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Tunneling time and stochastic-mechanical trajectories for the double-barrier potential

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ABSTRACT

Quantum motion of particles tunneling a double barrier potential is considered by using stochastic mechanics. Stochastic-mechanical trajectories give us information about complex motion of tunneling particles that is not obtained within the framework of ordinary quantum mechanics. Using such information, we calculate the tunneling times within each of the barriers which depend on the distance between them. It is found that the stochastic-mechanical tunneling time shows better asymptotic behavior than the quantum-mechanical dwell time and presence time.

1. Introduction

In the standard framework of quantum mechanics physical quantities are expressed by the self-adjoint operators called observables. There is an exception, however. *Time* is not an observable [1,2]. Indeed, as proved by Pauli, there is no self-adjoint time operator conjugate to a Hamiltonian bounded from below [3,4]. This means that expectation values related to the time variable cannot be calculated within the standard framework of quantum mechanics. As a result, there are some ambiguities in the calculation of tunneling times [2,5–8]. Although several definitions such as the phase time [9], the dwell time [10,11], the local Larmor time [12–14,11] and the complex time [15] have been proposed, tunneling times have remained to be controversial until now.

Recently, the use of stochastic mechanics [16] has been proposed for the calculations of the tunneling time [17,18], arrival time [19], and presence time [18,19]. These works have shown that stochastic mechanics may provide a unique numerical method of analyzing the time development of wave packets in spite of the fact that Nelson himself repudiated his original theory of stochastic mechanics [20]. Indeed, in stochastic mechanics the quantum motion of a particle is described as a set of sample paths or "stochastic-mechanical trajectories" [21]. Therefore, it becomes possible to define tunneling time in the similar way of classical mechanics [4]. This advantage of stochastic mechanics may give us information about tunneling time, which is not obtainable within ordinary quantum mechanics.

In this Letter we study numerically the resonance tunneling phenomena under a symmetric double barrier potential and compare the tunneling times between two barriers using stochastic mechanics as well as quantum mechanics. In the following sections, we show that the tunneling times for two barriers are different despite the symmetry of the double barrier potential. The ratio of these tunneling times changes as a function of the distance, height and width of two barriers. It is also shown that, in contrast to tunneling times based on the definition of the dwell time and presence time in quantum mechanics, the numerical result based on stochastic mechanics has a correct asymptotic behavior.

2. Theory

2.1. Model

We consider the simple symmetric double rectangular potential,

$$V = V_0\big(\mathrm{Rect}[-l/2-d,-l/2;x]+\mathrm{Rect}[l/2,l/2+d;x]\big), \tag{1}$$

where

$$\mathrm{Rect}[x_1,x_2;x] = \begin{cases} 0 & (x < x_1), \\ 1 & (x_1 \leqslant x \leqslant x_2), \\ 0 & (x_2 < x), \end{cases} \tag{2}$$

d is the width of a barrier, and l the distance between barriers. In this case the wave function for the stationary states becomes

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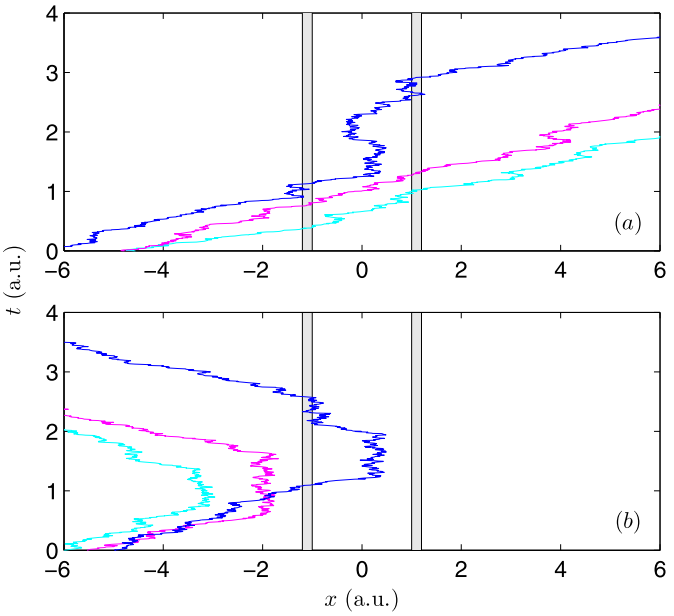

Fig. 1. (Color online.) Examples of (a) tunneled trajectories and (b) reflected trajectories calculated by stochastic mechanics ($V_0 = 1.5E_0$). The grey-color areas represent the region of the double-barrier potential.

$$\varphi_k(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & (x < -l/2 - d), \\ Ce^{\kappa x} + De^{-\kappa x} & (-l/2 - d \leqslant x < -l/2), \\ Fe^{ikx} + Ge^{-ikx} & (-l/2 \leqslant x < l/2), \\ He^{\kappa x} + Ie^{-\kappa x} & (l/2 \leqslant x < l/2 + d), \\ Ke^{ikx} & (l/2 + d \leqslant x) \end{cases} \tag{3}$$

where the amplitudes $A, B, \ldots, K$ are determined by the boundary conditions as usual, k is the wave number of the incoming plane wave, $\kappa = \sqrt{2m(V_0 - E)}/\hbar$, and $E = \hbar^2 k^2/(2m)$.

