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Fold points and singularity induced bifurcation in inviscid transonic flow

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ABSTRACT

Transonic inviscid flow equation of elliptic–hyperbolic type when written in terms of the velocity components and similarity variable results in a second order nonlinear ODE having several features typical of differential–algebraic equations rather than ODEs. These features include the fold singularities (e.g. folded nodes and saddles, forward and backward impasse points), singularity induced bifurcation behavior and singularity crossing phenomenon. We investigate the above properties and conclude that the quasilinear DAEs of transonic flow have interesting properties that do not occur in other known quasilinear DAEs, for example, in MHD. Several numerical examples are included.

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1. Introduction

It is well-known that traveling-wave solutions of systems of partial differential equations in two variables may result in differential–algebraic equations (DAEs) of quasilinear or semiexplicit forms [1–4]. Such DAEs often include trajectories crossing through *sonic* curves. The crossing phenomenon is possible thanks to the singularity induced bifurcation (SIB) [5]. Other points on the *sonic* curve are the *impasse* points where trajectories either terminate at or originate from. The SIB case comprises trajectories moving between the *supersonic* and *subsonic* sheets. Such transonic connections are possible because of the existence of special *single* or *double* SIB points (also known as *folded points*) allowing for trajectories of various degrees of smoothness to cross the *sonic* curve. Typical cases involve *folded saddle* and *folded node* SIB points [6–16].

This Letter falls in the area of qualitative analysis and modeling of DAEs. We analyze different type of solutions of certain problems in aerodynamics, namely the similarity solution of the small disturbance inviscid transonic flow equation. We look at the resulting equations from the DAE perspective. We shall show the existence of SIB and impasse points in such transonic flow DAEs and explain some known facts from the transonic flow theory using the language of DAEs.

The Letter is organized as follows. In Section 2 we present the concepts of singular algebraic and geometric points. The former behave typically as impasse points, while the later allow for trajectories to cross the singularity (sonic points). The singularity induced bifurcation (SIB) phenomenon is also described. In Section 3 we discuss a nonlinear DAE model of transonic flow and its similarity solution. Saddle and node fold points of the transonic flow DAEs are analyzed in Section 4. Conclusions are drawn in Section 5. Appendix A presents a few facts about matrix pencils and parameter-dependent polynomials used in the Letter.

2. Algebraic and geometric singular points of DAEs and the SIB phenomenon

Consider the DAEs in quasilinear form

$$A(u)u' = b(u) \tag{1}$$

where A and b represent sufficiently smooth matrix- and vector-valued functions, $u = u(\xi) \in R^n$ and the prime $'$ stands for $d/d\xi$. For transonic DAEs considered in this Letter we have $-\infty < \xi < +\infty$.

2.1. Singularities of DAEs

The singularities of quasilinear DAEs (1) occur for a noninvertible matrix $A(u^*)$ with u^* being the closure of the set of points for which $A(u)$ is invertible. We also assume that u^* is a *non-critical singular point*, which means that the condition $(\det A)'(u^*) \neq 0$ is satisfied. The $\det A$ denotes the determinant of $A(u)$. This implies that $\dim\{\ker A(u^*)\} = 1$ [15], where $\ker A(u^*)$ stands for the kernel of $A(u^*)$.

Geometric singularities are defined by the condition $b(u^*) \in Im\, A(u^*)$, while for algebraic singularities we have $b(u^*) \notin Im\, A(u^*)$.

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The $Im\,A(u^*)$ stands for the image of $A(u^*)$. It may be shown that for $\dim\{\ker A(u^*)\} = 1$ (*non-critical property*) we have the equivalent conditions: $\{adj\,A(u^*)\}b(u^*) = 0$ for geometric singularities and $\{adj\,A(u^*)\}b(u^*) \neq 0$ for algebraic singularities. The $adj\,A(u^*)$ is the adjoint matrix of $A(u^*)$. Under generic transversality assumption, it is shown in [15] that algebraic singularities behave as *impasse* points, where trajectories collapse (or originate from) in finite time with infinite speed. Thus, no crossing of the singularity is possible at an algebraic non-critical singularity.

