligned} \tag{3}$$

$$H_{\text{int}} = \int (g|\psi_1|^4 + g|\psi_2|^4 + 2g_{12}|\psi_1|^2|\psi_2|^2)d\vec{r}, \tag{4}$$

where $g = NU_1$ and $g_{12} = NU_{12}$ with $N = \int |\Psi(\vec{r})|^2 d\vec{r}$ being the number of total particles. Here we introduce four dimensionless parameters $b = \Omega/\omega$, $\kappa = \gamma/\sqrt{\hbar m\omega}$, $\mu = \alpha/\hbar\omega$ and $\nu = \beta/\hbar\omega$.

We first reveal some new features in the weak SO coupling regime in comparison to the strong SO coupling regime presented in Refs. [11,14] in the absence of external fields ($\mu, \nu = 0$). In the presence of the longitudinal field ($\mu \neq 0, \nu = 0$), we find that the number of vortices reduces as the field strength increases until there is only one vortex in one component and none in the other. In the presence of the transverse field ($\mu = 0, \nu \neq 0$), we find that, in addition to the number of vortices reducing to zero, the density packet moves to one direction and the ground state finally recovers the plane wave phase.

The paper is organized as follows. In Sec. 2 we present some new features in the ground state of weak SO coupled BECs without the external fields. In Sec. 3 we study the effects of the longitudinal field on the vortex states. In Sec. 4 the effects of the transverse field are studied. A brief summary is included in Sec. 5.

2. Vortex state without external fields

We discuss the ground states of the rotating BECs for the weak SO coupling in the absence of external fields ($\mu, \nu = 0$) and compare our results with existing works for strong SO coupling [11,14,23]. We perform the numerical simulations in a spatial grid of 401^2 by employing the Fourier domain method developed by Bao et al. [24]. Fig. 1 shows the vortex states for various rotation velocity $b = 0, 0.2, 0.4, 0.6$ at the SO coupling strength $\kappa = 3$. The intra-species and inter-species interactions are set as $g = 4$ and $g_{12} = 5$. In the absence of rotation (Fig. 1(a)), the ground state is stripe wave (SW) state ($g < g_{12}$) which is addressed in Ref. [18]. As the rotation velocity increases, a giant vortex forms at the center and the density profiles of both components display a ring structure. As shown in Fig. 1(b), each component piles into a button-shape density profile for small b. The vortex cores in the middle of the button merge together as b increases until they form a giant vortex (Fig. 1(c, d)). To reveal the underlying physics, we examine the energy spectrum of the free particles with the SO coupling,

$$H_{\text{fr}} = \frac{1}{2}(k_x + \kappa\hat{\sigma}_y)^2 + \frac{1}{2}(k_y + \kappa\hat{\sigma}_x). \tag{5}$$

The eigenvalue of this Hamiltonian is

$$E_\pm(k) = \frac{k^2}{2} + \kappa^2 \pm \kappa|k|, \tag{6}$$

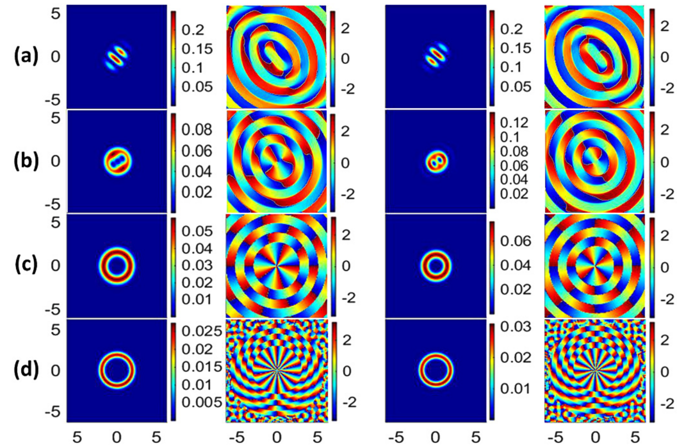

Fig. 1. The ground states for the weak SO coupling $\kappa = 3$ in the absence of external fields ($\mu, \nu = 0$). The interaction parameters are $g = 4$ and $g_{12} = 5$. From left to right: the density and phase profiles of spin-up and spin-down components, respectively. The rotation velocity is (a) $b = 0$, (b) $b = 0.2$, (c) $b = 0.4$, (d) $b = 0.6$. The colorbar on the right of each subplot shows the magnitude of corresponding density or phase. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where $|k| = \sqrt{k_x^2 + k_y^2}$. The energy minimum of the lower branch ($E_-(k)$) occurs on a ring of radius $|k| = \kappa$. It implies that a ring-shape density wall will be formed under rotation, as shown in Fig. 1(c, d). This density wall reversely forms an effective potential which is steeper than the harmonic external trap. The condensate in this effective potential tends to form a giant vortex than a lattice of vortices. When the angular momentum is large enough, more vortices trapped by the extremely steep ring potential enter the condensate and triangular lattices begin to form in the annular region, which is presented in Ref. [11].

The phase distributions have the windmill-like profiles which are surrounded by a series of concentric rings as b increases. There is a phase jump of $\Delta\phi = \pi$ between the adjacent concentric rings along the radial direction. In the angular direction the phase varies continuously. In contrast to the strong SO coupling regime, the condensates in the weak SO coupling regime by no means form domains but are replaced by a series of continuous phase rings. We also computed the strong SO coupling case for $\kappa = 10$ and indeed obtained the domain structures as in Ref. [11]. These results are also presented for $g > g_{12}$. In the latter case, where the two components are miscible, the ground state is the plane wave state at $b = 0$. As b increases, the condensates have the similar features as in the case of $g < g_{12}$. The difference from the sign of $g - g_{12}$ is also smeared out as the SO coupling is strong [11], which implies that the combination of rotation and SO coupling reduces the influence of the relative value of g and g_{12}.

The amount of phase windings differs exact one between the two components, independent of the rotation velocity. It roots in the SO coupling of the single particle problem. It is elucidated by solving the lowest Landau level (LLL) state of the non-interacting Hamiltonian H_0 [11,14,25]. $\hat{H}_0$ is represented in the second quantized representation as

$$\begin{aligned} \hat{H}_0 = (1+b)\hat{a}_+^\dagger\hat{a}_+ + (1-b)\hat{a}_-^\dagger\hat{a}_- \\ + \kappa\begin{pmatrix} 0 & \hat{a}_+^\dagger - \hat{a}_- \\ \hat{a}_-^\dagger - \hat{a}_+ & 0 \end{pmatrix}, \end{aligned} \tag{7}$$

where operators $\hat{a}_\pm = (\hat{a}_x \mp i\hat{a}_y)/\sqrt{2}$, $\hat{a}_\pm^\dagger = (\hat{a}_x^\dagger \pm i\hat{a}_y^\dagger)/\sqrt{2}$ with the annihilation and creation operators $\hat{a}_x = (x+\partial_x)/\sqrt{2}$, $\hat{a}_x^\dagger = (x-\partial_x)/\sqrt{2}$, $\hat{a}_y = (y+\partial_y)/\sqrt{2}$ and $\hat{a}_y^\dagger = (y-\partial_y)/\sqrt{2}$. Eq. (7) is solved by diagonalization which yields the wave function of the