For the calculation of tunneling times, we use the Gaussian-like wave packet $\psi(x,t)$ given by [17]

$$\psi(x,t) = \int_{-\infty}^{\infty} Ne^{-(k-k_0)^2/(2\sigma^2)} e^{-ikx_0} \varphi_k(x) e^{-iEt/\hbar}\, dk, \tag{4}$$

where N is the normalization factor, k_0 the mean value of the wave number for the superposition, σ^{-1} the width of the initial wave packet, and x_0 the central position of the wave packet at $t = 0$.

In stochastic mechanics [16], the forward time evolution of a stochastic-mechanical trajectory $x(t)$ is given by Ito's stochastic differential equation,

$$dx(t) = b\big(x(t), t\big)\, dt + dw(t), \tag{5}$$

where $b(x,t)$ is the drift velocity,

$$b(x,t) = \frac{\hbar}{m}(\mathrm{Re} + \mathrm{Im})\frac{\partial}{\partial x} \ln \psi(x,t), \tag{6}$$

and $dw(t)$ is the Wiener process satisfying

$$\big\langle dw(t) \big\rangle = 0, \qquad \big\langle dw(t)^2 \big\rangle = \frac{\hbar}{m}\, dt, \tag{7}$$

where $\langle \cdots \rangle$ represents the statistical average over all possible trajectories.

In Fig. 1 we show examples of the stochastic-mechanical trajectories calculated by using Eqs. (1)–(7) with the Monte Carlo method. We note that stochastic mechanics makes it possible to separate clearly the transmitted trajectories (Fig. 1(a)) from the reflected trajectories (Fig. 1(b)). For numerical calculations one must use a finite time Δt instead of infinitesimal time dt. We determine Δt so that the stochastic-mechanical calculations reproduce quantum-mechanical probability density $|\psi(x,t)|^2$. Examples of such agreements are shown in Fig. 2.

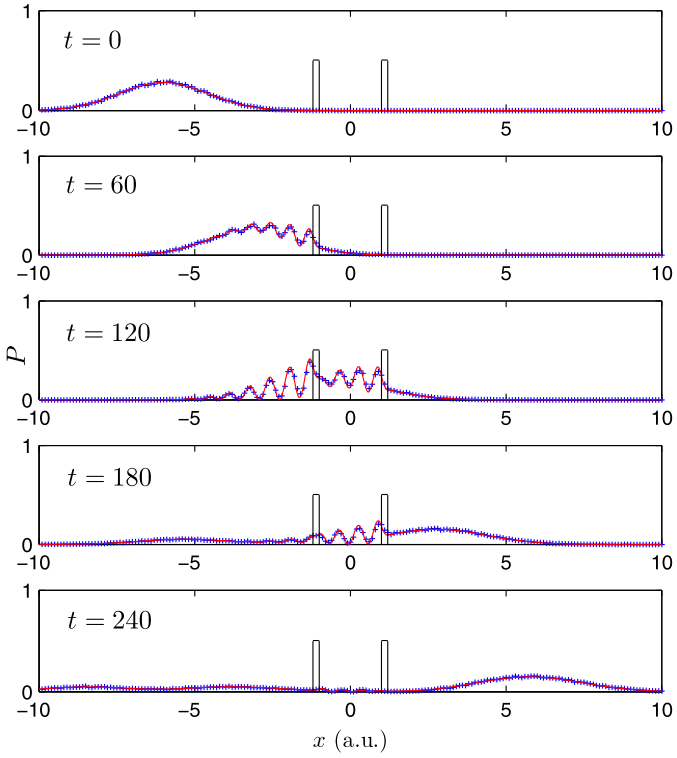

Fig. 2. (Color online.) The motion of a wave packet tunneling a double-barrier potential. The red solid lines represent $|\psi(x,t)|^2$. The blue crosses represent distributions derived from stochastic-mechanical trajectories.

2.2. Tunneling times

Following the definition of classical time spent in a certain region [4,18], the tunneling time $\tau(i)$ for the ith stochastic-mechanical trajectory $x_i(t)$ $(i = 1, 2, \ldots, n)$ passing through a barrier region $x_1 \leqslant x \leqslant x_2$ is defined by

$$\tau(i) = \int_0^{t_f} \mathrm{Rect}\big[x_1, x_2; x_i(t)\big]\, dt. \tag{8}$$

We introduce the stochastic-mechanical tunneling time τ^s [18] as

$$\tau^s = \sum_{i=1}^{n} \frac{\tau(i)}{n}. \tag{9}$$

It is noted that τ^s has been calculated by choosing only the tunneled trajectories passing through the barrier at $t \to \infty$, as in Fig. 1(a): the trajectories passing through barriers once but finally go back to the initial region have not been included.

For comparison, we have calculated four types of dwell times [5,10,11,22] and the presence time [4] within the framework of quantum mechanics. It is worthwhile to mention that the phase time is related to the dwell time for stationary wave functions [23–25] and hence we will not consider further the phase time in this work. For a stationary wave function $\varphi(x)$ in the region $x_1 \leqslant x \leqslant x_2$ the dwell time is given by [10,11]

$$\tau^{d0} = \frac{1}{j} \int_{x_1}^{x_2} \big|\varphi(x)\big|^2\, dx, \tag{10}$$