The singularity crossing phenomena can occur only through geometric singularities. Generically, such points define an $(n-2)$-dimensional manifolds, that is, a codimension one submanifold of the singular set. In planar problems (considered in this Letter) the geometric singularities are isolated points. The detailed analysis of the singularity crossing phenomena through such points is considered in [17] where Theorem 1 gives sufficient conditions for a well-defined C^1 solution through a geometric singular point u^*. An illustrative example is provided in Section 4 in this Letter. Also, a comprehensive discussion on impasse points in DAEs with tunnel diode circuits is given in [18].

2.2. The SIB phenomenon

The SIB phenomenon, originally presented in [5] and later improved in [10,11,14], is based on the fact that under certain sufficient conditions, a system of parameter-dependent linearized DAEs has eigenvalues diverging through infinity when a parameter, say λ, increases or decreases through a certain value, λ_0. Moreover, for $\lambda = \lambda_0$ an equilibrium point u^* of (1) is placed at the singularity causing the equilibrium and singularity to annihilate each other and allowing for a smooth C^1 trajectory to cross the singularity manifold. The linear DAE system is stable for $\lambda < \lambda_0$ and unstable for $\lambda > \lambda_0$ (or vice versa). The index (see Appendix A.1) of the linearized DAEs increases at a SIB point and a special factorization of the characteristic polynomial changes as described in Appendix A.2. This phenomenon has also been observed in the traveling-wave solutions in MHD [1–4] and power systems [5,8]. Fig. 2(a), (b) of Section 4 shows a typical movement of an equilibrium and its eigenvalues during the SIB phenomenon. An expanded discussion on SIB in inviscid transonic flow is also presented in Section 4.

3. DAE model of transonic flow

One of the most important equations in transonic inviscid aerodynamics is the following equation [19]

$$\left(F_0' - \lambda^2\xi^2\right)F_0'' - \lambda(5-5\lambda)\xi F_0' + (3-3\lambda)(3\lambda-2)F_0 = 0 \tag{2}$$

where $F_0(\xi)$ comes from the expansion of the velocity potential $\Phi(x,y)$ for $-\infty < \xi = x/y^\lambda < +\infty$, $\lambda > 0$ is a parameter and the $'$ denotes $d/d\xi$. The case $\lambda < 1$ for transonic far fields is considered in [20], while the case $\lambda > 1$ for inviscid transonic flow is analyzed in [21]. Axially symmetric transonic flow is described by (2) with $5-5\lambda$ and $3-3\lambda$ replaced by $4-5\lambda$ and $2-3\lambda$, respectively [20].

If we define a new variable χ through $d\chi = d\xi/(\lambda\xi)$, then we can write (2) as [21]

$$\begin{aligned} sgn(\xi)(f-1)\frac{df}{d\chi} &= 2f - 3g - 2\lambda f^2 + 3\lambda g \\ sgn(\xi)(f-1)\frac{dg}{d\chi} &= -2f^2 + 2\lambda f^2 + 3g - 3\lambda fg \end{aligned} \tag{3}$$

where $f(\xi) = F_0(\xi)/(\lambda^2\xi^2)$, $g(\xi) = G(\xi)/(\lambda^3\xi^3)$, $G(\xi) = (3\lambda - 2)F_0(\xi) - \lambda\xi F_0'(\xi)$ and $sgn(\xi)$ is the sign of variable ξ. The $F_0(\xi)$ defines asymptotic expansion of the potential function which takes the form

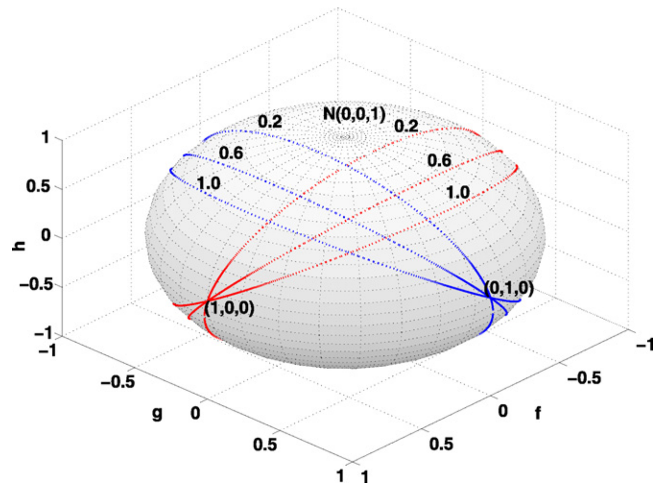

Fig. 1. Projection (5): vertical lines $f = const$ (blue) and horizontal lines $g = const$ (red) for $const = 0.2$, 0.6, and 1.0. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

$$\Phi(x,y) = x + \frac{1}{\gamma+1}y^{3\lambda-2}F_0(\xi) + \cdots \tag{4}$$

for $y \to 0$ and γ is the ratio of specific heats. The $sgn(\xi)$ causes the two systems in (3) (one for $\xi < 0$ and the other for $\xi > 0$) to differ only by the direction of their integral curves. The line $f = 1$ is a singularity for both systems. The half plane with $f < 0$ corresponds to subsonic velocities, while the half plane with $f > 0$ is for supersonic ones. If $\lambda \neq 1$, then no matter what the sign of ξ is, there exist three points in the (f,g) plane where both right-hand sides of (3) are zero, as follows: $EQ_1(0,0)$, $FP(1,2/3)$ and $EQ_2(1/\lambda^2, -2/(3\lambda^3))$. For any λ, EQ_1 is an unstable (for $\xi < 0$) and stable (for $\xi > 0$) node. If $\lambda > 1$ then EQ_2 is a saddle. For $\lambda < 1$, EQ_2 is an unstable node for $\xi < 0$ and a stable node for $\xi > 0$. If $\lambda > 1$, then FP is a folded node, while for $\lambda < 1$ it is a folded saddle. Instead of using two 2D phase planes to analyze (3) we chose to project the two variables (f,g) onto a sphere (see Fig. 1). We chose the following transformation (one of several possible ones)

$$\begin{pmatrix} f \\ g \\ h \end{pmatrix} \leftarrow \pm\frac{1}{\sqrt{f^2+g^2+1}}\begin{pmatrix} f \\ g \\ 1 \end{pmatrix} \tag{5}$$

where the *old* variables f and g on the right-hand side are those in (3), while the *new* (projected) variables f, g, plus a new variable $-1 \leqslant h = 1/\sqrt{f^2+g^2+1} \leqslant 1$ are on the left-hand side of (3). The reason for choosing such a transformation is as follows: a 3D geometry with the additional dimension added (vertical variable h in Fig. 1) allows us to represent points at infinity without the need to use the ∞ symbol. Thus, the two variables $(f(\xi), g(\xi))$ with infinite interval $-\infty < \xi < +\infty$ are mapped onto a 3D sphere (with radius 1) whose equator corresponds to the *old* variables (f,g) at $\xi = 0$. The northern hemisphere represents (3) for $sgn(\xi) < 0$, while the southern one is for $sgn(\xi) > 0$.

The above 3D projective hemisphere model (5) has the following basic properties (see Fig. 1):

(1) Straight lines from 2D space with *old* variables f and g map to great circles in projective space with the *new* variables f, g, plus variable h.
(2) Horizontal lines meet at $(\pm1, 0, 0)$ which are the points at infinity ($g \to \pm\infty$ in (3)).
(3) Vertical lines meet at $(0, \pm1, 0)$ which are the points at infinity ($f \to \pm\infty$ in (3)).
(4) Any point from 2D space in the *old* variables f and g with at least one variable approaching infinity is projected onto at

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